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Calculation of Capillary Pressure Curves from Data Obtained by the Centrifuge Method

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ABSTRACT

The various methods of reducing data collected by means of a centrifuge to obtain capillary pressure curves are described. A detailed comparison between the results obtained by Bentsen's parameter estimation technique, the Hassler and Brunner approximate technique, the van Domselaar method and a new method based on linear interpolation of the capillary pressure points is presented. This comparison is based on analysing simulated experimental data for a range of types of capillary pressure curves. The data is generated using the Bentsen method; therefore, this method provides the basis of comparison. It is found that the Hassler and Brunner method systematically underestimates the values of capillary pressure, while the van Domselaar method systematically overestimates the values of the capillary pressure. The linear interpolation method is in good agreement with the simulated results. Four sets of experimental data are analysed by the Bentsen and linear interpolation methods. In two cases, poor agreement is found between the results for the two methods. The cause of this behaviour is traced to the quality of the data. For all differential methods, the data must be obtained before the inner face of the core attains an irreducible saturation. Data analysis by more than one method is recommended in order to ensure proper interpretation of the results.

INTRODUCTION

The calculation of capillary pressure curves from centrifuge data has been problematic since the centrifuge method was first introduced over 40 years ago. The classic method, due to Hassler and Brunner (1945), was recognized by its authors as approximate. An 'exact' method, proposed by Hoffman (1963) was later show by Luffel (1964) to be in error. A second 'exact' method proposed by van Domselaar (1984) has also been shown to be an approximation (Rajan (1986) and later Melrose (1988)) .

The problem of data interpretation arises because the capillary pressure versus saturation curve is not measured directly. The measured data are the amounts of fluid produced and the rotational speeds. The cumulative amount of fluid produced at any speed can be used to calculate the mean saturation in the core , $\overline{S}$, using the equation

$$\overline{S} = \frac{(V_o - V_d)}{V_p} \quad , \qquad (1)$$

where V_o is the original volume (in the core) of the liquid being displaced from in the core, V_d is the measured cumulative volume of this liquid actually displaced, and V_p is the pore volume of the sample. In turn, the speed may be used to calculate the capillary pressure at the inside face of the core sample using the formula

$$P_{ci} = \frac{1}{2}\,\omega^2\,\Delta\rho\,(r_o^2 - r_i^2) \quad , \qquad (2)$$

where ω is the rotational speed, $\Delta\rho$ is the difference in density between the two phases, r_o is the radius to the outside face of the core, and r_i is the radius to the inside face of the core. Hassler and

Brunner showed that $\overline{S}$ and P_{ci} were related by the expression

$$\overline{S} = \frac{A}{P_{ci}} \int_0^{P_{ci}} \frac{S(P_c)}{\sqrt{1 - \frac{B\,P_c}{P_{ci}}}}\, dP_c\,, \tag{3}$$

where

$$A = \frac{(r_o + r_i)}{2\,r_o}\,, \tag{4a}$$

$$B = 1 - \left(\frac{r_i}{r_o}\right)^2\,, \tag{4b}$$

and

$$P_c = \frac{1}{2}\,\omega^2\,\Delta\rho\,(r_o^2 - r^2)\,, \tag{4c}$$

the capillary pressure at any location, r, along the core. This equation, which will be referred to as the Hassler-Brunner equation, has resisted exact solution. Hassler and Brunner obtained a first approximation by assuming that $A = 1$ and $B = 0$. The equation may then be rearranged and differentiated with respect to P_{ci} to obtain

$$S(P_{ci}) = \frac{d\,(\overline{S}\,P_{ci})}{dP_{ci}}\,. \tag{5}$$

This equation successfully relates saturations and capillary pressures at a single physical location (the top of the sample), thereby allowing the calculation of a capillary pressure curve.

