USING EXPLICIT AND IMPLICIT METHODS TO CALCULATE RELATIVE PERMEABILITIES FROM *IN SITU* MEASUREMENTS.

Kristofer Kolltveit , Jan-Erik Nordtvedt, Frank Hovland, Bjørn A. Kvanvik, Bente Lie
Department of Physics, University of Bergen, Norway.

Abstract. In a drainage experiment the water saturation is measured with a nuclear tracer method as a continuous function of time in two points on the core. The differential water pressure with respect to the outlet is measured at the same two points. It is shown how the pressure gradient can be derived with great accuracy, and how this result can be used to find the relative permeabilities as functions of the saturations by an explicit use of Darcy's equation. The relative permeabilities are also derived by an implicit method where a simulator is used to find relative permeabilities which give optimal fit to the experimental data.

INTRODUCTION

The relative permeabilities are important parameters in the theory of multiphase flow in porous media. Several methods have been introduced to determine the parameters experimentally, but a satisfactory method is still not developed. The JBN - method (Johnson *et al.*, 1959) is correct, except for the exclution of the capillary pressure, but since the detailed knowledge of the saturation distribution and the pressure in the fluids is not used, one is neglecting information that is important for the full understanding of the flow. The steady state method is mixing drainage and imbibition and is therefore not a sound method, though the results may still be very useful. Several attempts have been made to determine the relative permeabilities in two phase flow where the saturation distribution has been used(Islam *et al.*,1986;Chardaire *et al.*,1989).

A major difficulty in the different approaches has been the measurement of the pressure gradient, which has usually been taken as the pressure difference between two points on the core divided by the distance. This is a very bad approximation for the pressure gradient, which can been seen quite easily from pressure calculations in a simulator(Kvanvik,1989). The pressure is varying very rapidly around the front and careful measurement of the gradient is necessary to yield acceptable results. The relative permeabilities are of course inversely proportional to the pressure gradient and the errors will consequently be transferred to these.

In the present paper we will determine the two phase relative permeabilities from a drainage experiment. The saturation distribution in the core is measured, and we will show how the pressure gradient of the wetting phase may easily be determined to a very high degree of accuracy. From the saturation distribution the fractional flow of the two phases may be found. The Darcy's equation is then used to find the relative permeabilities. This method is called an explicit method.

We will also determine the relative permeabilities from an implicit method. Here, we assume the relative permeabilities to be known in a few points. These values are used to determine the relative permeabilities as continuous functions of the saturation by a polynomial interpolation. The two phase flow may now be simulated and the results may be compared to the experiments. The chosen values for the relative permeabilities are then varied until maximum overlap between experiment and simulation is achieved.

THE EXPERIMENT

At the flow rig at the University of Bergen one may study multiphase flow in long porous samples at reservoir conditions (Lien *et al.*,1988). The fluids are marked with radioactive tracers which may be detected from the outside of the core vessel by a germanium detector. In the present experiment laboratory conditions are used and only one phase, the waterphase, is marked. As tracer radioactive ^{22}Na is used. The positron emission to ^{22}Ne gives a gamma ray of 0.5 MeV by annihilation. This is energetic enough to penetrate the rock and the steel vessel around the core and soft enough so that the lead collimator functions properly. By moving the detector one can scan over the core, and the water saturation can be found as a function of position and time. The experimental set up is shown schematically in Figure 1.

