

ROCK MECHANICS CONTRIBUTION TO THE DETERMINATION OF FLUID FLOW PROPERTIES

Maurice J. Boutéca, Jean-Paul Sarda and Jérôme Laurent
Institut Français du Pétrole

Abstract Biot's theory, which couples fluid flow and rock deformation, is investigated from the petrophysicist standpoint. The rock deformation contribution to the diffusivity equation is discussed and experimental recommendations are made for the measurement of the compressibilities. An experimental procedure is explained and results obtained within our lab are presented and discussed. An equation is established to estimate the Biot's coefficient b for a carbonate of given porosity.

INTRODUCTION

From the petrophysicist standpoint, lab studies of fluid flow properties are based on the diffusivity equation:

$$\dot{P}_{pore} - \frac{k}{\Phi\eta c} \nabla^2 P_{pore} = 0 \quad (1)$$

Hence the properties to be determined are the porosity, the permeability and the compressibility. Since the rock deformation has to be taken into account, the compressibility is defined as:

$$c = c_{fl} + c_{pp} \quad (2)$$

in which c_{pp} is the matrix compressibility defined in reservoir engineering practice by:

$$c_{pp} = \frac{1}{V_p} \left[\frac{\partial V_p}{\partial P_{pore}} \right]_T \quad (3)$$

The lab test is usually performed at constant pore pressure, with a varying confining pressure. We will show that a variation of the pore pressure at constant external pressure is not equivalent to a variation of the external pressure at constant pore pressure.

BIOT'S THEORY FOR ELASTIC MEDIA

For a detailed description of Biot's theory, one should consult the article by Laurent and Quettier (1989) and refer to the articles by Biot (1941,1962) and Coussy (1989)

Let us assume a homogeneous isotropic medium subjected to a stress system σ_{ij} to infinity. The resulting displacement field is called u_i . Let us assume a fluid circulating in the porous medium. The displacement vector of the fluid is called U_i . For a given volume element, the relative displacement w_i of the fluid in relation to the solid is given by:

$$w_i = \Phi(U_i - u_i) \quad (4)$$

The equations of equilibrium are (Biot, 1962):

$$\nabla \cdot \sigma_{ij} = 0 \quad (5)$$

$$-\nabla \cdot w = \zeta \quad (6)$$

in which ζ is the variation in the fluid content. The behavior laws are written:

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + (\lambda_d + b^2M)tr(\varepsilon_{ij})\delta_{ij} - bM\zeta\delta_{ij} \quad (7)$$

$$\dot{P}_{pore} = bM\dot{e} - M\dot{\zeta} \quad (8)$$

$$\dot{w} = \frac{k}{\eta}\nabla P_{pore} \quad (9)$$

the third behavior law being Darcy's law. The second behavior law, can also be written in the form:

$$\frac{\dot{P}_{pore}}{M} = b\dot{e} - \dot{\zeta} \quad (10)$$

Assuming the isotropy of the material and elastic behavior, Biot worked out the following equations:

$$\mu \nabla^2 u_i + (\mu + \lambda_u) \nabla e - bM \nabla \zeta = 0 \quad (11)$$

$$\dot{\zeta} - \frac{k \cdot M_u}{\eta} \nabla^2 \zeta = 0 \quad (12)$$

in which

$$\lambda_u = \lambda_d + b^2 M \quad M_u = \frac{2\mu + \lambda_d}{2\mu + \lambda_u} M \quad (13)$$

POROELASTICITY AND FLUID FLOW PROPERTIES

In Equation (12) we recognize a diffusivity equation having a form that is familiar to hydraulics specialists. However, the variable is not the pressure but the variation of the fluid content, which includes a volumetric deformation term for the solid. For a detailed discussion of the relationship between Equation (12) and the pressure diffusivity equation familiar to hydraulics specialists one could consult the article by Boutéca and Sarda (1990). Let us study the diffusivity equation for two cases: a) constant isotropic stress tensor and b) constant volumetric deformation.

