STOCHASTIC APPROACH TO COMPUTATION OF UNCERTAINTIES IN PETROPHYSICAL PROPERTIES

by

Jude O. Amaefule

Dare K. Keelan

Core Laboratories, a Division of Western Atlas International

Houston, Texas

ABSTRACT

Laboratory and field determined petrophysical properties may differ from true values because of systematic and/or random errors. Data accuracy can be influenced by systematic errors which result from differences in analytical techniques, lack of proper equipment calibration or poor quality control. In contrast, random errors which result from chance and/or inherent equipment limitations affect precision and reproducibility of data.

Most petrophysical parameters are calculated from combinations of primary observable variables using non-exact empirical models. Uncertainties in these models and in the independent input variables can influence the accuracy and precision of the computed variables. These uncertainties can often be treated as random and, hence, computed stochastically. Knowledge of these uncertainties is important when comparing information from various sources (cores, logs and well tests) for use in quality control, reservoir definition/description, and/or sensitivity analyses in numerical simulation.

This paper presents mathematical procedures for computation of uncertainties at the various levels of scale (core, log, and well test). Examples are provided to show the magnitude of uncertainties associated with core-derived pore volume, porosity and formation factor, as well as log-derived porosity and well-test derived permeability.

INTRODUCTION

Stochastic models have been used quite extensively in ground water hydrology by Freeze (1975); Smith and Freeze (1979), and most recently in petroleum reservoir simulation by Cekirge and Kleppe (1981) and Smith and Brown (1982). A model is described as stochastic if it contains random variables which have probability distributions.

In the stochastic approach, uncertainties in input variables are simulated to predict uncertainties in output variables. Most stochastic models are solved by the Monte Carlo technique. This technique has been used by Walstrom (1967) to evaluate uncertainties in engineering calculations, and by Tao and Watson (1984) to investigate errors in relative permeability estimates from the JBN technique.

If the random variables are normally distributed, then the uncertainty in a predicted output variable can be determined from the uncertainties in the random input variables by using the less rigorous root mean square equation. This approach does not address systematic errors, but focuses on whether the analyst is "doing things right," and not whether the "right thing is being done."

MATHEMATICAL APPROACH

It can be shown that if a parameter Y is dependent on variables X_1, X_2...X_n, then the random error in Y can be designated as ΔY. This random error ΔY can in turn be calculated from the random errors existing in the X variables, when these errors are designated ΔX_1, ΔX_2 . . . ΔX_n. The following root mean square error equation strictly applies only if the independent variables are not related:

$$Y = f(X_1, X_2 \ldots X_n) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots .(1)$$

$$\Delta Y = \pm \left[\left(\frac{\delta Y}{\delta X_1} \Delta X_1 \right)^2 + \left(\frac{\delta Y}{\delta X_2} \Delta X_2 \right)^2 + \left(\frac{\delta Y}{\delta X_n} \Delta X_n \right)^2 \right]^{1/2} \quad \ldots \ldots .(2)$$

This approach to uncertainty calculation was previously applied to selected core measurements by Hook (1983). It was also applied to water saturations calculated from the Waxman-Smits equation by Freedman and Ausburn (1985) and to an analysis of the Archie equation components by Chen and Fang (1986).

Random errors in many different physical processes can be approximated by a normal distribution. While these errors arise by chance, they do involve probability and are hence stochastic. In this instance, errors ΔX_1, ΔX_2, ΔX_n and ΔY can be represented by the standard deviations σX_1, σX_2 and σX_n, and by σY. Use of the above equation requires the following:

1. An equation relating Y to its independent variables, i.e., $F = R_o/R_w$ $\quad$ . .(3)

2. Computation of the $\dfrac{\delta Y}{\delta X}$ i.e., $\dfrac{\delta F}{\delta R_o}$ and $\dfrac{\delta F}{\delta R_w}$ . . . . (4)

3. Substitution of the partial derivatives into error equation (2) and manipulation of the resulting equation to usable form, i.e.,

$$\sigma F = \pm F \left[\left(\frac{\sigma R_o}{R_o} \right)^2 + \left(\frac{\sigma R_w}{R_w} \right)^2 \right]^{1/2} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots (5)$$

or

$$\frac{\sigma F}{F} = \pm \left[\left(\frac{\sigma R_o}{R_o} \right)^2 + \left(\frac{\sigma R_w}{R_w} \right)^2 \right]^{1/2} \quad \ldots \ldots \ldots \ldots \ldots \ldots (6)$$

(In this equation, the ratio of the standard deviation (σF) to the value of the variable (F) is referred to as the relative standard deviation ($\sigma F/F$) and is reported as a fraction or percent.)

4. Designation of the level of uncertainty ($\sigma R_o/R_o$) and $\sigma R_w/R_w$ and calculation of the resultant relative standard deviation of the dependent variable ($\sigma F/F$).

In those cases where the uncertainty is expressed as the relative standard deviation with units of fraction or percent (i.e., $\sigma R_o/R_o$) no actual values of R_o are required to solve the equations. This approach has been used wherever possible. In other cases, such as in the calculation of pore volume by difference of bulk volume and grain volume, numeric values of V_B and V_G must be assigned. This is illustrated in Table 2, Equation 3, under Core Data Basic Properties.

STUDY APPROACH

Independent variables and the equations commonly employed to determine core data basic properties such as pore volume, bulk volume, grain volume and porosity are recorded in Table 1. Similar data relating to core residual oil saturation, electrical properties and specific and relative permeability are shown. Also given is information for well test data and log derived properties.

Partial derivatives have been determined for each independent variable, and the resultant uncertainty equation is shown for the computed variable in Table 2. Equations in Table 2 then become reference work equations. These allow insertion of any standard deviation or relative standard deviation desired into the uncertainty equation for subsequent calculation of uncertainty for the computed variable. For example, insertion of (1) the percentage uncertainty in R_o, which is represented by $(\sigma R_o/R_o)$, and (2) the percentage uncertainty in R_w, which is represented by $(\sigma R_w/R_w)$ allows calculation of the percentage uncertainty in F, represented by $(\sigma F/F)$ in the above example and illustrated on Table 5.