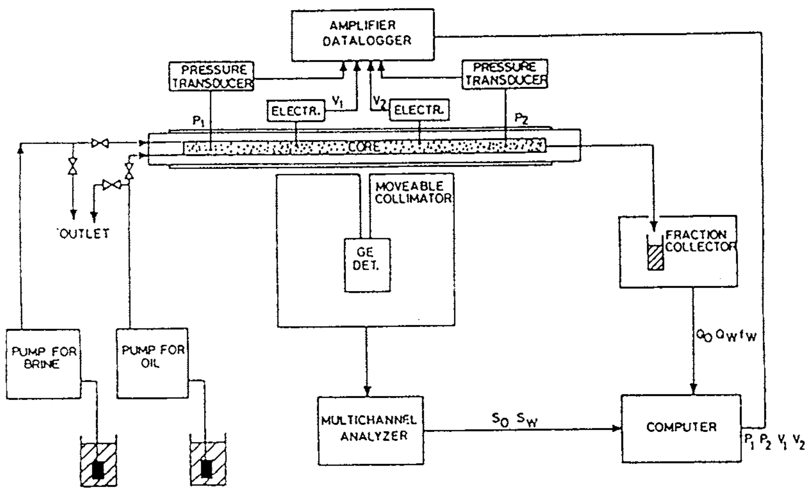

FIGURE 1. Schematic illustration of the flow-rig and connected equipment.

The statistical error of the measurement is proportional to the inverse of the square root of the number of disintegrations and will therefore be reduced with a long counting time. However, the slit in the collimator is 4 mm, and the counting time should therefore be shorter than the time it takes for the fluids to move this distance. Otherwise the image will be smeared out. We have found 2 min. to be a good compromise with an injection rate of 0.4 cm^3. The count corresponds to the water saturation of a 4 mm thick slice of the core. Scanning over the whole core wastes time by scanning areas which are unaffected by the front motion. An alternative would be to try to follow the front, but this proves to be difficult. Experience has taught us that the most efficient counting is to count in two points separated by a finite small length. We therefore measure the water saturation at two points A and B, 31.1 cm and 41.2 cm from the inlet, as a function of time. At the same two points we also measure the wetting phase pressure as a function of time. The pressure is also measured at the inlet. All pressures are measured as differential pressures with respect to the outlet.

Before the experiment the core is imbibed, drained, and imbibed again in order to study the residual saturations in the core under different conditions. The residual water saturation and the corresponding relative permeability for the oil phase prove to be very dependent on the injection rate. The results are shown in Figure 2.

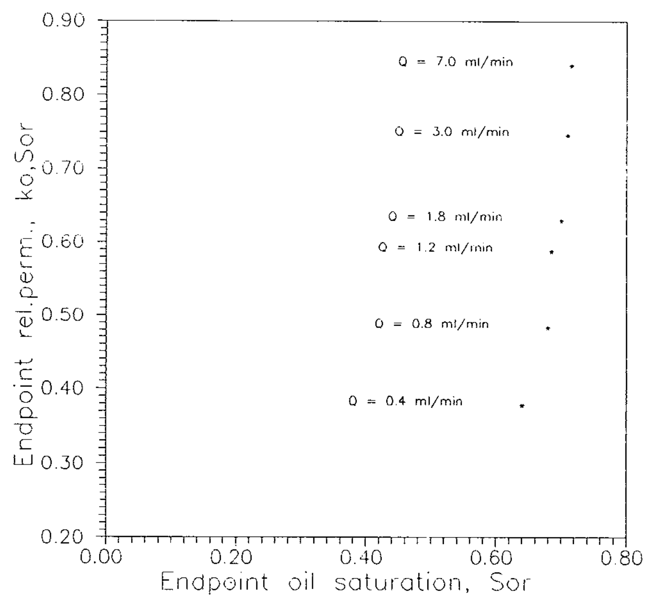

FIGURE 2. Maximum oil saturation and the corresponding relative permeability for oil at different injection rates, Q.

The chosen injection rate, 0.4 cm^3, gives a residual water saturation $S_{w,min}$=0.3. This is not the minimum water saturation one can reach, but this is the final water saturation with the chosen injection rate. No efforts have been made to determine the relative permeability for oil at a lower water saturation in this drainage experiment. Table 1 shows the basic parameters of the experiment.

TABLE 1 Basic data.