From Equations (6), (9) and (8) we obtain:

$$\frac{\dot{P}_{pore}}{M} - b\dot{e} = \frac{k}{\eta} \nabla^2 P_{pore} \quad (14)$$

Let us first consider that the isotropic part of the stress tensor is constant ($\dot{\sigma}_{kk} = 0$). Taking Equation (7) into account, we obtain:

$$0 = \frac{\dot{\sigma}_{kk}}{3} = \left[\frac{2}{3} \mu + \lambda_d + b^2 M \right] \dot{e} - bM \dot{\zeta} = K_u \dot{e} - bM \dot{\zeta} \quad (15)$$

Introducing into (14) and taking Equation (8) into account, we obtain:

$$\nabla^2 P_{por} = \frac{\eta}{k} \left[\frac{1}{M} + \frac{b^2}{K_d} \right] \dot{P}_{pore} \quad (16)$$

Hence, under constant isotropic stress conditions, the pressure diffusivity equation leads to a diffusivity coefficient defined by:

$$\frac{1}{K} = \frac{\eta}{k} \left[\frac{1}{M} + \frac{b^2}{K_d} \right] \quad (17)$$

Let us consider now that the volumetric deformation is constant ($\dot{\epsilon} = 0$). Introducing into Equation (14), we obtain:

$$\nabla^2 P_{pore} = \frac{\eta}{k} \left[\frac{1}{M} \right] \dot{P}_{pore} \quad (18)$$

Hence, under constant volumetric deformation conditions, the pressure diffusivity equation leads to a diffusivity coefficient defined by:

$$\frac{1}{K} = \frac{\eta}{k} \left[\frac{1}{M} \right] \quad (19)$$

Depending on the local condition ($\sigma_{kk} = 0$ or $\dot{\epsilon} = 0$) we obtain two expressions for the diffusivity coefficient:

$$\sigma_{kk} = 0 \quad \frac{1}{K} = \frac{\eta}{k} \left[\frac{1}{M} + \frac{b^2}{K_d} \right] \quad (20)$$

$$\dot{\epsilon} = 0 \quad \frac{1}{K} = \frac{\eta}{k} \left[\frac{1}{M} \right] \quad (21)$$

As shown by Boutéca and Sarda (1989), the Biot's coefficient M may be approximated by ($M \simeq \Phi c_{fl}$). On the other hand, lab results -see next sections- indicate that for weak rocks, the ratio (b^2/K_d) may be of the same order of magnitude as ($1/M$). Under these circumstances no approximation can be made as far as the rock deformation is concerned and the diffusivity equation should be solved together with the rock deformation equations.

Equations (11) and (12) show that one may solve any poroelastic problem when four elastic coefficients (μ, λ or K_d, b, M) and the rock permeability (k) are given. Deviatoric stress condition is needed to measure μ . The remaining poroelastic coefficients and the permeability may be determined under isotropic conditions.

Note that there is no need of measuring c_{pp} . However let us go back to this coefficient. Zimmerman et al (1986) defined 4 compressibilities, among which:

$$c_{pp} = \frac{1}{V_p} \left[\frac{\partial V_p}{\partial P_{pore}} \right]_{P_{conf}} \quad (22)$$

$$c_{pc} = -\frac{1}{V_p} \left[\frac{\partial V_p}{\partial P_{conf}} \right]_{P_{pore}} \quad (23)$$

According to Zimmerman et al. we have:

$$c_{pp} = c_{pc} - c_s \quad (24)$$

in which c_s is the skelton compressibility. Hence, one should be careful when measuring c_{pp} . This compressibility cannot be determined with a constant pore pressure condition unless c_s is known.

EXPERIMENTAL PROCEDURE AND RESULTS

Experimental Procedure

We will focus on the determination of K_d , b and M . Since they may be determined under isotropic stress condition the experimental set-up will only need to monitor two pressures: a) the confining pressure (P_{conf}) and the pore pressure (P_{pore}). The same experimental procedure is used for all the experiments (Figure 1).

The basic cycle goes as follows: the confining pressure is first increased while maintaining constant the pore pressure. We then maintain constant the confining pressure and increase the pore pressure. After several cycles we decrease the pressures using the same procedure.

The sample is coated with an impermeable material and saturated in the cell. The pore volume communicates with the external volumetric pump through a microwell.

Volumetric pumps coupled with pressure gauges are used to control and measure the confining pressure and the bulk volume on one side, the pore pressure and the pore volume on the other side (see Figure 2). The experimental set-up has been designed to reduce to a minimum the errors induced by the compressibility of the system. For a detailed discussion of the relative errors induced, one should refer to Laurent et al. (1990).