UNCERTAINTY ASSIGNMENT

The Equations in Table 2 can be used in various ways, and the uncertainty assigned can be of equal or unequal value for each of the independent variables. In many laboratory and field tests, certain variables are less rigidly controlled with larger standard deviations from the mean value, or technique limitations furnish values that vary within larger percentages than others. These differences can be accounted for in the uncertainty equation, and will yield an appropriate uncertainty value in the computed variable.

In those cases where the random data scatter and hence standard deviation of a variable has been determined by multiple measurements and statistical calculations, expected uncertainty is easily calculated. Where these measured standard deviations do not exist, reasonable levels of uncertainty must be assigned for sensitivity calculations.

Where a maximum uncertainty can be tolerated in a variable such as formation factor, the equation can be used to assess the maximum uncertainty that can be tolerated in any of the independent measured variables. For example, if Formation Factor $(\sigma F/F)$ must fall within $\pm 5\%$ of true value, and if the relative standard deviation in resistivity of $(\sigma R_o/R_o)$ can be measured to within $\pm 4\%$, the maximum relative deviation that can be tolerated in R_w represented as $(\sigma R_w/R_w)$ must be equal to or less than 3%.

APPLICATIONS OF UNCERTAINTY EQUATIONS

CALCULATION OF UNCERTAINTY IN PV BY SATURATION TECHNIQUE

Table 3 illustrates the uncertainty in the saturation pore volume $(\sigma V_p/V_p)$ resulting from various levels of uncertainty in dry weight, saturated weight, and density of the saturating fluid.

When working with the 1 inch diameter sample in the example, relative standard deviation in pore volume $(\sigma V_p/V_p)$ must be no greater than $\pm 2.8\%$ or the resulting porosity will exceed acceptable API limits of 0.5 porosity units, i.e., (18.5-19-19.5). With equal uncertainty in all variables of $\pm 0.1\%$, satisfactory PV and porosity accuracy is achieved.

In the laboratory, dry weight and saturated weight are not likely to be determined to within the same standard deviation. This results because the saturated weight is sensitive to the operator's technique of surface water removal after core saturation and prior to the measurement of saturated weight. Repeat measurements would determine the normal scatter in weight resulting from a combination of balance sensitivity and operator technique, and would improve calculation of uncertainty over that achieved from assuming $\pm 0.1\%$ in all variables.

Note that the calculation of $\sigma V_p/V_p$ requires assigned values for W_D and W_S, as the equation cannot be solved without these values, even when $\sigma W_S/W_S$ and $\sigma W_D/W_D$ are known.

CALCULATION OF UNCERTAINTY IN LAB DERIVED POROSITY

A common approach for determination of laboratory derived porosity is discussed by Thomas and Pugh (1989). Bulk volume is determined by mercury submersion, and measurement of grain volume by double-cell Boyle's Law technique. They concluded bulk volume error measurements were the most prevalent cause of unacceptable porosity data, and were commonly caused in a systematic manner by excessive mercury submersion depths.

Luffel and Howard (1988) indicated bulk volume relative uncertainty of $\pm 0.25\%$ for tight gas sands of 50cc bulk volume and 45cc grain volume. Substitution of these values into the uncertainty equation results in $\pm 4.2\%$ uncertainty for a calculated porosity. This represents $\pm 0.21\%$ porosity units for samples of this size. Experimental data indicated measured values ranged from -0.2 to +0.6 porosity units, again in reasonable agreement with deviation expected from the uncertainty equation.

Table 4 presents relative standard deviation of porosity for equal levels of uncertainty in V_B and V_G. This equation indicates the multiplier $V_G/(V_B-V_G)$ increases as V_G approaches V_B, indicating error is expected to be greater in low porosity rocks. At an equal uncertainty level of 0.5%, computed porosity exceeds the API accepted accuracy of ± 0.5 pu.

CALCULATION OF UNCERTAINTY IN LAB DERIVED FORMATION FACTOR

Formation factors determined from core measurements are subsequently used with log resistivity to calculate formation water saturation. The calculated formation factor uncertainty is $\pm 2.8\%$ when R_o and R_w vary by 2%, and increases to $\pm 7.1\%$ when R_o and R_w vary by 5%. This is illustrated in Table 5.

Repeat measurements on multiple non-stressed cores indicate R_o and R_w to have non-equal uncertainty with slightly higher relative standard deviation exhibited by R_o. Observed RSD for R_o and R_w was approximately $\pm 3\%$ and $\pm 2\%$ respectively. Substitution of these values into the uncertainty equation indicates formation factor

should exhibit a relative standard deviation of approximately ±4%. Repeat analysis of formation factor yielded a RSD of ±5%, which is in reasonable agreement with the predicted value.

CALCULATION OF UNCERTAINTY IN WELL TEST PERMEABILITY

Table 6 illustrates the ease of application of the uncertainty equations when the original equation involves only multiplication and division. When differences are computed in the numerator or denominator as illustrated in Tables 3 and 4, utilization of the equations are lengthy and more unwieldy.

The well test investigates a large area of the reservoir, and the resulting permeability is often used as the most appropriate reservoir value. It is generally recognized that permeability is log normally distributed, therefore, the calculation of $(\sigma K/K)$ requires the use of the following relationship: $(\sigma K/K) \approx [(e^{\pm \sigma} \ln K) - 1.0]$. The uncertainty equation illustrates the sensitivity of this computed value to its controlling variables, and focuses the need for good fluid property data (μ_o, B_o) as well as flow information.

Limitations of the calculated build-up permeability often exist because of non-random errors which are not accounted for in these equations. Incorrect selection of a proper thickness contributing to fluid flow can easily occur in partially perforated wells. If vertical cross flow is zero due to impermeable barriers, then the proper thickness is the perforated interval. If no vertical restrictions to flow exist, flow into the well bore may be spherical, and the effective reservoir thickness contributing to flow will exceed the perforated interval.

