THE VISCOUS-CAPILLARY PARADOX IN 2-PHASE FLOW IN POROUS MEDIA

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ABSTRACT

In the context of multi-phase flow in porous media, one of the questions is if the flow is dominated by viscous or by capillary forces. It appears as a paradox that the structure of Darcy's law is that of a viscous law but that mobilization of oil trapped in pores of water-wet rock is controlled by capillary numbers that are typically of the order of 10^{-6} - 10^{-4} indicating a capillary dominated behaviour.

The confusion originates from two the structure of Darcy's law and from the definition of the capillary number. Darcy's law and it's 2-phase extension have a structure of a viscous flow law. Capillary effects come into play via the capillary pressure function, which can often be ignored in reservoir engineering simulations. However, capillary pressure effects and viscous coupling between the two phases are lumped into the relative permeabilities. Darcy's 2-phase extension appears to be a purely viscous law, but this is incorrect.

The capillary number relates viscous forces to capillary forces. On a macroscopic level, it is unclear what the magnitude and direction of these forces are. On a microscopic level, however, the capillary number can be clearly defined. The capillary number for residual, disconnected phases is of the order of 10^{-6}, but for continuous phases, it is shown that capillary forces and viscous forces are of the same order of magnitude. The paradox seizes to exist: in equilibrium situations, for continuous phases, viscous forces are in balance with capillary forces and the capillary number equals unity, while during flooding, viscous forces exceed capillary forces. In Darcy's law, this is all lumped into the relative permeability and into the capillary pressure.

INTRODUCTION

One of the questions that emerges very often in the context of multi-phase flow in porous media is whether the flow is controlled by viscous or by capillary forces. An apparent paradox is that the structure of Darcy's law is that of a viscous law but typical capillary numbers of 10^{-6} - 10^{-3} indicate a capillary dominated behaviour. This is an apparent contradiction, which is sometimes named the ***viscous-capillary paradox***. In this paper, we approach this apparent contradiction from two sides: from a macroscopic picture and from a microscopic picture. When comparing both pictures, inconsistencies and incomplete descriptions are found which are in the end responsible for the paradox. A key figure in our investigation is the force balance between viscous and capillary forces in multiphase flow.

THE MACROSCOPIC PICTURE

The flow of a single phase in a porous medium is described via Darcy's law:

$$v = \frac{Q}{A} = -\frac{K}{\mu}\frac{\partial p}{\partial x}, \tag{1}$$

where v is the Darcy velocity, which is the flux Q normalized by the cross sectional area A. K is the permeability of the porous medium, μ is the viscosity of the fluid, and $\partial p / \partial x$ is the pressure gradient. In a strict sense, Darcy's law is valid for slow, steady, one-dimensional single-phase flow through a homogeneous and isotropic sand [1].

Darcy's law from Eq. (1) is a macroscopic model valid on the scale that is large compared with the individual pore scale [2] but smaller than the gradients of the averaged velocity or pressure [3]. The rock appears as a continuous medium [4] with the properties porosity, φ, and permeability, K. Darcy's law employs continuum mechanics concepts and the parameters like porosity and permeability but also the velocity are the result of a volume average [2] over the representative elementary volume (REV) [4].

Darcy's law has the ***structure of a viscous law*** very similar to that of the Poiseuille flow through a pipe with circular cross section and radius R [5] for which

$$\frac{Q}{A} = -\frac{R^2}{8\mu}\frac{\partial p}{\partial x}. \tag{2}$$

In pore network modeling [20], the flow on the pore scale is abstracted to flow in channels with geometric cross sections, which is essentially Poiseuille type of flow [5]. Even though the flux through pore channels can be described by an expression like in Eq. (1), the term K would then be a local hydraulic conductivity like $R^2/8$ like in Eq. (2) for a cylindrical channel and only become a rock permeability when averaged over a volume that is large compared to the pore scale [2].

