NON-DARCY FLOW IN CORE PLUGS: A PRACTICAL APPROACH

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Abstract Rapid turnaround in conventional core analysis usually demands only single point gas permeability measurements. The flow rate and mean pressure are often set arbitrarily, and the data are reported without correction for slippage. To perform the Klinkenberg correction properly, the single point permeability measurement must not be significantly affected by non-Darcy flow. A permeability-dependent limit has been put on the flow rate used in a test to ensure this. The criterion for the onset of significant non-Darcy pressure loss is defined by the purposes of the measurement: the Forchheimer equation is a continuous function, with no 'critical' flow rates to signal transition from one flow regime to another. Results from a UKCS gas reservoir are presented to illustrate the method, on a facies basis.

NON-DARCY FLOW IN CONVENTIONAL PERMEABILITY MEASUREMENT

The work presented in this paper resulted from efforts to find a method to transform routine core analysis permeability (k_g) into equivalent in-situ permeability for the Morecambe Bay gas reservoirs. The difference between the two values is a result of contrasting conditions: routine k_g is measured in a dry state ($S_w = 0$) under low confining stress; reservoir permeability, however, is a function of an increased stress state and the relevant water saturation, S_w.

To achieve this, a correction factor (CF) is required, which is a function of permeability and is defined as:

$$CF = \frac{\text{Permeability at restored overburden stress and } S_w}{\text{Routine } k_g} \qquad (1)$$

However, routine k_g depends on mean pore pressure (Klinkenberg, 1941) and is therefore a function of its abitrary selection by an operator. In order to overcome this problem, it was decided to make the transformation in two stages. Firstly, the routine k_g is extrapolated to an equivalent liquid permeability (k_l). Then this Klinkenberg permeability is converted to a restored state value using a function in the form of equation 1. The transform function is determined by measuring the Klinkenberg permeability under restored conditions for a set of plugs, along with their k_l in a normal, routine dry state at a net confining pressure of 400 psi.

The South Morecambe reservoir in the Irish Sea Basin has a crest at only -2300 ft ss with a gas water contact at -3750 ft ss, so the restoration of reservoir net overburden (~1770 psi) is relatively straightforward. With routine core plugs being taken at intervals of around one per foot, however, large numbers are turned around in a short period of time, necessitating a rapid way of converting k_g to k_l. This is done by means of a slip factor correlation and the Klinkenberg equation:

$$k_g = k_l \left(1 + \frac{b}{P_m}\right) \qquad (2)$$

Therefore, knowing the mean pore pressure (P_m) at which k_g is measured, and estimating b from a correlation with permeability, k_l can be determined in an iterative fashion.

Unfortunately, this technique is invalid if the initial k_g value is significantly affected by non-Darcy effects. A procedure to ensure that non-Darcy effects are insignificant had to be found.

Forchheimer Equation

Forchheimer (1901) presented an extension to the Darcy equation, shown here in its differential form:

$$-\frac{dP}{dl} = \frac{\mu v}{k} + \beta \rho v^2 \qquad (3)$$

This equation states that the pressure drop per unit length of porous medium has two components. The first term on the right hand side represents the viscous, or Darcy, pressure drop and includes a proportionality constant k, the permeability. The second term describes the pressure drop due to kinetic, or non-Darcy, effects. These are caused by deviation from capillary flow, for example as a result of pore tortuosity and aspect ratio. The second term also contains a proportionality constant β, which represents to the non-Darcy term what k represents to viscous flow. It is a constant for a given medium, in a given saturation and stress condition, like permeability.

Some authors have suggested that it is a function of the flowing medium (Tiss and Evans, 1989), but this need not be included in the subsequent analysis since nitrogen gas is used throughout.

Often, when the velocity is low, the second term is much smaller than the first and essentially Darcy's Law applies. This is also the case when viscosity is high, as in liquid flow. However, we require to know the conditions under which the second term becomes significant in order to ensure that a gas permeability can be measured without its undue influence.

Methods exist for measuring β (Dranchuk and Kolada, 1968) and indeed correlations exist relating it to permeability (Noman et al, 1985). Recently, however, confusion has come to light over the point at which non-Darcy effects become significant in equation 3.