Core material:	Clashack sandstone
Core length:	77.8 cm
Core diameter:	5.16 cm
Transducer position:	0, 31.1,41.2 cm
Permeability:	0.952 Darcy
Porosity:	0.183
Viscosity oil:	1.31 cp
Viscosity water:	1.07 cp
Maximum water saturation:	0.65
Minimum water saturation:	0.30
Injection rate:	0.4 cm^3/min

The experimental data are shown in Figures 3 - 5.

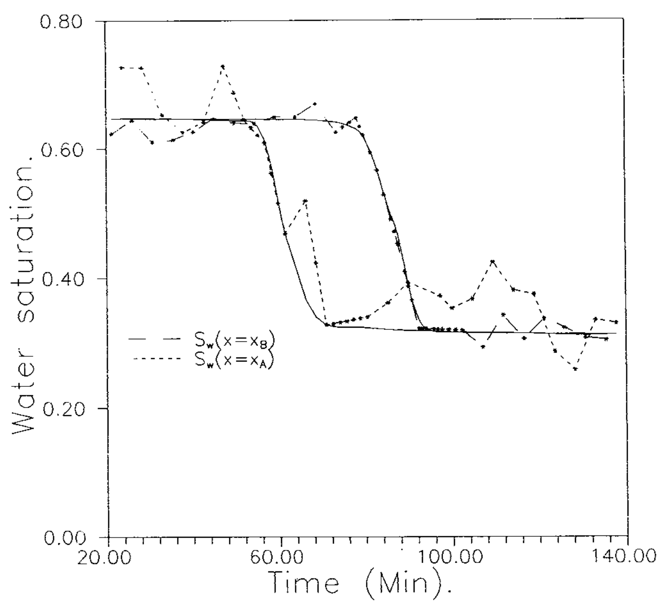

FIGURE 3. The water saturation in points A and B as function of time. The full curves show the smoothed best fit to the data points.

In Figure 3 the experimental water saturation in point A and B is drawn together with a smoothed curve determined by a spline fit. The spline fit is constructed by third order polynomials through some chosen spline knots. By a least square fit one may optimize the construction, but as no criteria may be given for the saturation profile, the fit is eventually based upon eyesight. The curve and the two first derivatives are continuous. The initial water saturation $S_{w,Max}$ equals 0.65 and the final water saturation $S_{w,min}$ equals 0.30. From the data we find that the time for a certain value of the water saturation to move from A to B is constant for all $0.33 < S_w < 0.63$. For values closer to the endpoint saturations the experiment can not decide. We will use a constant average saturation velocity v_S = 0.0067 cm/sec throughout.

The measured pressures in the three transducers, T_1 at the inlet, T_A and T_B at points A and B, are presented in Figure 4.

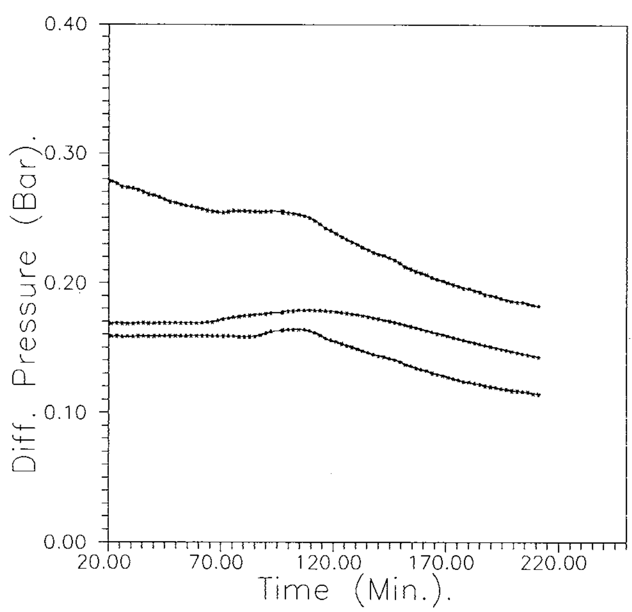

FIGURE 4. From top to bottom. The differential pressure with respect to the outlet as measured in the transducers at the inlet and at points A and B.