CALCULATION OF UNCERTAINTY IN LOG DERIVED BULK DENSITY POROSITY

Errors are often introduced in calculated porosities from log bulk density by usage of incorrect matrix or fluid densities. Many engineers assume a formation grain density of 2.65 g/cm^3 when a sample is described as sandstone, when in reality minerals in the rock such as siderite, pyrite, or calcite can easily drive grain density up to values approaching that of limestone (2.71 g/cm^3). Producing sandstones with grain densities of 3.0 g/cm^3, caused by secondary deposition of siderite, have been observed. Use of low grain density results in calculation of erroneously low porosity. This type of error is systematic, commonly encountered, and not addressed by random uncertainty.

The effect of random errors in bulk, matrix and fluid densities on the relative standard deviation of porosity is presented on Figure 1. This figure illustrates another use of the uncertainty equation, and that is to identify the dominant contributor to the computed relative standard deviation. This figure indicates the primary contributor to random error is log bulk density, followed by matrix density and fluid density. The total contribution is calculated from the ϕ (density) equation in "Log Derived Properties" in Table 2. Individual independent variable contribution is calculated from the portion of the Table 2 equation describing the error in each variable of bulk $(\sigma_{\rho b}/\rho b)$, matrix $(\sigma_{\rho ma}/\rho ma)$ and

fluid ($\sigma_{\rho\,f}/\rho_f$) density, multiplied by the appropriate equation terms. Note that it is not correct to add the relative standard deviation of each component to calculate the total, since the sum of the square root of several terms is not equal to the square root of the sum of each term squared.

This figure indicates that at a relative standard deviation of 1% in the independent variables, the RSD for porosity is about 20% for the rock with 10% porosity and 10% for the rock with 20% porosity. Therefore, while the lower porosity rock exhibits twice the RSD of the higher porosity rock, the standard deviation in porosity will be ±2 porosity units for both samples.

CONCLUSIONS

1. Uncertainty equations utilizing the root mean square equation have been developed for laboratory core measurements as well as for selected well test and down hole log data.

2. The uncertainty equations are generic solutions that yield an uncertainty envelope for dependent variables (such as porosity) for any assigned value of uncertainty in the independent variables (such as grain or bulk volume).

3. The uncertainty equations can be used to define the maximum level of uncertainty that can be tolerated in any independent variable (such as R_o) if the maximum uncertainty to be tolerated in the dependent variable (such as formation factor) is known or assigned.

4. The equations can be used to evaluate the relative contribution to uncertainty of each independent variable (such as bulk, matrix and fluid density) to the total uncertainty of the dependent variable (porosity). This allows identification of the independent variables contributing most to uncertainty, and, hence, where to target uncertainty reduction efforts.

5. Agreement between observed and calculated uncertainty for laboratory derived porosity and formation factor support the equations used to theoretically assess uncertainty in a dependent variable when reasonable values of uncertainty in the independent variables can be assigned.

ACKNOWLEDGEMENT

NOMENCLATURE

a	= Intercept value of F at $S_w = 1.0$
A	= Cross sectional area; cm^2
B_o	= Oil formation volume factor, res BBL/STB
CEC	= Cation exchange capacity, milliequivalents/100 grams
ΔP_b	= Pressure drop (base permeability)
ΔP	= Pressure drop
F	= Formation factor (R_o/R_w)
h	= Formation thickness, feet
K_L	= Liquid permeability, darcies
K_g	= Gas permeability, darcies
K_{ro}	= Oil relative permeability, fraction
K_{rw}	= Water relative permeability, fraction
K_{BU}	= Build up permeability, md
K_o	= Oil permeability, darcies
L	= Length, cm
m	= Cementation exponent (formation factor equation)
m	= Slope of buildup curve, psi/cycle
n	= Saturation exponent
Pa	= Atmospheric pressure
P_1	= Initial V_R pressure for Boyle's Law grain volume, psig
P_2	= Final V_R pressure for Boyle's Law grain volume, psig
P_1	= Upstream pressure for gas permeability, atmosphere
P_2	= Downstream pressure for gas permeability, atmosphere
pu	= Porosity units (%)
P_{wf}	= Flowing wellbore pressure, psig
P_{1h}	= Ideal wellbore pressure after one hour shut in, psig
Q_v	= Cation exchange capacity/unit pore volume, milliequivalents/cc
q_a	= Gas flow rate at atmospheric pressure, cm^3/s
q_L	= Liquid flow rate, cm^3/s
q_o	= Oil flow rate, cm^3/s
q_t	= Total flow rate ($q_o + q_w$), cm^3/s
q_w	= Water flow rate, cm^3/s
R	= Radius, cm
r_e	= Drainage radius, ft
r_w	= Wellbore radius, ft
RI	= Resistivity index (R_t/R_o)
R_o	= Resistivity of 100% water saturated rock, ohm-m
R_w	= Resistivity of saturating brine, ohm-m
R_t	= Resistivity of partially water saturated rock, ohm-m
RSD	= Relative standard deviation ($\sigma X/X$), fraction
S_{or}	= Residual oil saturation; fractional pore volume
S_w	= Water saturation, fractional pore volume

t_L = Acoustic travel time of formation, micro s/ft

t_{ma} = Acoustic travel time of matrix rock, micro s/ft

t_f = Acoustic travel time of fluid, micro s/ft

V_p = Pore volume, cc

V_B = Bulk volume, cc

V_G = Grain volume, cc

V_R = Reference volume, cc

V_{mc} = Matrix cup volume, cc

V_{oi} = Initial oil volume, cc

V_{op} = Produced oil volume, cc

W_D = Dry weight, g

W_I = Immersed weight, g

W_S = Saturated weight, g

ϕ = Porosity, fraction

ρ_b = Bulk density, g/cm^3

ρ_f = Fluid density, g/cm^3

ρ_s = Saturating fluid density, g/cm^3

ρ_{ma} = Matrix density, g/cm^3

ρ_{Hg} = Mercury density, g/cm^3

ρ_{wc} = Volume correction of distilled water to brine volume in sample, fraction > 1.0