Melrose has presented a solution of the Hassler-Brunner equation that attempts to account for at least part of the effect of A and B varying from 1 and 0 respectively. Expanding the square root in powers of the argument:

$$\sqrt{1 - \frac{B\,P_c}{P_{ci}}} = 1 + \frac{1}{2}\frac{B\,P_{ci}}{P_{ci}} + \frac{3}{8}\left(\frac{B\,P_{ci}}{P_{ci}}\right)^2 + \frac{5}{16}\left(\frac{B\,P_{ci}}{P_{ci}}\right)^3 + \cdots\,. \tag{6}$$

After substituting into the Hassler-Brunner equation and differentiating, Melrose obtained an equation that is equivalent to

$$S(P_{ci}) = \sqrt{1-B}\,\sum_{n=1}^{\infty}\left(\frac{n\,B^n}{P_{ci}^{n+1}}\,\frac{\prod\limits_{1\le j\le n}(2j-1)}{\prod\limits_{1\le j\le n}2j}\int_0^{P_{ci}} S(P_{ci})\,P_{ci}^n\,dP_{ci}\right) + \frac{\sqrt{1-B}}{A}\,\frac{d\,(\overline{S}\,P_{ci})}{dP_{ci}}\,. \tag{7}$$

Melrose proceeded to solve Equation 7 by making use of the assumption

$$\int_0^{P_{ci}} S(P_{ci})\,P_{ci}^n\,dP_{ci} = \overline{S}\int_0^{P_{ci}} P_{ci}^n\,dP_{ci}\,, \tag{8}$$

and obtained van Domselaar's equation:

$$S(P_{ci}) = \frac{\sqrt{1-B}}{A}\,\frac{d\,(\overline{S}\,P_{ci})}{dP_{ci}} + \overline{S}\left(\frac{\sqrt{1-B}}{A}\right)\,. \tag{9}$$

The solution procedures discussed above may all be classed as differential methods because they rely on obtaining experimental estimates of the gradient of $(\overline{S}\,P_{ci})$ with P_{ci}. Because they involve the calculation of differences, differential methods are in general highly sensitive to experimental error . An alternate approach to reducing the data is provided by the parameter estimation technique put forward by Bentsen and his co-workers (Bentsen and Anli (1977), Golaz and Bentsen (1980)). In this method, a functional form is assumed between P_c and S. One form that will be used in the present paper is

$$P_c = a + b\,\ln\left(\frac{S - S_{r1}}{1 - S_{r1} - S_{r2}}\right)\,, \tag{10}$$

where a and b are constants to be found during data analysis, and the subscripts on S refer to the residual values for the two phases (phase 1 is assumed to be the phase being displaced). If a and b were known, then the Hassler-Brunner equation could be solved (if not exactly, at least numerically to any degree of accuracy) to give the experimentally observed $\overline{S}$ versus P_{ci} behaviour. However, a and b are not know. In order to apply the method, a and b are assumed, and a simulated production history is calculated. By comparing the simulated production history with the actual production history, an iteration scheme may be used to find the values of a and b that give the best approximation of the experimental values of production.

In the present paper, comparisons are drawn between results derived using the Hassler-Brunner approximation, the van Domselaar equation, the parameter estimation technique of Bentsen and a new method which is based on replacing the assumption made by Melrose in Equation 8 with a more general, experimentally based assumption. This method is based on linear interpolation between the calculated capillary pressure versus saturation points, and will be referred to as the linear interpolation method (LIM).