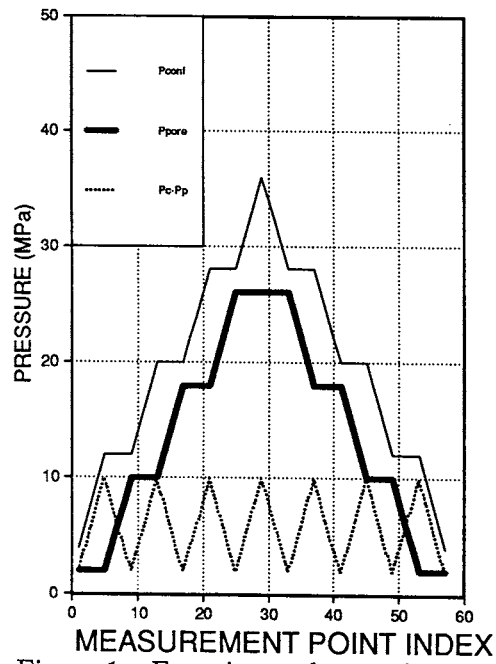


Figure 1 : Experimental procedure

APPARATUS

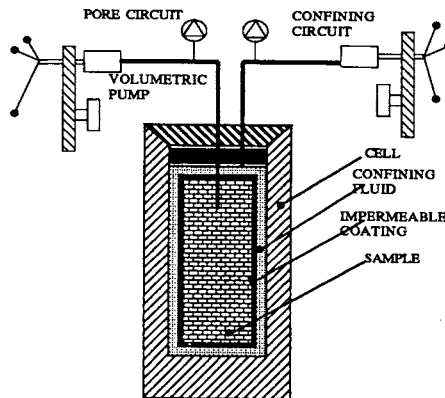


Figure 2 : Experimental set-up

Experimental Determination of the Coefficients

Since we are operating under isotropic stress conditions the equations previously obtained may be simplified.

Let us first determine b , using Equation (10). ζ may be experimentally determined using the relationship (Geerstma, 1957):

$$-\zeta = \frac{\partial V_p}{V_b} - \frac{\partial V_{fl}}{V_b} \quad (25)$$

Under constant pore pressure condition this equation leads to:

$$-\zeta = \frac{\partial V_p}{V_b} \quad (26)$$

and Equation (8) leads to:

$$be = \zeta \quad (27)$$

in which e is experimentally measured using the relationship:

$$e = -\frac{\partial V_b}{V_b} \quad (28)$$

Hence b may be determined as:

$$b = \left. \frac{\partial V_p}{\partial V_b} \right|_{P_{pore}} \quad (29)$$

Let us now determine K_d . Equation (7) may be written as:

$$\frac{\dot{\sigma}_{kk}}{3} = \dot{P}_{conf} = K_u \dot{e} - bM\dot{\zeta} \quad (30)$$

taking Equation (10) into account we obtain:

$$\dot{P}_{conf} - b\dot{P}_{pore} = K_d \dot{e} \quad (31)$$

\dot{e} is the volumetric variation and $P_{conf} - bP_{pore}$ is the effective stress. Using the relationship (31) one can determine K_d .

Finally, using Equations (8) and (25), one can determine M .

Results

Five carbonates have been studied, their description is given in table 1. The results for K_d , b and M are given on table 2, together with the approximation $M \simeq \Phi c_{f\mu}$. In the table, Sdev stands for Standard Deviation. An example is given for the determination of K_d on figure 3.

Table 1 : Description of the carbonates

Name	porosity (%)	density	permeability (mD)	comments
Larrys	4.2 - 4.4	2.58-2.59	0.1	micrite pores 20-50 μ
Tavel	9.9-10.2	2.41-2.42	0.2	micrite pores up to 50 μ
Vilhonneur	13.1-14.8	2.31-2.34	0.4	oolites
Lavoux	23.3-23.9	2.02-2.06	3-5	pellets
Estailades	28.9		100	bioclasts

From Table 2 one may see that the approximation for M sounds reasonable.