We note that the pressure drop per unit time in the three transducers eventually reaches a final common value. The rate of change of the pressure in T_1 is just the difference between the pressure drop over a length v_m with watersaturation $S_{w,min}$ at the inlet minus the pressure drop over a length v_M with watersaturation $S_{w,Max}$ at the outlet, where v_m and v_M are the front velocities of the minimum and the maximum watersaturation. When the front has passed the other transducers they will register the same rate. Thus asymptotically:

$$\frac{\Delta P_1}{\Delta t} = \frac{\Delta P_A}{\Delta t} = \frac{\Delta P_B}{\Delta t} = -v_m \frac{\partial P_{S_{w,min}}}{\partial x} + v_M \frac{\partial P_{S_{w,Max}}}{\partial x} \qquad 1$$

In transducer T_B we see that the pressure is constant until the front reaches point B when it rises to a maximum until it again falls until the rate of change has reached the asymptotic value.

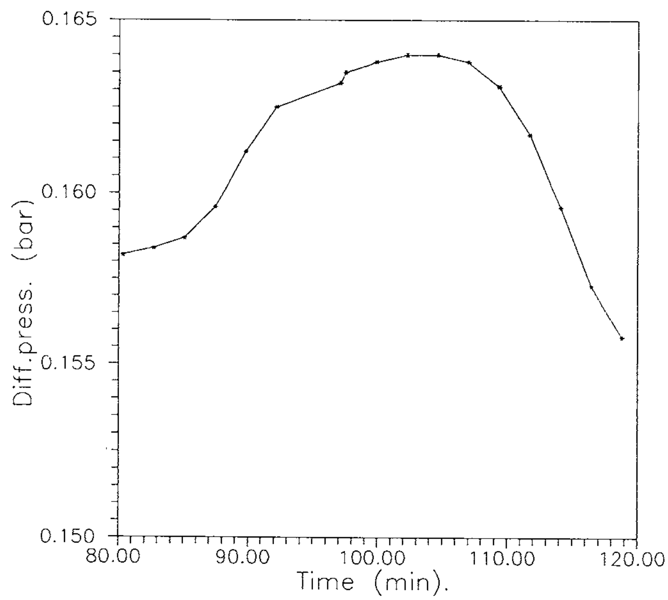

FIGURE 5. The measured pressure in the transducer at point B. The scale is blown up to show the variation in the pressure when the front passes.

In Figure 5 the pressure in transducer T_B is shown in a blown up scale. The transducer T_A shows similar behavior, but due to some inhomogeneities in the core the measurements here do not correspond to the expected results from an assumed homogeneous core.

The saturation and the pressure measurements represent the experimental data for the analysis.

EXPLICIT METHOD

The basic equation of motion for the problem is the Darcy's equation.

$$u_i = -\frac{k\ k_{ri}}{\mu_i}\frac{\partial P_i}{\partial x} \qquad 2$$

The quantities we want to calculate are the relative permeabilities. Unknown quantities in the equation are the fractional flow rates and the pressure gradients. The fractional flow rates are trivial to find when the saturation is known as function of time at the two neighbouring points A and B. The same oil saturation is measured in point x and x+Δ x at times t and t+Δt. Thus:

$$S_o(x+\Delta x, t+\Delta t) = S_o(x,t) \qquad 3$$

or:

$$\Delta S_o = S_o(x+\Delta x, t+\Delta t) - S_o(x,t) = 0 \qquad 4$$

but:

$$dS_o = \frac{\partial S_o}{\partial x}dx + \frac{\partial S_o}{\partial t}dt \qquad 5$$

and to first order:

$$\Delta S_o = \frac{\partial S_o}{\partial x}\Delta x + \frac{\partial S_o}{\partial t}\Delta t \qquad 6$$

Combining equations 4 and 6 we find:

$$\frac{\partial S_o}{\partial t} = -\frac{\partial S_o}{\partial x}\frac{\Delta x}{\Delta t} = -\frac{\partial S_o}{\partial x}v(S_o) \qquad 7$$