LINEAR INTERPOLATION METHOD

Figure 1 shows the typical behaviour of $S(P_{ci})$, thatis, the saturation at the top of the core as a function of the capillary pressure at the top of the core. During an experiment, only a discrete number of points on the curve are actually realized. Therefore, the simplest shape of the curve can be obtained by drawing straight lines joining the experimental data points. Of course these points are not known, and are in fact the information being sought in the experiment. However, if they were known, then the saturation could be found at any value of capillary pressure between the points by the interpolation equation

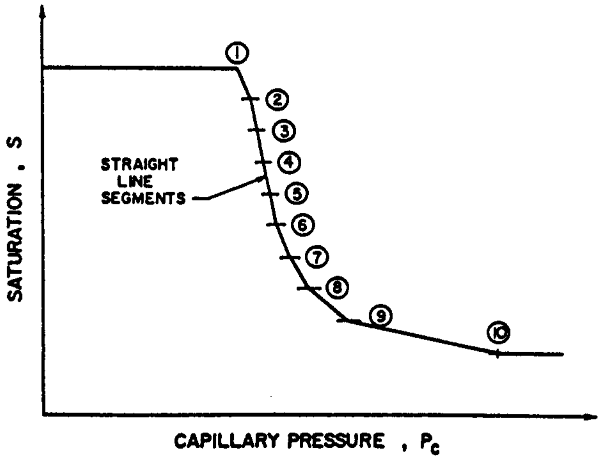

Figure 1 Experimental Capillary Pressure Curve

$$S(Z) = S_{k-1} + \frac{S_k - S_{k-1}}{Z_k - Z_{k-1}} (Z - Z_{k-1}) \ , \qquad (11)$$

where

$$Z = P_{ci} \ , \qquad (12)$$

and the subscript k denotes the experimentally derived values of Z and the corresponding, calculated values of S. This interpolation formula may be used to evaluate the integral derived by Melrose. The experimental data consists of a set of matched $\overline{S}_k - Z_k$ data. The integral must therefore be evaluated for each pair of data points. However, for each new pair of points, the previous values of S_k are known. The only unknown is the value of the saturation that corresponds to the current value of Z_k. On substituting the interpolation formula into the integral:

$$\begin{aligned} \int_0^{Z_k} S(Z)\, Z^n \, dZ = & \int_0^{Z_0} Z^n \, dZ \\ & + \sum_{j=1}^{k} \Big[S_{j-1} \int_{Z_{j-i}}^{Z_j} Z^n dZ \\ & + \ (S_j - S_{j-1}) \int_{Z_{j-i}}^{Z_j} \frac{Z - Z_{j-1}}{Z_j - Z_{j-1}} \, Z^n \, dZ \Big] \ , \end{aligned} \qquad (13)$$

where the first integral accounts for the case where a finite threshold capillary pressure exists. For that case, Z_0 is the value of the threshold capillary pressure, otherwise, Z_0 is zero (ie. the first data point is assumed to be at a fully saturated condition and capillary pressure is assumed to be zero). Performing the integrations

$$\begin{aligned} \int_0^{Z_k} S(Z)\, Z^n \, dZ = & \frac{Z_0^{n+1}}{n+1} \\ & + \sum_{j=1}^{k} \Big[S_{j-1} C_{nj} + (S_j - S_{j-1}) D_{nj} \Big] \ , \end{aligned} \qquad (14)$$

where

$$C_{nj} = \frac{Z_j^{n+1} - Z_{j-1}^{n+1}}{n+1} \qquad (15a)$$

and

$$D_{nj} = \frac{(n+1)\, Z_j^{n+1}(Z_j - Z_{j-1}) - Z_{j-1}\,(Z_j^{n+1} - Z_{j-1}^{n+1})}{(n+1)(n+2)(Z_j - Z_{j-1})} . \qquad (15b)$$

For any application of Equation 14, the values of $S_{j<k}$ are known; therefore, the equation may be cast in the form

$$\int_0^{Z_k} S(Z)\, Z^n \, dZ = S_k D_{nk} + K_{nk} \; , \qquad (16)$$

where

$$K_{nk} = \frac{Z_0^{n+1}}{n+1} + S_0(C_{n1} - D_{n1}) + \sum_{j=1}^{k-1} \left[S_j(D_{nj} + C_{n\,(j+1)} - D_{n\,(j+1)}) \right] . \qquad (17)$$