BIOT'S COEFFICIENT b AS A FUNCTION OF THE CARBONATE POROSITY

Biot's coefficient b can be computed using the relationship:

$$b = 1 - \frac{K_d}{K_s} \quad (32)$$

Theoretical research work in mechanics have been devoted to the determination of upper and lower bounds for the properties of a composite material from the properties of the basic components. A synthetic work has been presented by Watt *et al.* (1976). We have been working on the basis of Hashin and Shtrikman's work (1961) and on the basis

Table 2 : Experimental results

Name	K_d (MPa)	Sdev (MPa)	b	Sdev	M (MPa)	Sdev (MPa)	$\frac{1}{\Phi_{cft}}$
Larrys	33 850	0.9	0.34	0.04	41 350	2 200	46 200
Tavel	19 500	0.7	0.63	0.03	19 260	1 200	23 000
Vilhonneur	19 800	0.3	0.71	0.05	13 500	580	15 200
Lavoux	8 600	0.4	0.81	0.03	8 400	260	9 500
Estailades	5 400	0.5	0.88	0.02	7 200	330	7 800

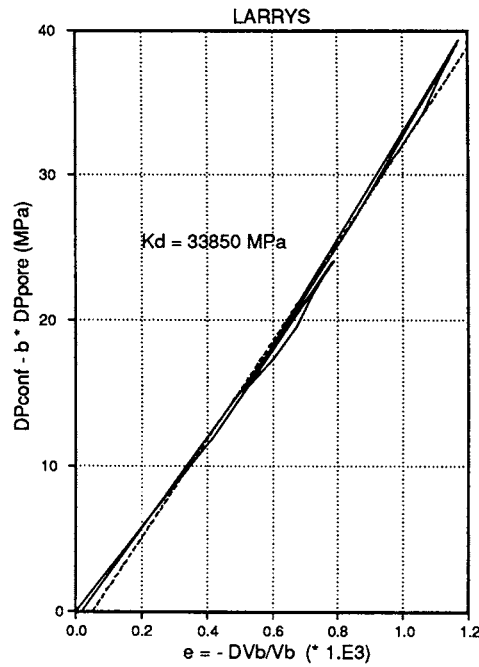


Figure 3 : Determination of K_d

of Miller's work (1968). In our case one of the components is the calcite and the other one is the void. Under such circumstances, in both theoretical work, the upper limit for b is 1. We will thus study herein the lower limit. Based on Hashin and Shtrikman's work Zimmerman *et al.* (1986) obtained:

$$b = 1 - \frac{K_d}{K_s} = 1 - (1 - \Phi) \frac{1}{\left[1 + \frac{\Phi(1+\nu_s)}{2(1-2\nu_s)}\right]} \quad (33)$$

Using the same procedure we derived from Miller's work:

$$b = 1 - (1 - \Phi) \left[1 - \frac{\Phi}{\Phi + 3 \frac{1-2\nu_s}{1+\nu_s} \left[1 - \frac{4}{3}\Phi - 3(1-2\Phi)G \right]} \right] \quad (34)$$

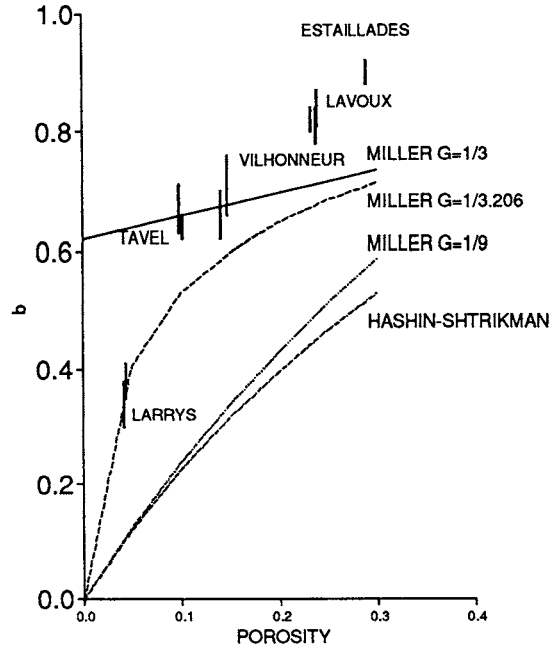
In which G is a geometrical constant varying in the range of 1/9 to 1/3. $G = 1/9$ corresponds to an average spherical grain geometry and $G = 1/3$ to an average plate geometry.