For 2-phase flow, the description of Darcy's law is extended to a set of 2 equations for water (w) and oil (o)

$$v_w = -\frac{k_{r,w} K}{\mu_w} \frac{\partial p_w}{\partial x}, \tag{3}$$

$$v_o = -\frac{k_{r,o} K}{\mu_o} \frac{\partial p_o}{\partial x}, \tag{4}$$

where $k_{r,w}$ and $k_{r,o}$ are the (phenomenological) relative permeabilities for water and oil, respectively. These functions are dependent on the saturation of their phase. Since the oil saturation $S_o = 1 - S_w$, the saturation dependence can be expressed via the water saturation S_w only, i.e. $k_{r,w} = k_{r,w}(S_w)$ and $k_{r,o} = k_{r,o}(S_w)$. Even though the relative permeability parameters are phenomenological, their dependencies on rock and fluid parameters like wettability can be understood as a volume average over a complicated microscopic behaviour that depends on the spatial distribution of the two phases inside the pore structure.

In the case of 2-phase flow, the viscous structure of the flow equations has not changed compared to single phase flow. When considering the example of a steady state experiment with a constant saturation of oil and water and continuous fluid phases for oil and water, there is a constant pressure drop over the oil and the water phase, and fluid flow in oil and water is still subject to a viscous flow equation. For each phase, the viscous forces are balanced by the externally applied pressure drops. In the Darcy formalism there is no viscous coupling between the two fluid phases. Indirectly, the impact of viscous coupling of the two fluid phases are contained in the relative permeability functions [1].

Compared to single phase flow, two-phase flow is subject to viscous and capillary forces. The capillary forces originate from the microscopic curvature between the oil and water phase which will be discussed in the microscopic picture in the next section in more detail. On the macroscopic scale, this curvature does not exist as individual pores are not visible (continuum medium approach). To stay consistent in the macroscopic picture, in Eqns. (3) and (4), the relation of the phase pressures of oil, p_o, and water, p_w, is defined as the capillary pressure saturation function

$$p_c = p_o - p_w = p_c(S_w), \tag{5}$$

which is dependent on the water saturation S_w. Phenomenological macroscopic capillary pressure functions like that by Brooks and Corey [6] do not contain microscopic quantities. While for specific cases the capillary pressure function is important[1], for many reservoir engineering problems the macroscopic capillary pressure function can be neglected and the flow through the reservoir can be described by Darcy’s law in combination with conservation of mass.

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[1] In specific situations the capillary pressure cannot be neglected, such as for counter current imbibition [7], for tight gas (near-wellbore multiphase flow damage [18]), for oil wet reservoirs (water hold up by oil wet capillary threshold pressure), for gas reservoirs which produce water (mobile transition zone during production) and for gas condensate fields (banking [19]).

If we consider an oil-filled reservoir at connate water saturation, gravity (buoyancy) forces are balanced by capillary forces. The hydrostatic pressure balances the capillary pressure, which is linked to saturations via the capillary pressure curve. In the horizontal direction, there is no flow and thus there are no net horizontal forces.

If the reservoir is produced by water injection, oil will start flowing due to the external pressure drop. At the water-oil displacement front, saturations are rapidly changing. The saturation at the displacement front can be obtained from the fractional flow equation, and is in the end a function of the relative permeabilities of oil and water. The transient flow can be described using Eq. (3) and (4).

When the reservoir has been flooded and saturations have become constant, the fluids are flowing at their end-point relative permeabilities and we have reached an equilibrium situation. By increasing the pressure drop, no additional oil will be produced once true residual oil saturation has been reached.

The capillary pressure and relative permeability saturation functions limit the accessible saturation range of this formalism to $S_{w,c} \le S_w \le 1 - S_{o,r}$ where $S_{w,c}$ is the connate water saturation and $S_{o,r}$ is the residual oil saturation. In this phenomenological description, the saturation at which phases get disconnected is not defined. While in water wet sandstones, the water phase is believed to be always connected, oil gets disconnected at some point. Saturation ranges beyond $S_{w,c}$ and $S_{o,r}$ cannot be accessed via this formalism in a direct way. But it is well known that an increase in flow rate, a more viscous displacing fluid (e.g. polymer water-flooding) and reduction in interfacial tension by surfactants can produce additional oil beyond the initial $S_{o,r}$ (capillary de-saturation [8]). In the conventional formalism, this phenomenon can only be parameterized by defining new capillary pressure and relative permeability functions and adjusting $S_{o,r}$ to the new conditions.

The capillary de-saturation curve [8,9,10] describes the relationship between residual saturation of the wetting and non-wetting phase and the capillary number [8] as displayed in Figure 1.