Substituting this equation into Equation 7 and rearranging

$$S(Z_k) = \frac{\frac{\sqrt{1-B}}{A} \left. \frac{d(\overline{S}Z)}{dZ} \right|_k + \sqrt{1-B} \sum_{n=1}^{\infty} E_{nk} K_{nk}}{1 - \sqrt{1-B} \sum_{n=1}^{\infty} E_{nk} D_{nk}} \qquad (18)$$

where

$$E_{nj} = \frac{n\, B^n}{Z_k^{n+1}} \frac{\prod_{1 \leq j \leq n} (2j-1)}{\prod_{1 \leq j \leq n} 2j} . \qquad (19)$$

The linear interpolation method allows the sequential calculation of the saturation at the top of the sample from the experimental data and preceeding values of the S_k. In order to apply it, the only assumption that must be made is the intital data point (Z_0 and $\overline{S}_0$), which would be the threshold capillary pressure and fully saturated for a strongly water wet rock, or the saturation of the rock at capillary pressure equal to zero for a mixed wettability condition. Either of these data pairs may be estimated from the measured data.

A COMPARISON OF THE METHODS

The various methods of analysing centrifuge capillary pressure data were compared by first generating simulated capillary pressure data using Equation 10. Because the Bentsen parameter estimation technique is used to calculate these results, the following discussion uses the Bentsen method as a basis for comparison. The data were generated for an a of 12.0 kPa, and four values of b: -96 kPa, -24 kPa, -6 kPa and -1.5 kPa. These values represent rocks with an increasingly wide distribution of pore throat sizes. The Bentsen method was used to predict the speeds and displaced phase productions when the capillary pressures at the top of the core were

$$Z_k = 12.0 \times 1.05^k, \quad k = 0 \text{ to } 23 . \qquad (20)$$

The remaining parameters used in the analysis are presented in Table 1. These data are representative of a typical experiment obtained by using a Beckman PIR-16.5 rotor. The simulated production versus speed results were analysed by the Hassler-Brunner approximation, the van Domselaar equation, and the linear interpolation method. Figures 2 through 5 show the results. The Hassler-Brunner predictions are systematically low, with the quality of the prediction deteriorating as b decreases. The van Domselaar predictions are systematically high. Both of these observations agree with other published work (see for example the previously quoted works by Rajan and Melrose). For the first two cases, the linear interpolation method provides estimates that are in excellent agreement with the real curve. However, for $b = -6\, kPa$ there is a slight departure from the real curve in the region of

TABLE 1 Test Parameters for the Model

Parameter	Value	Units
Length	5.08	cm
Diameter	3.18	cm
Distance to Bottom	9.38	cm
Porosity	0.15	
Phase 1 Density	1.00	gm/cc
Phase 2 Density	0.85	gm/cc
S_{r2}	0.15	
S_{r1}	0.00	