The authors (Noman and Kalam, 1990) assert that the 'critical' Reynolds number at which non-Darcy effects become significant is a function of rock properties. In addition, they and others (Tiss & Evans, 1989) become unduly concerned about the meaning of the Reynolds number in porous media and what should be considered the 'characteristic length'. Its meaning has been the subject of considerable debate, with some even offering an average grain diameter as a candidate (Green and Duwez, 1951). The reason for such conjecture is the extension of classical fluid flow in large diameter pipes to flow in porous media. The crucial difference is that the mean free path of gas flowing in a large diameter conduit is insignificant in comparison to the flow area. This is not the case with gas flow in a porous medium. The flow structures present on the macro scale do not occur when gas is flowing through rock pores: the distance between the molecules is significant with respect to the size of the conduit. The problem is a simpler one, however.

'Critical' Rate Criterion

Rearranging equation (3) gives:

$$-\frac{dP}{dl} = \frac{\mu v}{k}\left(1 + \frac{\beta \rho v k}{\mu}\right) \tag{4}$$

All this manipulation has done is to take out a common factor, such that the relationship of the second term to the first in equation 3 is identical to that of the group $\beta \rho v k/\mu$ to unity in equation 4. Since this group closely resembles a Reynolds number, many have fallen into the trap of reading too much into the term (βk). This is a red herring. The name we give to the group in parentheses in equation 4 is not important - it is only an indicator of the relative magnitude of non-Darcy and Darcy contributions to the pressure gradient. Indeed each of the terms in the dimensionless group - call it modified Reynolds number if you will - is measurable:

$$Re' = \frac{\beta \rho v k}{\mu} \tag{5}$$

Given this definition, it is evident that equation 4 becomes:

$$-\frac{dP}{dl} = \frac{\mu v}{k}(1 + Re') \tag{6}$$

The crux of our problem is that we have to determine when non-Darcy effects become *significant*, i.e. when Re' becomes significant in comparison to unity. When Re' becomes significant in equation 6, then there is a *significant* non-Darcy contribution to the pressure gradient. Hence the definition of 'significance' defines the 'critical' Reynolds number at which *significant* non-Darcy flow occurs.

The use of the word 'critical' implies a discontinuity in the pressure gradient/flowrate relationship. It is accepted by many, however, that the Forchheimer equation (equation 3) describes the gradually increasing influence of the non-Darcy term over the whole range of flow velocities (Dullein, 1979; Ahmed and Sunada, 1969). Since non-Darcy effects can result from deviations from streamline flow, for example (Noman and Archer, 1987; Ahmed and Sunada, 1969), it is concluded that a continuous relationship applies for flow in porous media.

The consequence of this reasoning is that the definition of significance is subjective. A degree of non-Darcy contribution to the pressure gradient which is acceptable for one application may not be in another. If, for example, the tolerance of non-Darcy contribution for our present purposes - routine k_g measurement - is 1%, then Re'_c = 0.01. In light of this, it is better to term this definition the 'limiting' condition as it is imposed for a set of standards defined by the purposes of the measurement.

Since the velocity term in equation 5 is the Darcy velocity, Q/A, then the limiting flowrate which will ensure that the contribution of non-Darcy effects to the pressure gradient is 1% is given by:

$$Q_L = \frac{0.01 A \mu}{\beta \rho k} \tag{7}$$

The dimensions of a plug can be measured; fluid physical properties can be measured or estimated from published correlations.

However, for our applications, k is initially unknown and it is not practical to measure β each time a plug sample is tested. Therefore, use can be made of correlations which relate β to permeability (and perhaps porosity). These are generally of the form:

$$\beta = ak^c \qquad (8)$$

and examples have been published in the literature (Firoozabadi and Katz, 1979; Tek et al,1962; Noman et al, 1985; Jones, 1987; Tiss and Evans, 1989). Substituting equation 8 into equation 7 and using a general variable T to denote the chosen tolerance yields:

$$Q_L = \frac{TA\mu}{a\rho k^{(c+1)}} \qquad (9)$$

As a result, knowledge of the physical properties of the system in addition to the permeability allows the estimation of the appropriate limiting flow rate for a plug.

When measuring a single point k_g, however, the knowledge of permeability is only approximate. However, given the error involved in using β vs k correlations, then equation 9 is sufficient provided the tolerance criterion, T, is stringent enough.

For nitrogen at standard conditions, μ= 0.017 cP and ρ= 1.251 kg m^{-3}.

Using a 1% tolerance limit, and converting for units transforms equation 9 into (for 1" diameter plugs):

$$Q_L = \frac{1.28 \times 10^{12}}{ak^{(c+1)}} \qquad (10)$$

(k in mD; Q_L in $cm^3 min^{-1}$). This expression will be used later to illustrate limiting flowrate for Morecambe Bay plug samples.