From the equation of continuity we then find:

$$\int_x^L \frac{\partial u_o}{\partial x} dx = -\Phi \int_x^L \frac{\partial S_o}{\partial t} dx = \Phi \int_x^L v(S_o) \frac{\partial S_o}{\partial x} dx =$$

$$-v_{ave} \Phi(S_o(x,t) - S_o(L,t)) = u_o(L,t) - u_o(x,t) \quad 8$$

The main approximation is to introduce an average velocity for the front. From previous analyses, where we have compared integration over the saturation, we know that the approximation is good.

A major difficulty in the determination of the relative permeabilities is the measurement of the pressure gradient. By measuring the pressure and the saturation in two points A and B a distance Δx= 10.1 cm apart, we know the pressures for equal saturation in the two points. This corresponds to the two times t_1=t and t_2=t+Δt.

$$\Delta P_w = P_w^S(x + \Delta x, t + \Delta t) - P_w^S(x,t) \quad 9$$

ΔP_w is determined experimentally for all times and all saturations.

$$dP = \frac{\partial P}{\partial x} dx + \frac{\partial P}{\partial t} dt \quad 10$$

To first order we may write:

$$\Delta P = \frac{\partial P}{\partial x} \Delta x + \frac{\partial P}{\partial t} \Delta t \quad 11$$

Combining equations 9,10, and 11, we find:

$$\frac{\partial P_w}{\partial x} = \frac{P_w^S(x + \Delta x, t + \Delta t) - P_w^S(x,t)}{\Delta x} - \frac{\partial P}{\partial t} \frac{\Delta t}{\Delta x} \quad 13$$

where all quanteties on the right side are known.

If the front is approximately stationary, then the only difference in the physical situation for the two fronts is a column of water of length $\Delta x = v_{S_{w,M}} \Delta t$ at the outlet. Thus the pressure difference for two points with equal saturation is the pressure drop over this water column. Thus:

$$\Delta P_w = P_w^S(x + \Delta x, t + \Delta t) - P_w^S(x,t) = -\frac{u \mu_w}{k \; k_{r,S_{wM}}} \Delta x \quad 9$$

giving an alternative expression:

$$\frac{\partial P_w}{\partial x} = -\frac{u\mu_w}{k\ k_{r,S_{wM}}} - \frac{\partial P}{\partial t}\frac{\Delta t}{\Delta x} \tag{14}$$

where Δt is the time it takes for the water saturation S_w to move from transducer A to B. This way of measuring the pressure gradient is tested against several simulated experiments and has proved to be very accurate even for very rapidly changing pressures. For the oil phase the capillary pressure must be included. We have measured this before the experiment as function of water saturation and the gradient of the capillary pressure may therefore easily be added to the water pressure gradient. All quantities in the Darcy's equation needed for the evaluation of the relative permeabilities are then known.

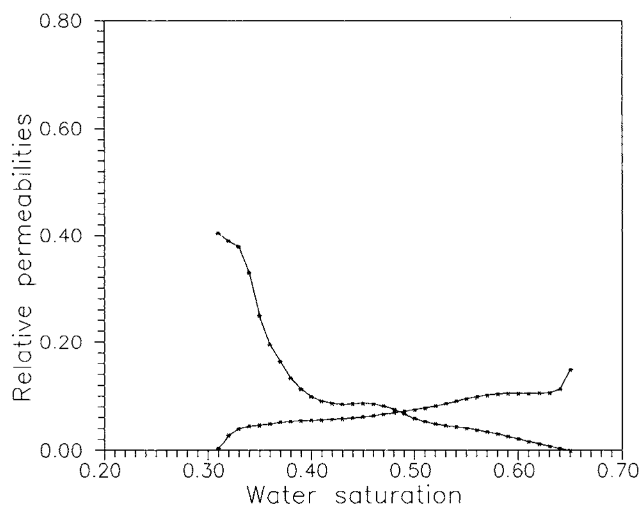

FIGURE 6. The oil and water relative permeabilities as calculated with the explicit method.