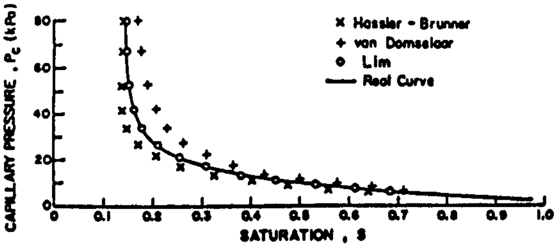

Figure 2 Calculated Curve for $b = -96$

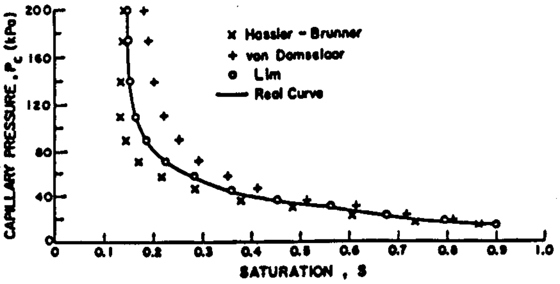

Figure 3 Calculated Curve for $b = -24$

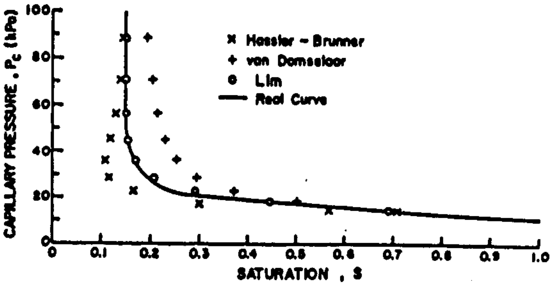

Figure 4 Calculated Curve for $b = -6$

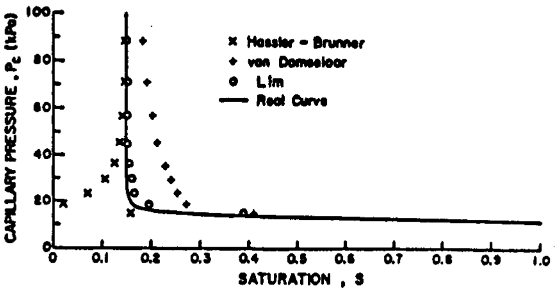

Figure 5 Calculated Curve for $b = -1.5$

irreducible water saturation; this disagreement increases for $b = -1.5\,kPa$. (It should be noted that, in all cases investigated, if the curves were analysed to very high capillary pressures, all of the methods eventually agreed with the real curve.)

In order to investigate this disagreement further, some modified data runs were performed. The results for extending the radius to the bottom of the core by a factor of 10 are shown in Figure 6. Here

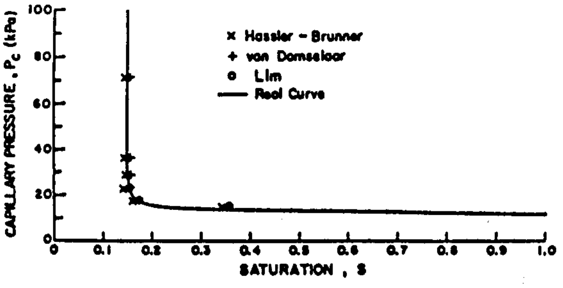

Figure 6 Calculated Curve for $b = -1.5$ and $r_o = 93.8\,cm$

all of the methods agree well with the real curve. This is as expected because, with this long of a radius, the original Hassler-Brunner assumptions for A and B are very good.

The actual cause for the disagreement between the linear interpolation results and the real curve was traced to problems in evaluating the derivative

$$\left.\frac{d(\overline{S}\,Z)}{dZ}\right|_k .$$

By using a data set with approximately three times as many data pairs in the lower capillary pressure range, the results in Figure 7 were obtained. The greater number of data points allowed a more accurate determination of the slope of the experimental data. The improvement in the linear interpolation results is obvious, while the predictions of the other methods are essentially unchanged.

In order to further evaluate the linear interpolation method, the test data published by Skuse (n.d), as well as three sets of new data, were analysed by both the linear interpolation method and the Bentsen method. The results for Skuse's data are presented in Figure 8. Overall, the agreement between the two data analysis methods is excellent. The Bentsen results appear to give a best fit curve through the points predicted by the linear interpolation method.