It is of some interest to note that both formulations (33) and (34) may be expressed as:

$$b = 1 - (1 - \Phi) \left[\frac{A(\Phi)}{\beta\Phi + A(\Phi)} \right] \quad (35)$$

in which $A(\Phi)$ is a linear function of Φ . Both limiting values for b may be found out of this formulation. When the porosity tends towards zero b is equal to zero. When the porosity tends towards one b is equal to one.

In Figure 4 we have plotted our experimental results together with the theoretical bounds ($\nu_s = 0.3$). The vertical bars correspond to the standard deviation. We have plotted Miller's bound for 3 values of G . For $G = 1/9$ Hashin and Shtrikman's bound and Miller's bound are very similar. For $G = 1/3$ Miller's bound leads to a straight line. This curve cannot be accepted since it corresponds to a lower bound for b while it is strongly greater than the experimental value for $\Phi = 0.05$. Finally we plotted Miller's bound for $G = 1/3.206$ which fits with the experimental value for $\Phi = 0.05$ (Larrys carbonate). The curve underestimates all the other values. It leads to the best theoretical lower bound for our experiments.

Figure 4 : Theoretical bounds for $b(\Phi)$

IFP PHENOMENOLOGICAL MODEL

Let us assume that the rock may be described as an assembly of cubic blocks with Young's modulus E_s and Poisson's ratio ν_s . The space around the blocks constitute the pore volume. We assume that the cement between the blocks has a Poisson's ratio equal to 0 and a Young's modulus E_c . Let us then compute the bulk volume variation and the pore volume variation for a given confining pressure variation. We obtain:

$$\frac{\Delta V_b}{V_{bo}} = 3 \frac{\delta P_{conf}}{E_s} (1 - 2\nu_s)(1 - \Phi) + 3 \frac{\delta P_{conf}}{E_c} \Phi \quad (36)$$

$$\frac{\Delta V_p}{V_{bo}} = 3 \frac{\delta P_{conf}}{E_c} \Phi \quad (37)$$

In which V_{bo} is the initial bulk volume. The b coefficient is then given by:

$$b = \frac{\Delta V_p}{\Delta V_b} = \frac{\Phi}{\Phi + \frac{E_c}{E_s}(1 - 2\nu_s)(1 - \Phi)} \quad (38)$$

Note that Equation (38) is in agreement with Equation (35), in which:

$$A = \frac{E_c}{E_s}(1 - 2\nu_s) \quad \beta = 1 - A \quad (39)$$

In order to estimate the ratio $\frac{E_c}{E_s}$, we used Equation (38) for Larrys carbonate. The ratio is then 0.19. We kept this ratio constant for all the carbonates. The resulting curve is plotted on Figure 5. The theoretical curve fits reasonably well with the experimental data.

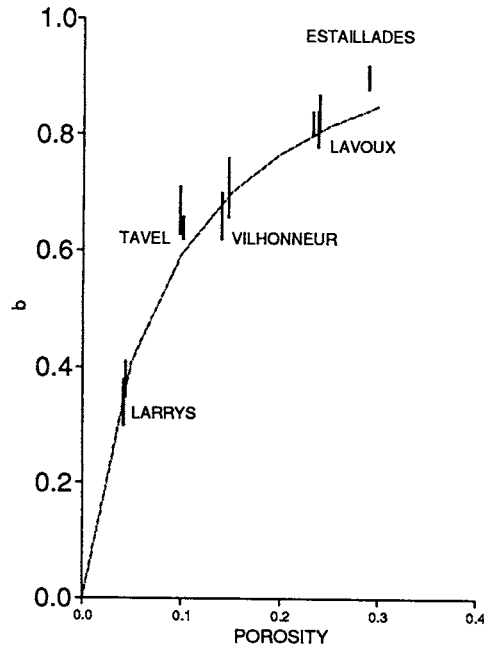


Figure 5 : IFP model for $b(\Phi)$

CONCLUSION

Using Biot's theory of poroelasticity we have shown that, depending on the mechanical assumption we obtain different expressions for the diffusivity coefficient of the pressure diffusivity equation as defined in reservoir engineering:

$$\sigma_{kk} = 0 \quad \frac{1}{K} = \frac{\eta}{k} \left[\frac{1}{M} + \frac{b^2}{K_d} \right] \quad (40)$$

$$\dot{\epsilon} = 0 \quad \frac{1}{K} = \frac{\eta}{k} \left[\frac{1}{M} \right] \quad (41)$$

Using the experimental results of Table 2, one may see that the two coefficients may differ by a factor of 2 for weak rocks (the Estailades carbonate for instance).