μ_g = Viscosity to gas, cp

μ_L = Viscosity to liquid, cp

μ_o = Viscosity to oil, cp

μ_w = Viscosity to water, cp

σX = Standard deviation in value of X (units same as X)

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Table 1

COMPUTED VARIABLE	INDEPENDENT VARIABLES	MODEL

CORE DATA

Basic Properties:

1. V_P (Saturation) W_S, W_D, ρ_S $V_P = \dfrac{(W_S - W_D)}{\rho_S}$

2. V_P (Boyle's Law Direct Injection) P_1, P_2, V_R $V_P = \dfrac{V_R(P_1 - P_2)}{(P_2)}$

3. V_P (Difference) V_B, V_G $V_P = V_B - V_G$

4. V_B (Caliper) R, L $V_B = \pi R^2 L$

5. V_B (Hg Immersion) W_I, ρ_{Hg} $V_B = \dfrac{W_I}{\rho_{Hg}}$

6. V_G (Saturation) W_S, W_I, ρ_S $V_G = \dfrac{(W_S - W_I)}{\rho_S}$

7. V_G (Boyle's Law Matrix Cup) P_1, P_2, V_R, V_{mc} $V_G = \dfrac{V_R(P_2 - P_1) + P_2 V_{mc}}{P_2}$

8. ϕ_1 V_B, V_G $\phi_1 = \dfrac{(V_B - V_G)}{V_B}$

9. ϕ_2 V_P, V_B $\phi_2 = \dfrac{V_P}{V_B}$

Table 1 Cont'd.

10. ϕ_3 V_P, V_G $\phi_3 = \dfrac{V_P}{(V_P + V_G)}$

11. ρ_{ma} W_D, V_G $\rho_{ma} = \dfrac{W_D}{V_G}$

Saturations:

1. S_{or} (Volumetric Balance) V_{oi}, V_{op}, V_p $S_{or} = \dfrac{(V_{oi} - V_{op})}{V_P}$

2. S_{or} (Dean Stark) $W_S, W_O, \rho_O, \rho_{WC}$ $S_{or} = \dfrac{(W_S - W_D - V_W \rho_{WC})}{V_P \rho_O}$

3. S_W (Gravimetric) W_S, W_D, ρ_W, ρ_H $S_W = \dfrac{(W_S - W_D) - \rho_H}{V_P(\rho_W - \rho_H)}$

Electrical Properties:

1. F R_0, R_W $F = \dfrac{R_0}{R_W}$

2. RI R_t, R_0 $RI = \dfrac{R_t}{R_0}$

3. m a, F, ϕ $m = \dfrac{Log\ a - Log\ F}{Log\ \phi}$

4. n R_t, R_0, S_W $n = \dfrac{Log\ R_0 - Log\ R_t}{Log\ S_W}$

Table 1 Cont'd.

5. Q_v CEC, ϕ, ρ_{ma} $Q_v = \dfrac{CEC(1-\phi)\rho_{ma}}{\phi \times 100}$

Specific Permeability:

1. K_L $q_L, A, \Delta P, \mu_L, L$ $K_L = \dfrac{q_L \mu_L L}{A\ \Delta P}$

2. K_g $q_a, A, P_1, P_2, \mu_g, L, P_a$ $K_g = \dfrac{2q_a \mu_g L P_a}{A(P_1^2 - P_2^2)}$

Relative Permeability (Gravimetric Steady State):

1. K_{ro} $q_t, q_o, \Delta P_b, \Delta P$ $K_{ro}{}^* = \dfrac{q_o\ \Delta P_b}{q_t\ \Delta P}$

2. K_{rw} $q_t, q_w, \Delta P_b, \Delta P, \mu_w, \mu_0$ $K_{rw}{}^* = \dfrac{q_w\ \Delta P_b\ \mu_w}{q_t\ \Delta P\ \mu_o}$

*   Equation requires q_t be constant throughout test.

WELL TEST DATA

1. K_{BU} q_0, μ_0, B_0, m, h $K_{BU} = \dfrac{162.6\ q_0 \mu_0 B_0}{mh}$

LOG-DERIVED PROPERTIES

1. ϕ (Acoustic) t_L, t_{ma}, t_f $\phi = \dfrac{t_L - t_{ma}}{t_f - t_{ma}}$

Table 1 Cont'd.

2. ϕ (Density) $\qquad\qquad \rho_{ma}, \rho_f, \rho_b \qquad\qquad\qquad \phi = \dfrac{\rho_{ma} - \rho_b}{\rho_{ma} - \rho_f}$

3. S_W $\qquad\qquad\qquad a, \phi, m, R_W, R_t, n \qquad\qquad S_W = \left[\dfrac{a\ R_W}{\phi^m R_t} \right]^{1/n}$

Table 2	
COMPUTED VARIABLE	**UNCERTAINTY EQUATION**

<div align="center">CORE DATA</div>

<u>Basic</u> <u>Properties</u>:

1. V_P (Saturation)

$$\frac{\sigma_{V_P}}{V_P} = \pm \left[\left(\frac{\sigma_{W_S}}{W_S - W_D} \right)^2 + \left(\frac{\sigma_{W_D}}{W_S - W_D} \right)^2 + \left(\frac{\sigma_{\rho_S}}{\rho_S} \right)^2 \right]^{1/2}$$

2. V_P (Boyle's Law)

$$\frac{\sigma_{V_P}}{V_P} = \pm \left[\left(\frac{\sigma_{V_R}}{V_R} \right)^2 + \left\{ \left(\frac{\sigma_{P_2}}{P_2} \right) \left(\frac{P_1}{(P_1 - P_2)} \right) \right\}^2 + \left(\frac{\sigma_{P_1}}{P_1 - P_2} \right)^2 \right]^{1/2}$$

3. V_P (Difference)

$$\frac{\sigma_{V_P}}{V_P} = \pm \left[\left(\frac{\sigma_{V_B}}{V_B - V_G} \right)^2 + \left(\frac{\sigma_{V_G}}{V_B - V_G} \right)^2 \right]^{1/2}$$

4. V_B (Caliper)

$$\frac{\sigma_{V_B}}{V_B} = \pm \left[\left(\frac{2\sigma_R}{R} \right)^2 + \left(\frac{\sigma_L}{L} \right)^2 \right]^{1/2}$$

5. V_B (Hg Immersion)

$$\frac{\sigma_{V_B}}{V_B} = \pm \left[\left(\frac{\sigma_{W_I}}{W_I} \right)^2 + \left(\frac{\sigma_{\rho_{Hg}}}{\rho_{Hg}} \right)^2 \right]^{1/2}$$

Table 2 Cont'd.