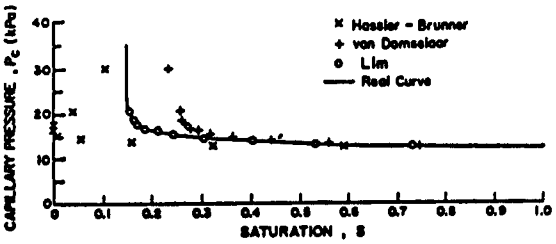

Figure 7 Calculated Curve for $b = -1.5$ and Dense Data

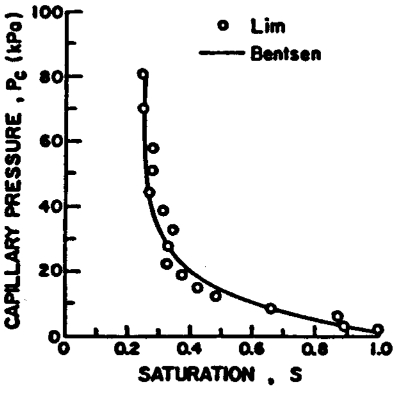

Figure 8 Skuse's Data

The three new data sets were all obtained during a single centrifuge run. The properties of the samples are listed in Table 2; they represent a wide variation of rock type and permeability. No attempt was made to optimize the experiment — a speed schedule was specified, and the schedule was followed regardless of the productions that were observed. Even with this non-optimal test procedure, the results for the two analysis methods, presented in Figures 9 through 11, show a fair amount of agreement.

TABLE 2 Sample Parameters

#	Type	k (md)	ϕ
1	Dolostone	18.05	0.297
2	Brownstone	1703	0.244
3	Berea Grit	83.77	0.168

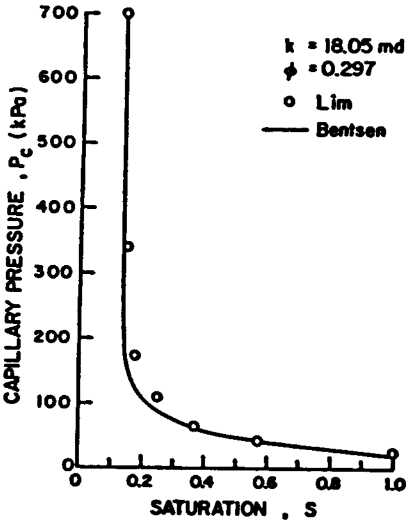

Figure 9 GEOTECH Dolostone

For the dolostone, agreement is good except for the region near the irreducible saturation. The reason for this, as previously discussed, is the estimation of the derivative. The brownstone sample shows much poorer agreement. The shape of this curve is indicative of the low b value cases; based on previous discussion, poorer agreement is expected. For exactly the same reason, the Berea grit data also shows departure near the irreducible water value.

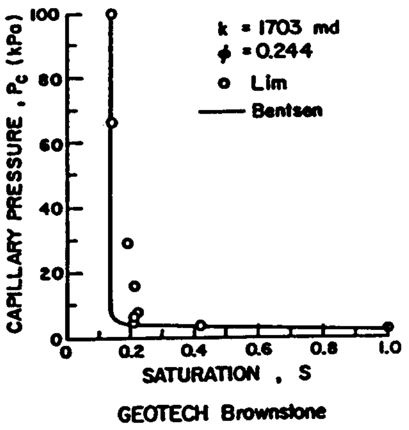

Figure 10 GEOTECH Brownstone

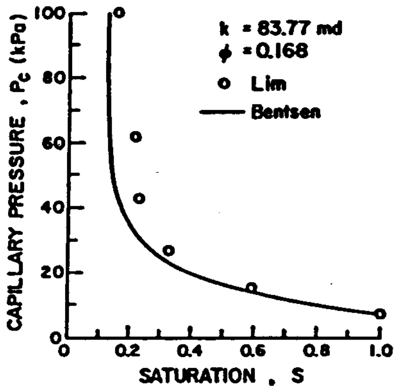

Figure 11 GEOTECH Berea Grit

Further analysis of the simulated data lead to a better understanding of some of the discrepancies. It was found that the quality of the linear interpolation method results improves significantly if all of the data points are taken before the water saturation at the inner face of the core sample approached its irreducible value. For the experimental samples, this was true in only the dolstone case. On the other hand, the Bentsen parameter estimation technique was found to be far less sensitive to this restriction, giving similar curves regardless of when the data was taken relative to the inner face water saturation, provided that some of the data points are taken at high saturations. However, predicted curves were more consistent if more data with high inner face water saturations were used.