EXPERIMENTAL WORK

While published β vs. k correlations are available, it is much better to use functions which are field-specific. To that end, non-Darcy coefficients have been measured on forty-six plug samples from Morecambe Bay. The sample set reflects the relative importance and occurrence of certain facies present in the Sherwood reservoir sequence in the area. The main facies elements present are channel sands (coded A), sheetflood deposits (C) and aeolian sand (F). In addition, diagenetic platy illite is present in some samples, resulting in the six groups represented in the sample set (Table 1, below). The presence of illite is known to cause a decrease in permeability of up to two orders of magnitude.

TABLE 1 Morecambe Bay study sample set

	Number of samples		
Facies	Illite Free	Illite Affected	Total
Channel (A)	9	10	19
Sheetflood (C)	7	6	13
Aeolian (F)	6	8	14
	22	24	46

All of the plugs used in this study had one inch nominal diameter, as is common for routine permeability samples. The length and diameter of each sample was measured in addition to the atmospheric pressure. A net confining pressure of 400 psi was used. A series of at least six upstream pressures was applied to each sample, with the outlet at atmospheric pressure. This method was employed to maximise the flowrate at each mean pore pressure, to achieve the maximum non-Darcy pressure loss. Flow and pressure measurements were taken when both parameters were stable: equilibrium could take over an hour for tight samples (<0.5 mD). A flow diagram of the rig used in the study is shown in Figure 1.

The data analysis employed software which supplies Klinkenberg permeability (k_l), slip factor (b) and non-Darcy coefficient (β) as output. It uses an iterative form of the Dranchuk and Kolada analysis (1968). It should be noted, however, that reliable results require the presence of significant non-Darcy effects and smooth data. The need for great care and accuracy in data acquisition is imperative.

Results of Morecambe Bay Study

The results of the Morecambe Bay study are shown in Table 2. Correlations of slip factor (b) and non-Darcy coefficient (β) are given in Table 3. The coefficients of the regression equations correspond to the following forms:

$$\log \beta = a_1 + a_2 \log k \quad (11)$$

$$\log \beta = a_3 + a_4 \log k + a_5 \log \phi \quad (12)$$

$$\log b = a_6 + a_7 \log k \quad (13)$$

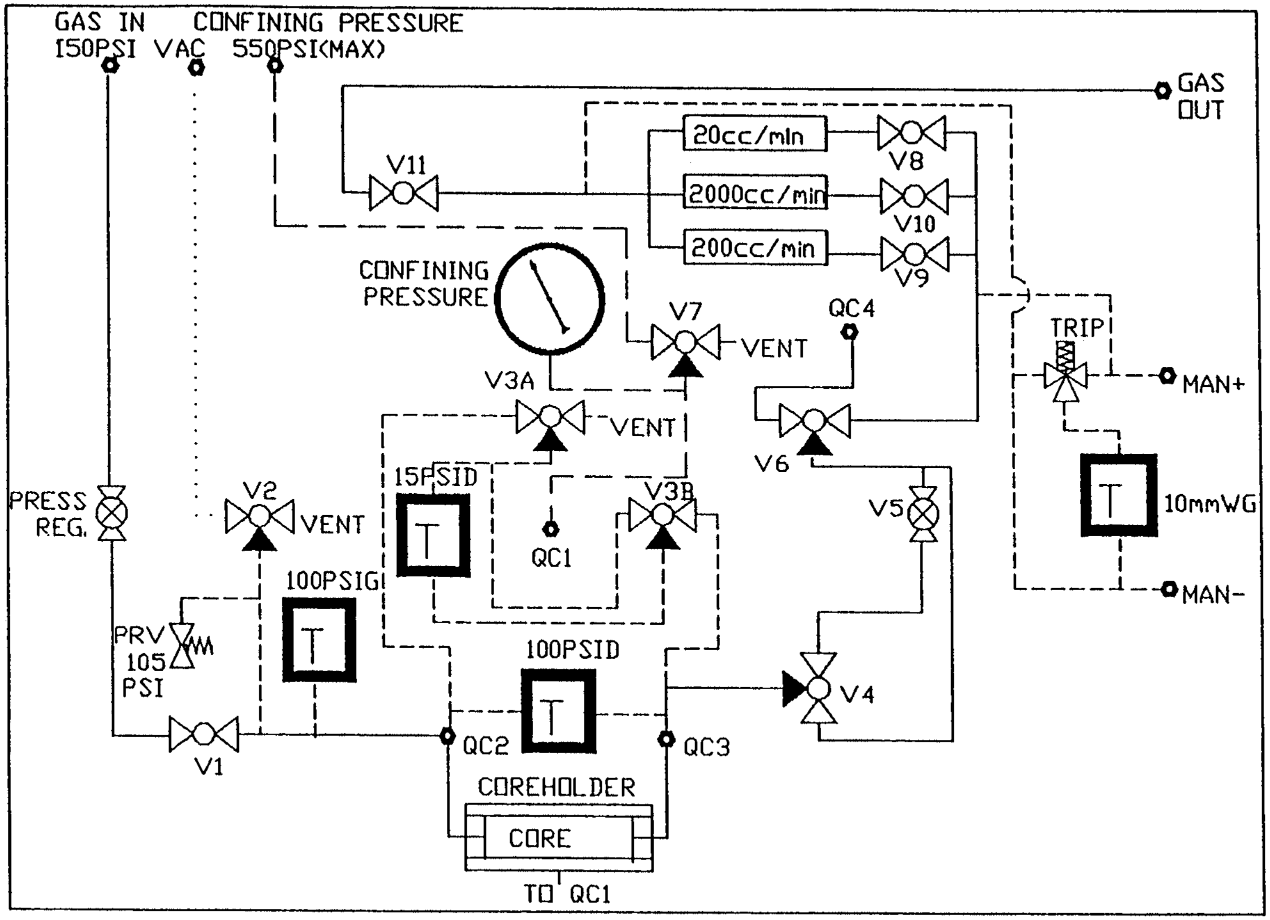

FIGURE 1 Experimental rig (Courtesy EPS)

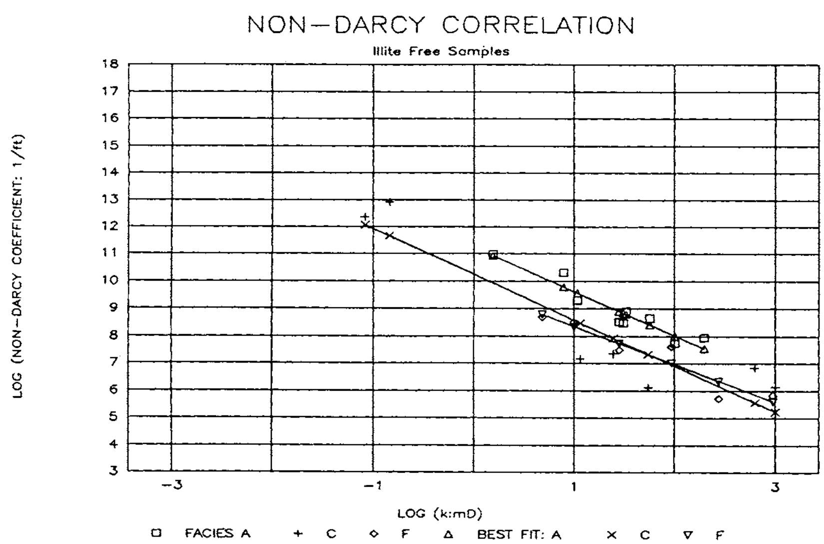

FIGURE 2 Non-Darcy correlation by facies

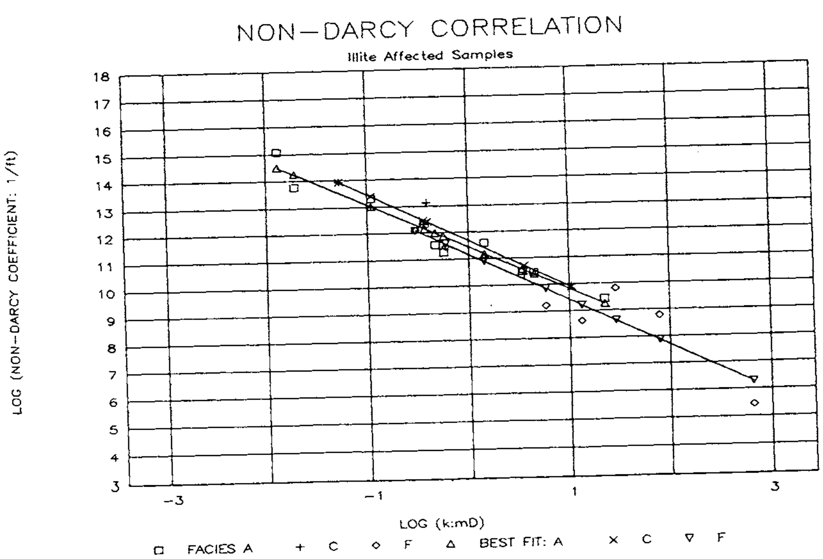

FIGURE 3 Non-Darcy correlation by facies (illite-affected samples)