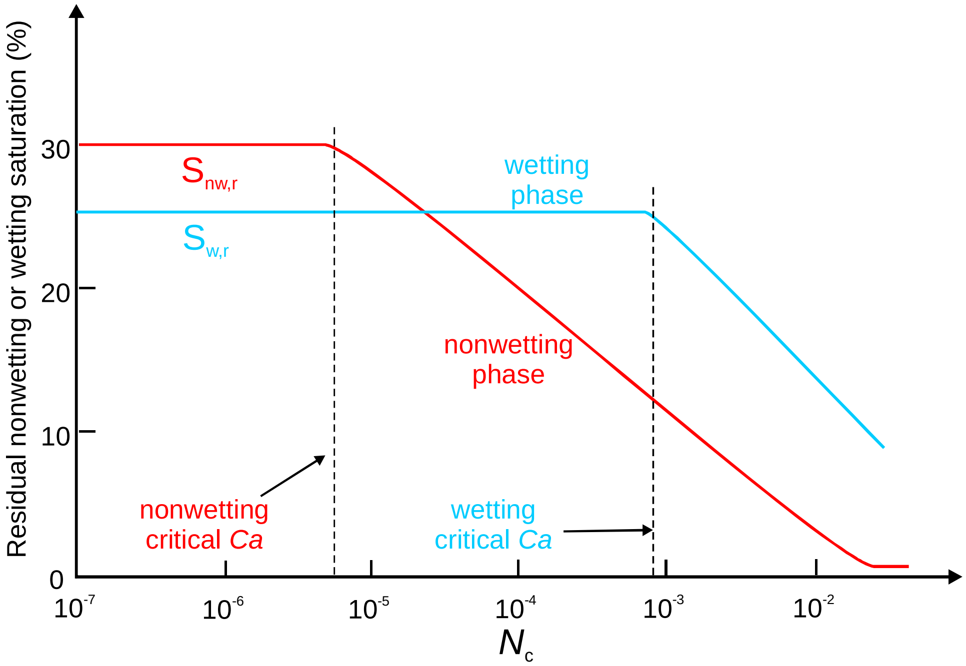

Figure 1: Capillary de-saturation curve: Residual saturation of the wetting and non-wetting phase as function of the capillary number N_c (sketch after [7]).

There is no consensus on how to define the capillary number. A contributing factor to this is that the physical argumentation for defining the capillary number is often absent. A very frequently used definition of the capillary number depends on the viscosity μ, the Darcy velocity v and the interfacial tension σ [3,11] [2]:

$$N_c = \frac{\mu v}{\sigma}. \tag{6}$$

Sometimes the porosity is included in the capillary number $N_c = \mu v/(\varphi\sigma)$ [5,12], and sometimes the contact angle $N_c = \mu v/(\sigma\cos\theta)$ [13]. From a macroscopic point of view, including a contact angle is somewhat questionable, since contact angles are not defined. By substitution of the Darcy velocity using Darcy's law one obtains

$$N_c = \frac{\mu v}{\sigma} = \frac{k_{r,w} K \Delta p / l}{\sigma}. \tag{7}$$

Sometimes, the relative permeability term is excluded in Eq. (7) [12]. Other definitions for the capillary numbers found in literature are

$$N_c = \sqrt{\frac{k}{\varphi}}\frac{|\nabla p|}{p_c}, \tag{8}$$

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[2] There are various notations for the capillary number. The most popular ones are *Ca* and N_c.

$$N_c = \frac{k|\nabla p + \rho g|}{\sigma \cos\theta}, \tag{9}$$

$$N_c = \frac{k|\nabla p|}{\sigma}, \tag{10}$$

from [14] [15] and [16] respectively. The capillary numbers only compare pressures and a conversion to forces acting in the reservoir is not straightforward, if possible at all. It is impossible to determine the magnitude and direction of these forces on fluid-fluid interphases on the basis of 'upscaled' quantities such as the Darcy velocity and capillary pressure.

Capillary numbers range from 10^{-5} to 10^{-7} for typical pressures, permeabilities and interfacial tensions, depending on the definition that is being used.
But this is in contradiction with the previous statement that capillary forces can often be neglected in reservoir engineering problems. Based on a capillary numbers of 10^{-5}, the capillary forces are much greater than the viscous forces!