The Bentsen method and the linear interpolation method are based on different philosophies. The former recognizes explicitly the inherent uncertainty of any evaluation technique and assumes a smoothed curve from the outset. This approach has the advantage of being less sensitive to data scatter and experimental error; it has the disadvantage of being insensitive to capillary pressure curves which have an inherently different shape than that assumed. The linear interpolation method is more sensitive to errors in the data, but does not presuppose a shape. In the differences between the methods, lies the reason for always analysing the data by both methods and comparing the results. In this way a more complete evaluation of the data can be made. Systematic variations between the predicted curves, particularly at high water saturations, may indicate shapes of the curve which do not conform to the assumed Bentsen model. Scatter of the linear interpolation method results about the Bentsen curve, such as in the case of Skuse's data, is a reflection of scatter in the experimental data.

An evaluation of the accuracy of the various data analysis methods can be made by using the capillary pressure curves resulting from the analyses to predict new sets of simulated production versus speed data. The differences between these predicted data sets and the original simulated data set provide a direct measure of experimental error. A root-mean-squared error can be calculated and compared with the error of observation in reading the centrifuge (±0.15 cc). In Figure 12, the results of such an analysis are shown as functions of the curve shape parameter (b/a). The seemingly good performance of the Hassler-Brunner method at low (b/a) values is actually an artifact of two large errors cancelling each other. For all cases, the linear interpolation method yields results which are 'exact' to within the experimental error.

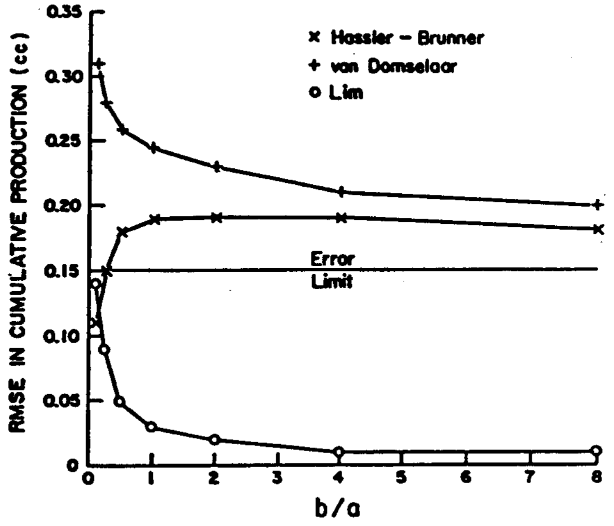

Figure 12 RMSE as a Function of Shape

CONCLUSIONS

The present work supports the following conclusions:

1. The linear interpolation method provides an accurate solution to the Hassler-Brunner equation.
2. The linear interpolation method is a differential method and the results are sensitive to the quality of the data upon which the differentials are based.
3. The linear interpolation method requires that all of the data be taken before the inner face of the core attains the irreducible water saturation.
4. The Bentsen technique does not have the limitation of Conclusion 3. However, data quality is improved if most of the data is taken before the inner face of the core attains the irreducible water saturation.
5. Data analysis should not rely on one method, but should make use of different methods to ensure that a proper interpretation of the data is achieved.

ACKNOWLEDGEMENTS

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NOMENCLATURE

Symbol	Defintion
a, b, c	Constants
A, B	Constants
C_{ij}, D_{ij}, E_{ij}	Constants
P_c	Capillary pressure
P_{ci}	P_c at r_i
r	Radius from center of rotation
r_i	r at top of sample
r_o	r at bottom of sample
S	Saturation
$\overline{S}$	Mean saturation
S_{r1}	S of displaced phase
S_{r2}	S of injected phase
V_d	Displaced volume
V_o	Original volume of displaced phase
V_p	Pore volume
Z	P_{ci}
$\Delta\rho$	Density difference
ω	Rotational speed

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