Figure 6 shows the resulting curves. There is one serious problem in the measurements. The response in the transducers when the front reaches the transducers is immediate, but the rise in the pressure and the fall as the front passes is delayed. This is a feature which is encountered also elsewhere, the response of a pressure change is translated slowly. We have changed the time scale of the pressure measurements so that the asymptotic value for the pressure drop coincides with maximum oil saturation. If this is not done the crossing point of the oil and water relative permeability will be lowered and moved towards higher oil saturation.

IMPLICIT METHOD

In the explicit method the relative permeabilities are found directly from Darcy's equation. They are proportional to the inverse of the pressure gradient and proportional to the fractional flow. All uncertainties in the measurements are directly transferred. In the implicit method we use a simulator to simulate the experiment. The simulator used is developed by Guo (Guo,1988).The relative permeabilities are input data that may be varied to give an optimal overlap between calculated results and experimental data. Traditionally the comparison between simulator and experiment has been restricted to effluent data(Archer and Wong, 1988,Watson *et al.*,1986). More recently saturation distribution has been included, which gives a much better comparison(Chardaire *et al.*,1989). However, the saturation distribution is mainly sensitive to the end point relative permeabilities. Of course, this means that the reservoir flow is also mostly dependent on these. When three phase flow is to be studied,however, it will be necessary to know the relative permeabilities also for intermediate saturations. Fortunately the pressure gradient is sensitive to these. When the front passes the transducer the pressure gradient will show a maximum for minimum combined mobility of the phases. This is shown clearly if we add the Darcy's equations for the two phases:

$$u = \Sigma u_i = -k\left(\frac{k_{r,w}}{\mu_w}\frac{\partial P_w}{\partial x} + \frac{k_{r,o}}{\mu_o}\frac{\partial P_o}{\partial x}\right) \quad 15$$

In regions where the capillary pressure may be neglected, small S_o, this becomes:

$$u = -k(\frac{k_{r,w}}{\mu_w} + \frac{k_{r,o}}{\mu_o})\frac{\partial P_w}{\partial x} \qquad 16$$

In the optimalization procedure some values of the relative permeabilities are chosen as initial input data. From the requirements that the relative permeability curves and the derivatives must be continuous the whole curve is found by piecewise to interpolate with a third order polynomial, a spline procedure (Nordtvedt,1989). The simulator calculate the saturation distribution, the pressures, and the pressure gradients. These values are compared with the experimental data through a chi-square procedure, and the initial values for the relative permeabilities are changed to minimize the chi-square values.

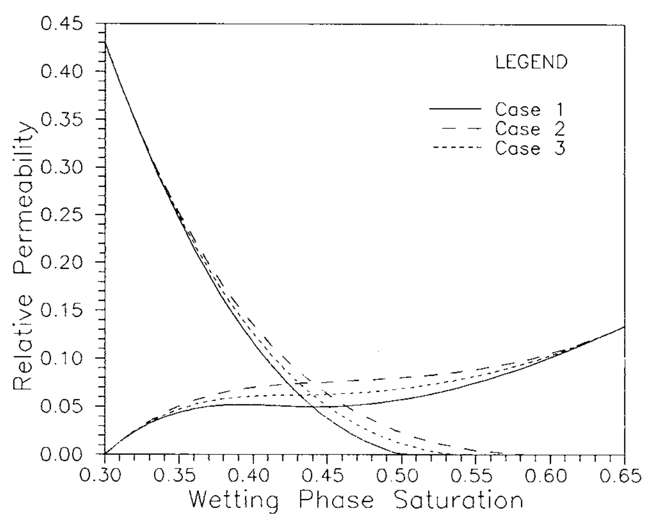

FIGURE 7. Three different oil and water relative permeability curves used for testing the sensitivity of the optimalization with respect to pressure variations.