At first sight, the interfacial tension does not play a role in the Darcy description. It is implicitly hidden in the capillary pressure function and in most cases capillary pressure can be neglected. But for enhanced oil recovery (EOR) mechanisms, the important parameter is increasing the capillary number, which can be tweaked by increasing the viscosity or by decreasing the interfacial tension. A direct consequence of this is a lower residual oil saturation, and straighter relative permeability functions [8]. This example shows that capillary forces are lumped in the relative permeability curves.

In short, in the conventional macroscopic formalism viscous forces appear to dominate capillary forces, which can be seen from the structure of Darcy's law: an applied pressure drop results in viscous flow that disturbs the capillary equilibrium. This picture is inconsistent with the typical values for the capillary number, which show that the capillary forces are much larger than the viscous forces.

The first argument for explaining the paradox comes from the first assumption that Darcy's law is a viscous law which is invalid. In the Darcy description, viscous and capillary forces are not clearly separated. While Darcy's law does not contain viscous coupling or capillary pressure, these factors are embedded in the relative permeability functions. The microscopic picture presented in the next section provides the second argument that solves the paradox.

THE MICROSCOPIC PICTURE

On the microscopic scale, e.g. inside a pore, the continuum description of Darcy's law is not valid as discussed in detail in the previous section. In the tortuous network of pores and throats, the actual flow field is very complex and subject to a description via Stokes flow [17]. Some of the macroscopic properties from the continuum description can be obtained by e.g. averaging *microscopic* models and quantities over a *macroscopic* volume V.

Other properties are only available on the microscopic scale because an explicit microscopic length scale is involved in the picture. One of these quantities is the capillary pressure. On the microscopic scale, the capillary forces are approximated via the capillary pressure as a result of interfacial tension σ between the two immiscible fluids and a curvature κ that is caused by the fluid distribution inside the pore structure. The curvature can be expressed as $\kappa = 1/R$ where R is an effective curvature radius that is on the order of the pore scale. The capillary pressure then becomes [8]

$$p_c \propto \sigma\kappa \propto \frac{\sigma}{R}. \tag{11}$$

The radius of curvature is a true microscopic quantity that cannot be scaled relative to a unit volume.

In fluid mechanics, for phenomena involving drops, bubbles and flow phenomena involving interfaces similar to those in a porous medium, the capillary number

$$N_c = \frac{F_{\text{viscous}}}{F_{\text{capillary}}}, \tag{12}$$

which is the ratio of viscous over capillary forces is a relevant scaling group [3,5].

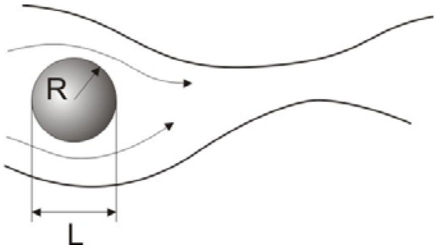


Figure 2: Residual oil blob sitting in the middle of a water wet pore

If we consider an oil blob that is snapped of during water flooding (Fig. 2), we can estimate the viscous and the capillary forces on the stationary oil droplet. The viscous force of a liquid on a solid sphere is given by Stokes' law [14].

$$F_v = 6\pi\mu Ru. \tag{13}$$

The capillary force that resists shape change is determined by Young's-Laplace equation

$$F_c = P_c\, A = \frac{2\sigma}{R} 4\pi R^2 = 8\pi\sigma R\,, \tag{14}$$

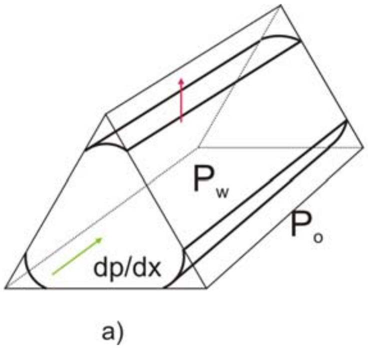
with R the radius of the droplet, A the cross-sectional area and σ the interfacial tension between the two fluids. The droplet starts to break-up once

$$\frac{F_v}{F_c} > \frac{6\pi\mu u R}{8\pi\sigma R} = \frac{3}{4}\frac{\mu u}{\sigma} = \frac{3}{4\varphi}\frac{\mu v}{\sigma} \propto N_c\,, \tag{15}$$

which is (within an order of magnitude) the microscopic capillary number, where it has to be pointed out that the interstitial velocity in the pores u is related to the Darcy velocity v via $u=v/\varphi$.