6. V_G (Saturation)

$$\frac{\sigma_{V_G}}{V_G} = \pm \left[\left(\frac{\sigma_{W_S}}{W_S - W_I} \right)^2 + \left(\frac{\sigma_{W_I}}{W_S - W_I} \right)^2 + \right.$$

$$\left. \left(\frac{\sigma_{\rho_S}}{\rho_S} \right)^2 \right]^{1/2}$$

7. V_G (Boyle's Law)

$$\frac{\sigma_{V_G}}{V_G} = \pm \frac{P_2}{(V_R(P_2 - P_1) + P_2 V_{mc})} \left[\left(\sigma_{V_R} \left\{ 1 - \frac{P_1}{P_2} \right\} \right)^2 + \cdots \right.$$

$$\cdots + \left(\sigma_{P_1} \cdot \frac{V_R}{P_2} \right)^2 + \left(\sigma_{P_2} \cdot \frac{V_R P_1}{P_2^2} \right)^2 + \cdots$$

$$\left. \cdots + \left(\sigma_{V_{mc}} \right)^2 \right]^{1/2}$$

8. ϕ_1

$$\frac{\sigma_{\phi_1}}{\phi_1} = \pm \frac{V_G}{V_B - V_G} \left[\left(\frac{\sigma_{V_G}}{V_G} \right)^2 + \left(\frac{\sigma_{V_B}}{V_B} \right)^2 \right]^{1/2}$$

9. ϕ_2

$$\frac{\sigma_{\phi_2}}{\phi_2} = \pm \left[\left(\frac{\sigma_{V_P}}{V_P} \right)^2 + \left(\frac{\sigma_{V_B}}{V_B} \right)^2 \right]^{1/2}$$

10. ϕ_3

$$\frac{\sigma_{\phi_3}}{\phi_3} = \pm \frac{V_G}{V_P + V_G} \left[\left(\frac{\sigma_{V_P}}{V_P} \right)^2 + \left(\frac{\sigma_{V_G}}{V_G} \right)^2 \right]^{1/2}$$

Table 2 Cont'd.

11. ρ_{ma}

$$\frac{\sigma_{\rho_{ma}}}{\rho_{ma}} = \pm \left[\left(\frac{\sigma_{W_D}}{W_D} \right)^2 + \left(\frac{\sigma_{V_G}}{V_G} \right)^2 \right]^{1/2}$$

Saturations:

1. S_{or} (Volumetric Balance)

$$\frac{\sigma_{S_{or}}}{S_{or}} = \pm \left[\left(\frac{\sigma_{V_P}}{V_P} \right)^2 + \left\{ \left(\frac{\sigma_{V_{Oi}}}{V_{Oi}} \right) \left(\frac{V_{Oi}}{V_{Oi} - V_{OP}} \right) \right\}^2 + \circ\circ\circ \right.$$

$$\circ\circ\circ \left. + \left\{ \left(\frac{\sigma_{V_{OP}}}{V_{OP}} \right) \left(\frac{V_{OP}}{V_{Oi} - V_{OP}} \right) \right\}^2 \right]^{1/2}$$

2. S_{or} (Dean Stark)

$$\frac{\sigma_{S_{or}}}{S_{or}} = \pm \left[\left(\frac{\sigma_{V_P}}{V_P} \right)^2 + \left(\frac{\sigma_{\rho_O}}{\rho_O} \right)^2 + \circ\circ\circ \right.$$

$$\circ\circ\circ + \left(\frac{\sigma_{W_S}}{W_S} \cdot \frac{W_S}{W_S - W_D - V_W \rho_{WC}} \right)^2 + \circ\circ\circ$$

$$\circ\circ\circ + \left(\frac{\sigma_{W_D}}{W_D} \cdot \frac{W_D}{W_S - W_D - V_W \rho_{WC}} \right)^2 + \circ\circ\circ$$

$$\circ\circ\circ \left. + \left(\frac{\sigma_{V_W}}{V_W} \cdot \frac{V_W}{W_S - W_D - V_W \rho_{WC}} \right)^2 \right]^{1/2}$$

Table 2 Cont'd.

3. S_W

$$\frac{\sigma_{S_W}}{S_W} = \pm \left[\left(\left\{ \frac{\sigma_{W_S}}{W_S} \right\} \left\{ \frac{W_S}{W_S - W_D - \rho_H} \right\} \right)^2 + \circ \circ \circ \right.$$

$$\circ \circ \circ + \left(\left\{ \frac{\sigma_{W_D}}{W_D} \right\} \left\{ \frac{W_D}{W_S - W_D - \rho_H} \right\} \right)^2 + \circ \circ \circ$$

$$\circ \circ \circ + \left(\left\{ \frac{\sigma_{\rho_H}}{\rho_H} \right\} \left\{ \frac{\rho_H}{W_S - W_D - \rho_H} + \frac{\rho_H}{\rho_W - \rho_H} \right\} \right)^2 + \circ \circ \circ$$

$$\circ \circ \circ + \left. \left(\left\{ \frac{\sigma_{\rho_W}}{\rho_W} \right\} \left\{ \frac{\rho_W}{\rho_W - \rho_H} \right\} \right)^2 + \left(\frac{\sigma_{V_P}}{V_P} \right)^2 \right]^{1/2}$$