The relevant coefficients have been experimentally determined and the relationship between Biot's coefficient b and the carbonate porosity has been studied. A general relationship has been established (35) for the theoretical lower bound based on Hashin and Shtrikman's work and Miller's work:

$$b = 1 - (1 - \Phi) \left[\frac{A(\Phi)}{\beta\Phi + A(\Phi)} \right] \quad (42)$$

We finally derived a phenomenological model which leads to a similar expression (38) and fits with the experimental results:

$$A = \frac{E_c}{E_s}(1 - 2\nu) \quad \beta = 1 - A \quad \frac{E_c}{E_s} = 0.19 \quad (43)$$

NOMENCLATURE

- c_{f1} compressibility of the fluid (counted positively)
- c_{pp} matrix compressibility as defined in reservoir engineering practice
- c_s compressibility of the skeleton (grains + cement + occluded porosity) (counted positively)
- $e = \nabla \bullet u = tr(\epsilon_{ij}) =$ volumetric deformation (counted positively for a compression)
- k permeability
- K diffusivity
- $K_d = \frac{2}{3}\mu + \lambda_d$
- K_s incompressibility modulus of the skeleton
- $K_u = K_d + b^2M$
- M poroelastic coefficient
- M_d undrained poroelastic coefficient
- P_{pore} pore pressure
- P_{conf} confining pressure
- u displacement of the rock (positive for a compression)
- U displacement of the fluid (same sign as u)
- V_b bulk volume

∂V_{fl} fluid volume variation due to the fluid compressibility (positive for expelled fluid)

V_p pore volume

w relative displacement of the fluid in relation to the solid

b Biot's coefficient

$\varepsilon_{ij} = \frac{1}{2} [u_{i,j} + u_{j,i}]$

λ_d Lamé's coefficient for drained conditions

λ_u Lamé's coefficient for undrained conditions

μ shear modulus

ν_s Poisson's ratio (drained)

Φ open (or connected) porosity

σ_{ij} stress (positive for a compression)

ζ variation of fluid content (positive for expelled fluid)

η fluid viscosity

REFERENCES

BIOT M.A. (1941) - General Theory of Three Dimensional Consolidation, *J. Appl. Phys.*, 12, 155-164

BIOT M.A. (1962) - Mechanics of Deformation and Acoustic Propagation in Porous Media, *J. Appl. Phys.*, 33, 1482-1498

BOUTECA M., SARDA J.P. (1990) - Fluid Flow in Porous Media and Related Rock Mechanics Problems. In *2nd European Conference on the Mathematics of Oil Recovery*, edited by D. guérillot and O. Guillon, pp. 205-210 Technip

COUSSY O. (1989) - Thermomechanics of Saturated Porous Solids in Finite Deformation. *European Journal of Mechanics, A/Solids*, vol. 8, n. 1, 1-14

GEERSTMA J.A. (1957) - Problems of Rock Mechanics in Petroleum Production Engineering. *Trans. AIME and Petro.*, TP 4678, vol 210

HASHIN Z., SHTRIKMAN S. (1961) - Note on a variational approach to the theory of composite elastic materials, *J. Franklin Inst.*, 271, 336-341

LAURENT J., QUETTIER L. (1989) - Comportement des milieux poreux consolidés dans le domaine élastique, *Rock at Great Depth*, edited by D. Fourmaintraux and V. Maury, 2, 915-921, Balkema.

LAURENT J., BOUTÉCA M. and SARDA J.P. (1990) - Pore Pressure Influence in the Poroelastic Behaviour of Rocks: Experimental studies and Results. *paper SPE 20922*, Europec 90.

MILLER M. N. (1968) - Bounds for Effective Bulk Modulus of Heterogeneous Materials. *J. Math. Phys.*, vol. 10, N. 11, pp 2005-2013

WATT J.P., DAVIES G. F., O'CONNELL R.J. (1976) - The Elastic Properties of Composite Materials. *Reviews of Geophysics and Space Physics* Vol. 14, N. 4, 541-563

ZIMMERMAN R.W., SOMERTON W.H. and KING M.S. (1986) - Compressibility of Porous Rocks. *J. of Geophys. Res.*, 91, B12, 12,765-12777.

Electrical Properties