The procedure is in principle straight forward, but great care must be taken when the least square method is used, otherwise one may find that the optimalization is only sensitive to data that are dependent on trivial information as flow rates and absolute permeabilities. For example, in a piston like displacement the arrival time of the front to the measuring point is given by trivial data and can be calculated. If this does not coincide with the measurements, this must be due to inhomogeneities in the core, and corrections should be made.

The end point relative permeabilities are measured before and after the experiment and these are therefore kept fixed. If we concentrate the optimalization routine on the shape of the saturation distribution we find a strong dependence on the values of the permeabilities near the end point values. This is fortunate since this is where the explicit method is most uncertain.

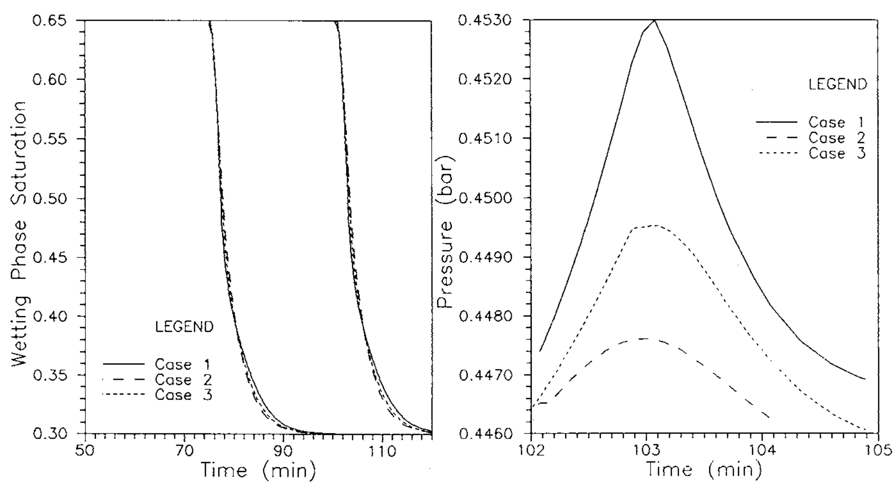

FIGURE 8. Variation in saturation distribution and pressure with the three different relative permeabilities.

If the optimalization routine is concentrated on the pressures and their gradients, we find the method to be sensitive to the intermediate region with low total mobility. This is a region which is very little studied and where studies of the saturation distribution can not give the answer.

In figure 7 we show three sets of relative permeability curves that are used in the simulator to test the sensitivity of the pressures. The end point permeabilities and the variation near the end points are kept fixed. In figure 8 the resulting water saturation in point A and B, and the differential pressure in the same two points are shown as function of time. We see clearly how sensitiv the pressure in point B is to variations in the lowest total mobility.

Finally figure 9 shows the relative permeability obtained with the explicit method together with the best fit from the implicit method.

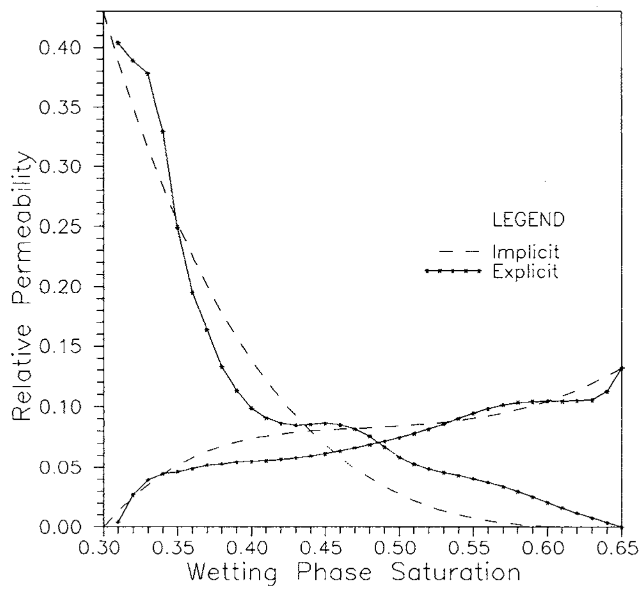

FIGURE 9. Comparison of the relative permeabilities calculated with the explicit and the implicit methods.