Electrical Properties:

1. F

$$\frac{\sigma_F}{F} = \pm \left[\left(\frac{\sigma_{R_0}}{R_0} \right)^2 + \left(\frac{\sigma_{R_W}}{R_W} \right)^2 \right]^{1/2}$$

2. RI

$$\frac{\sigma_{RI}}{RI} = \pm \left[\left(\frac{\sigma_{R_t}}{R_t} \right)^2 + \left(\frac{\sigma_{R_0}}{R_0} \right)^2 \right]^{1/2}$$

3. m

$$\frac{\sigma_m}{m} = \pm \frac{1}{\ln (a/F)} \left[\left(\frac{\sigma_a}{a} \right)^2 + \left(\frac{\sigma_F}{F} \right)^2 + \circ \circ \circ \right.$$

$$\circ \circ \circ + \left. \left(\frac{\ln (a/F)}{\ln \phi} \cdot \frac{\sigma_\phi}{\phi} \right)^2 \right]^{1/2}$$

Table 2 Cont'd.

4. n
$$\frac{\sigma_n}{n} = \pm \left[\left(\frac{\sigma_{S_W}}{S_W} \cdot \frac{1}{\ln S_W} \right)^2 + \left(\frac{\sigma_{R_t}}{R_t} \cdot \frac{1}{\ln (R_t/R_0)} \right)^2 + \circ \circ \circ \right.$$

$$\left. \circ \circ \circ + \left(\frac{\sigma_{R_0}}{R_0} \cdot \frac{1}{\ln (R_t/R_0)} \right)^2 \right]^{1/2}$$

5. Q_V
$$\frac{\sigma_{Q_V}}{Q_V} = \pm \left[\left(\frac{\sigma_{CEC}}{CEC} \right)^2 + \left(\frac{\sigma_{\rho_{ma}}}{\rho_{ma}} \right)^2 + \left(\frac{\sigma_\phi}{\phi(1-\phi)} \right)^2 \right]^{1/2}$$

Specific Permeability:

1. K_L
$$\sigma_{\ln K_L} \cong \pm \left[\left(\frac{\sigma_{q_L}}{q_L} \right)^2 + \left(\frac{\sigma_A}{A} \right)^2 + \left(\frac{\sigma_{\Delta P}}{dP} \right)^2 + \left(\frac{\sigma_{\mu_L}}{\mu_L} \right)^2 + \circ \circ \circ \right.$$

$$\left. \circ \circ \circ + \left(\frac{\sigma_L}{L} \right)^2 \right]^{1/2}$$

Note:
$$\frac{\sigma_{K_L}}{K_L} \cong \left(\left(e^{\pm \sigma_{\ln K_L}} \right) - 1 \right)$$

Table 2 Cont'd.

2. K_g

$$\sigma_{\ln K_g} \cong \pm \left[\left(\frac{\sigma_{q_a}}{q_a} \right)^2 + \left(\frac{\sigma_A}{A} \right)^2 + \left(\frac{\sigma_{P_1}}{P_1} \cdot \frac{2P_1^2}{P_1^2 - P_2^2} \right)^2 + \circ \circ \circ \right.$$

$$\circ \circ \circ + \left\{ \left(\frac{\sigma_{P_2}}{P_2} \cdot \frac{2P_2^2}{P_1^2 - P_2^2} \right) \right\}^2 + \left(\frac{\sigma_{P_a}}{P_a} \right)^2 + \left(\frac{\sigma_{\mu_g}}{\mu_g} \right)^2 + \circ \circ \circ$$

$$\left. \circ \circ \circ + \left(\frac{\sigma_L}{L} \right)^2 \right]^{1/2}$$

Note: $\dfrac{\sigma_{K_g}}{K_g} \cong \left(\left(e^{\pm\sigma_{\ln K_L}} \right) - 1 \right)$

Relative Permeability (Gravimetric Steady State):

1. K_{ro}

$$\frac{\sigma_{K_{ro}}}{K_{ro}} = \pm \left[\left(\frac{\sigma_{q_t}}{q_t} \right)^2 + \left(\frac{\sigma_{q_0}}{q_0} \right)^2 + \left(\frac{\sigma_{\Delta P_b}}{\Delta P_b} \right)^2 + \left(\frac{\sigma_{\Delta P}}{\Delta P} \right)^2 \right]^{1/2}$$

2. K_{rw}

$$\frac{\sigma_{K_{rw}}}{K_{rw}} = \pm \left[\left(\frac{\sigma_{q_t}}{q_t} \right)^2 + \left(\frac{\sigma_{q_w}}{q_w} \right)^2 + \left(\frac{\sigma_{\Delta P_b}}{P_b} \right)^2 + \left(\frac{\sigma_{\Delta P}}{\Delta P} \right)^2 + \circ \circ \circ \right.$$

$$\left. \circ \circ \circ + \left(\frac{\sigma_{\mu_w}}{\mu_w} \right)^2 + \left(\frac{\sigma_{\mu_0}}{\mu_0} \right)^2 \right]^{1/2}$$

Table 2 Cont'd.