Substituting typical values for interstitial velocity v (1 ft/day $\equiv 3\cdot10^{-6}$ m/s), viscosity of the flowing phase ($1\cdot10^{-3}$ Pas), porosity (0.25) and interfacial tension ($30\cdot10^{-3}$ N/m) gives indeed numbers of the order of 10^{-7}. So, viscous forces need to be orders of magnitude higher to mobilize the oil droplet through the pore throat. In reality, the oil blob is produced at lower interstitial velocities (lower capillary numbers), which can be seen from the capillary desaturation curve (see Figure 1), where the residual oil saturation already decreases at capillary numbers of 10^{-3} for water wet media. Droplets will start to break-up once the critical capillary number is exceeded. For viscosity ratios between 1-10, this critical number lies between 0.1 and 1 [21]. The capillary number at which droplets start to deform can be lower and oil blob will be produced at lower capillary numbers due to the interaction of the position and deformation of the oil blob and the flow field in the pores. Summarizing: for residual oil blobs, the definition of the capillary number and its order of magnitude are in agreement with what has been published in literature.

Let us now consider the (rather hypothetical) case of an oil-wet reservoir that is being waterflooded. Oil is flowing in the corners and water is flowing in the middle of the pores, see a schematic picture in Figure 3. To account for two phases being present in a 'pore' we have chosen a triangular duct [20]. The water exerts a drag force on the oil film (viscous coupling), and oil will be produced until a molecular thin layer is present on the pore surface. During the process, the difference in phase pressure, P_o-P_w, is adapted so that it matches the curvature of the oil water interphase. While oil is produced, the viscous forces exceed the capillary forces and the capillary number is larger than 1. *In equilibrium* the capillary forces are in balance with the viscous forces (for connected phases only) and hence the capillary number is equal to 1.



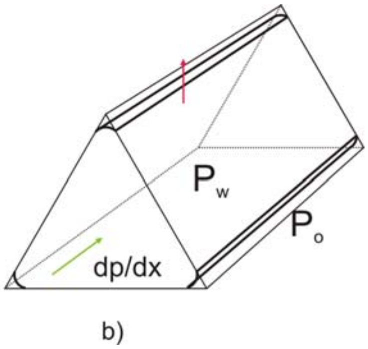


Figure 3: Schematic picture of water flowing through an oil-wet duct. Oil is produced via film flow, as the viscous forces always exceed the capillary forces.

DISCUSSION AND CONCLUSION

A viscous flow law from Eqns. (3) and (4) and a capillary number $N_c << 1$ are apparently contradictory. The contradiction arises because of two invalid assumptions.

First, although Darcy's law seems to be a purely viscous law of two non-interacting fluid phases, viscous coupling and capillary effects are lumped into the relative permeabilities. For very low interfacial tensions (near-miscible floods) the relative permeabilities become straight lines. In the macroscopic view, capillary forces play a significant role through the relative permeabilities (microscopic fluid distributions are determined via the capillary equilibrium, and the fluid distributions have a major impact on the hydraulic conductivities). In the macroscopic description there is no clear separation between viscous and capillary effects.

Second, the definition of the capillary number is valid for the situation where residual, disconnected blobs of fluid are surrounded by another fluid and typical values are 10^{-7} to 10^{-5}. Prior to snap-off of these residual ligaments, the capillary forces exceeded the viscous forces and $N_c<1$. For cases where both phases are still connected, in equilibrium the viscous forces are in balance with capillary forces and capillary numbers are of the order of unity. During displacement of one phase with another, viscous forces are larger than capillary forces and $N_c>1$. In the microscopic view, capillary forces determine the equilibrium distribution of the fluids in each pore, depending on the actual phase pressures.

The authors want to stress that the use of the capillary number is extremely useful, for example in the capillary desaturation curve. At the same time one should care to use rather undefined quantities to explain very complex fluid mechanical processes. As the oil industry is going towards more complex production mechanisms such as low salinity/designer water flooding, polymer flooding and surfactant flooding, fundamental understanding of the processes occurring at the pore scale is needed as well as the translation to common reservoir engineering quantities as relative permeability and capillary pressure.

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