CONCLUSIONS

In this work we have studied the two phase relative permeabilities using an explicit and an implicit method.

In the explicit method the relative permeabilities are found directly from Darcy's law. This method is dependent on good measurements of the pressure gradients, and special care has been taken to determine these. A simple and efficient method has been developed to measure the pressure gradients to a high degree of accuracy. It is noticed, however, that the response in the pressure measurements are delayed, an experimental fact that one must overcome in order to trust the method fully. The capillary pressure is included, but as no additional information is given by this, we have not discussed the capillary pressure explicitly.

In the implicit method the comparison between a simulator and the experiment is used to find the relative permeabilities through an optimalization procedure. The simulator is not built for this method and the analysis is therefore time consuming. We have consequently concentrated the work on finding the regions of sensitivity for the relative permeabilities. The method proves to be very good. The optimalization procedure, when carefully used, is sensitive to different areas of interest, and can be focused on problems of interest. More work is now done on building out the simulator and the optimalization routine.

The methods we have used are developed for long cores where the pressures may be studied before breakthrough at the outlet. We are convinced that also for shorter cores, pressure data will prove to be of greater importance for studies of multiphase flow in porous media.

ACKNOWLEDGEMENTS

The authors wish to acknowledge The Norwegian Research Council (NAVF) for financial support.

REFERENCES:

ARCHER, J.S., WONG, S.W. (1973). *Use of a reservoir Simulator to Interpret Laboratory Waterflood Data*, SPEJ, Vol.13, p.343

CHARDAIRE, C., CHAVENT, G., JAFFRE, J., LIU, J., BOURBIAUX, B. (1989). *Simultaneous Estimation of Relative Permeabilities and Capillary Pressure*, SPE 19680, SPE 64th Annual Technical Conference and Exhibition, San Antonio, TX, Oct. 8-11, 1989

GUO, Y. (1988). *Centrifuge Experiment and Relative Permeabilities,* Dr.Eng. Dissertation, Norwegian Institute of Technology, Trondheim, Norway.

ISLAM, M.R., BENTSEN, R.G. (Jan. 1986). *A Dynamic Method For Measuring Relative Permeability,* J.Cnd.Pet.Tech.

JOHNSON, E.F., BOSSLER, D.P., NAUMANN, V.O. (1959). *Calculation of Relative Permeability From Displacement Experiments,* Trans. AIME, Vol. 216, p.370

KERIG, P.D., WATSON, A.T. (1986). *A new Algorithm for Estimating Relative Permeabilities From Displacement Experiments,* SPERE, Vol.2, p.175

KVANVIK, B.A. (1989). *Bestemmelse av relative permeabiliteter ved fortrengningsprosesser i lange kjerner; tofase systemer,* Thesis for the degree of cand.scient., Department of Physics, University of Bergen, Norway.

LIEN, J.R., GRAUE, A., KOLLTVEIT, K. (1988). *A Nuclear Imaging Technique for Studying Multiphase Flow in a Porous Medium at Oil Reservoir Conditions,* Nuclear Instruments and Methods in Physics Research, A271 pp. 693-700.

NORDTVEDT, J.E. (1989). *Using the Inverse Problem Formulation to Calculate Characteristic Properties of the Porous Medium,* Ph.D. Dissertation, Department of Physics, University of Bergen, Bergen, Norway.

WATSON, A.T., KERIG, P.D., RICHMOND, P.C., TAO, T.M. (April 2-4, 1986). *An Improved Method for Estimating Relative Permeability From Displacement Experiments,* paper SPE 15064 presented at the SPE 56th California Regional Meeting, Oakland, CA.