WELL TEST DATA

1. K_{BU}

$$\sigma_{\ln K_{BU}} \cong \pm \left[\left(\frac{\sigma_{q_0}}{q_0} \right)^2 + \left(\frac{\sigma_{\mu_0}}{\mu_0} \right)^2 + \left(\frac{\sigma_{B_0}}{B_0} \right)^2 + \circ \circ \circ \right.$$

$$\left. \circ \circ \circ + \left(\frac{\sigma_m}{m} \right)^2 + \left(\frac{\sigma_h}{h} \right)^2 \right]^{1/2}$$

Note:

$$\frac{\sigma_{K_{BU}}}{K_{BU}} \cong \left(\left(e^{\pm \sigma_{\ln K_{BU}}} \right) - 1 \right)$$

LOG-DERIVED PROPERTIES

1. ϕ (Acoustic)

$$\frac{\sigma_\phi}{\phi} = \pm \left[\left\{ \left(\frac{\sigma_{t_L}}{t_L} \right) \left(\frac{t_L}{t_L - t_{ma}} \right) \right\}^2 + \left\{ \left(\frac{\sigma_{t_f}}{t_f} \right) \left(\frac{t_f}{t_f - t_{ma}} \right) \right\}^2 + \circ \circ \circ \right.$$

$$\left. \circ \circ \circ + \left\{ \left(\frac{\sigma_{t_{ma}}}{t_{ma}} \right) \left(\frac{t_{ma}}{t_f - t_{ma}} \right) \left(\frac{t_L - t_f}{t_f - t_{ma}} \right) \right\}^2 \right]^{1/2}$$

Table 2 Cont'd.

2. ϕ (Density)

$$\frac{\sigma_\phi}{\phi} = \pm\left[\left\{\left(\frac{\sigma_{\rho_b}}{\rho_b}\right)^2\left(\frac{\rho_b}{\rho_{ma} - \rho_b}\right)\right\}^2 + \left\{\left(\frac{\sigma_{\rho_f}}{\rho_f}\right)\left(\frac{\rho_f}{\rho_{ma} - \rho_f}\right)\right\}^2 + \circ\circ\circ\right.$$

$$\circ\circ\circ + \left.\left\{\left(\frac{\sigma_{\rho_{ma}}}{\rho_{ma}}\right)\left(\frac{\rho_{ma}}{\rho_{ma} - \rho_b}\right)\left(\frac{\rho_b - \rho_f}{\rho_{ma} - \rho_f}\right)\right\}^2\right]^{1/2}$$

3. S_w

$$\frac{\sigma_{S_w}}{S_w} = \pm\frac{1}{n}\left[\left(\frac{\sigma_a}{a}\right)^2 + \left(\frac{\sigma_{R_w}}{R_w}\right)^2 + \left(\frac{\sigma_{R_t}}{R_t}\right)^2 + \circ\circ\circ\right.$$

$$\circ\circ\circ + m^2\left(\frac{\sigma_\phi}{\phi}\right)^2 + (m \cdot \ln\phi)^2\left(\frac{\sigma_m}{m}\right)^2 + \circ\circ\circ$$

$$\circ\circ\circ + \left.(n \cdot \ln S_w)^2\left(\frac{\sigma_n}{n}\right)^2\right]^{1/2}$$

TABLE 3

Example calculation of percent uncertainty ($\sigma V_p/V_p$) in pore volume resulting for uncertainty in core dry weight, saturated weight and density of saturating fluid.

Equations:

$$V_p = \frac{W_S - W_D}{\rho_s} \quad \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \text{(TABLE 1)}$$

$$\frac{\sigma V_p}{V_p} = \pm \left[\left(\frac{\sigma W_S}{W_S - W_D} \right)^2 + \left(\frac{\sigma W_D}{W_S - W_D} \right)^2 + \left(\frac{\sigma \rho_s}{\rho_s} \right)^2 \right]^{1/2} \quad \cdot \cdot \cdot \cdot \text{(TABLE 2)}$$

$$\frac{\sigma V_p}{V_p} = \pm \left[\left(\frac{\sigma W_S}{W_S} \cdot \frac{W_S}{W_S - W_D} \right)^2 + \left(\frac{\sigma W_D}{W_D} \cdot \frac{W_D}{W_S - W_D} \right)^2 + \left(\frac{\sigma \rho_s}{\rho_s} \right)^2 \right]^{1/2}$$

VARIABLE	MEAN VALUE	UNCERTAINTY IN BEST ESTIMATE ($\sigma X/X$)		
		±0.1%	±1%	±2%
W_D	27.1076	.0271	.2711	.5422
W_S	29.5100	.0295	.2951	.5902
ρ_s	1.001	.001	.01	.020
$\dfrac{\sigma V_p}{V_p}$	--	±1.67%	±16.7%	±33.5%
V_p	2.402	±.040*	±.400	±.800

* AT EQUAL UNCERTAINTY LEVEL OF ±0.1% IN ALL DEPENDENT
　VARIABLES, 2.36 < V_p < 2.44
　　　　　18.7 < ϕ < 19.3

TABLE 4

Example calculation of percent uncertainty (σ_ϕ/ϕ) in porosity resulting from uncertainty in core bulk and grain volumes.

Equations:

$$\phi = \frac{V_B - V_G}{V_B}$$

$$\frac{\sigma_\phi}{\phi} = \pm \left[\frac{V_G}{(V_B - V_G)}\right] \left[\left(\frac{\sigma V_G}{V_G}\right)^2 + \left(\frac{\sigma V_B}{V_B}\right)^2\right]^{1/2}$$

VARIABLE	MEAN VALUE	UNCERTAINTY IN BEST ESTIMATE ($\sigma X/X$)		
		.1%	0.5%	1%
V_B	50.	.05	.25	.5
V_G	45.	.045	.225	.45
$\dfrac{\sigma_\phi}{\phi}$	--	±1.3%	±6.4%	±12.7%
ϕ	10.0	±.13	±.64*	1.27

* AT EQUAL UNCERTAINTY LEVEL OF ±0.5% IN ALL DEPENDENT
   VARIABLES, $9.4 < \phi < 10.6$

TABLE 5

Example calculation of percent uncertainty ($\sigma F/F$) in formation factor (F) for uncertainty in resistivity of 100% saturated sample resistivity (R_o) and water resistivity (R_w).

Equations:

$$F = \frac{R_o}{R_w}$$

$$\frac{\sigma F}{F} = \pm \left[\left(\frac{\sigma R_o}{R_o}\right)^2 + \left(\frac{\sigma R_w}{R_w}\right)^2 \right]^{1/2}$$

VARIABLE	MEAN VALUE	UNCERTAINTY IN BEST ESTIMATE ($\sigma X/X$)		
		1%	2%	5%
R_o	12.0	0.12	0.24	0.6
R_w	.3	.003	.006	0.015
$\dfrac{\sigma F}{F}$	--	±1.41%	±2.83%	±7.1%
F	40	±.56	±1.13	±2.84*

* AT EQUAL UNCERTAINTY LEVEL OF ±5% IN ALL DEPENDENT
  VARIABLES, 37.2 < F < 42.8

TABLE 6

Example calculation of percent uncertainty ($\sigma K_{BU}/K_{BU}$) in well test build-up permeability for uncertainty in flow rate, viscosity, formation volume factor, slope and thickness.

Equations:

$$\ln K_{BU} = \ln \left(\frac{162.6\, q_o \mu_o B_o}{mh} \right)$$

$$\sigma \ln K_{BU} \approx \pm \left[\left(\frac{\sigma q_o}{q_o} \right)^2 + \left(\frac{\sigma \mu_o}{\mu_o} \right)^2 + \left(\frac{\sigma B_o}{B_o} \right)^2 + \left(\frac{\sigma m}{m} \right)^2 + \left(\frac{\sigma h}{h} \right)^2 \right]^{1/2}$$

VARIABLE	MEAN VALUE	UNCERTAINTY IN BEST ESTIMATE ($\sigma X/X$)		
		±2%	±5%	±10%
q_o	1000.	20.	50.	100.
μ_o	0.32	.0064	.0160	.032
B_o	1.37	.0274	.0685	.137
m	70.	1.4	3.50	7.0
h	50.	1.0	2.5	5.0
$\sigma (\ln K_{BU})$	--	.045	.112	.224
$\sigma K_{BU}/K$**	--	±4.6%	±11.9%	±25.1%
K_{BU}	20.4	19.5-21.8	18.2-22.8	16.3-25.5*

* AT EQUAL UNCERTAINTY LEVEL OF ±10% IN ALL VARIABLES,
  $16.3 < K_{BU} > 25.5$

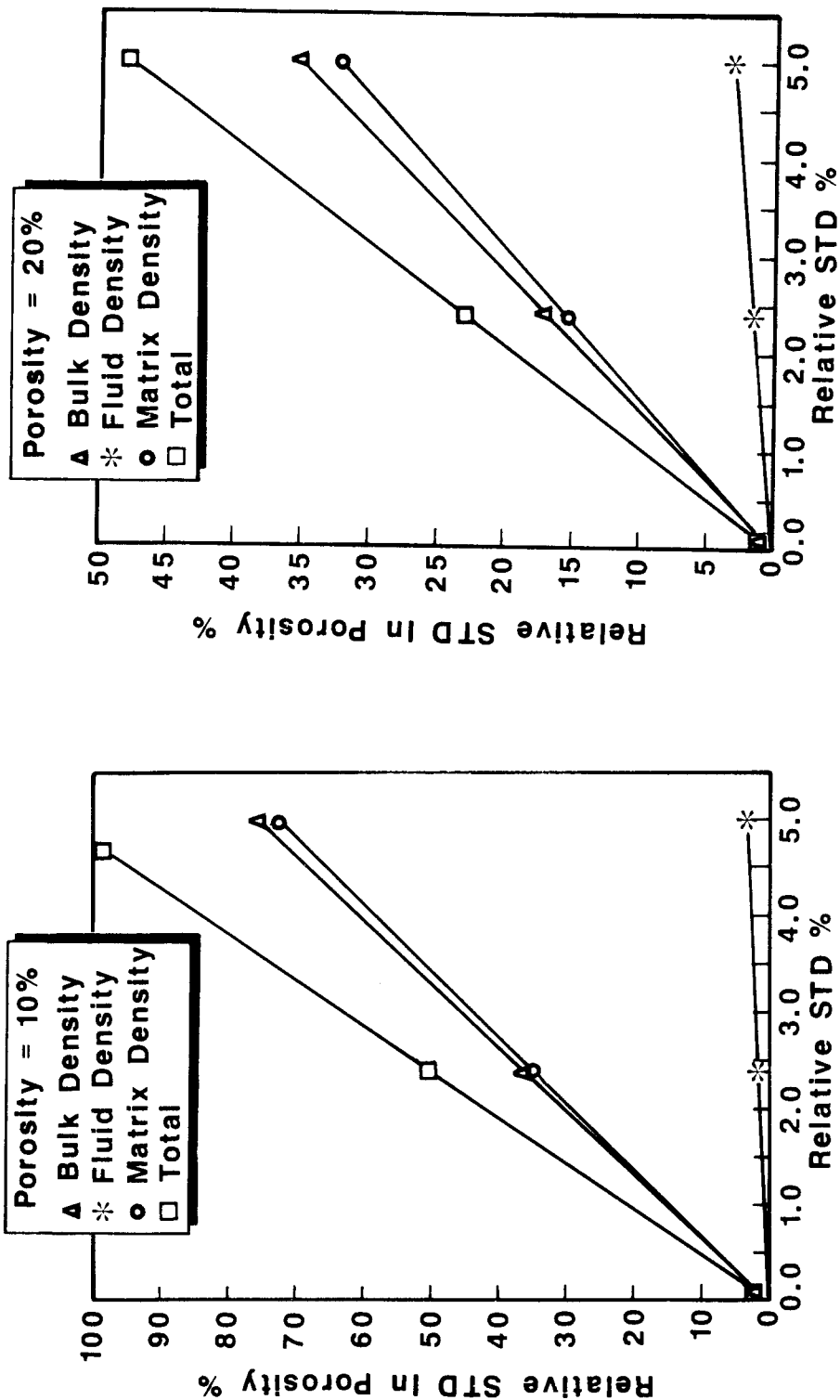**$\sigma K_{BU}/K \approx (e^{\pm \sigma \ln K_{BU}})$

Figure 1

Effect of Relative Standard Deviation of Bulk, Matrix and Fluid Density on the Relative Standard Deviation of Porosity For 10% and 20% Porosity Rock.