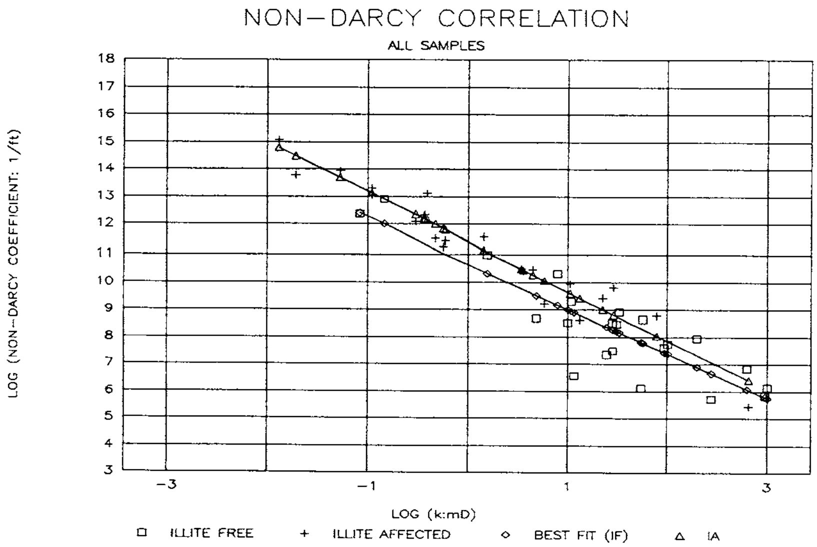

FIGURE 4 Illite-free versus illite-affected

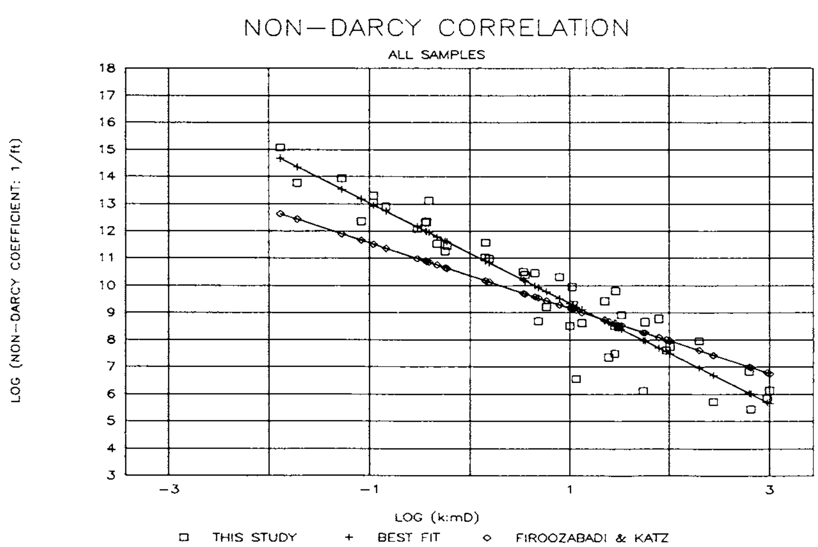

FIGURE 5 Comparison of this study with Firoozabadi correlation

$$\log b = a_8 + a_9 \log k + a_{10} \log \phi \tag{14}$$

The corresponding correlation co-efficients are denoted R_1 to R_4 respectively.

Comparison of R_1 with R_2 and R_3 with R_4 shows that the inclusion of porosity does little to improve each correlation. This may be due to the narrow range of porosity in comparison with permeability. Therefore only equations of the form of (11) and (13) above will be discussed further. There is a remarkable consistency in the value of a_2, indicating that the β/k relationship is applicable on a field-wide basis. This is illustrated better by examining Figures 2 to 4.

Figure 2 shows that in general, β is greater for facies A (Channel sand) illite free material, followed by facies C (sheetflood) then facies F (Aeolian) rock. Figure 3 shows that the original depositional environment is of little importance as far as the β vs k relationship is concerned: the presence of platy illite dominates its behaviour. Figure 4 shows that illite affected material has a higher non-Darcy coefficient than illite free. This would be expected, but the magnitude of the difference is small. It is shown to be insignificant when the data from this study is compared to a correlation from the literature (Firoozabadi and Katz, 1979), Figure 5.

The slip factor comparison illustrated in Figures 6 and 7 may be heavily influenced by scatter on the data: the regression coefficients are worse than for the β vs k relationships. A comparison of illite free and illite affected results (Figure 8) shows little difference. Indeed, the correlation for the whole sample set is very similar to one from the literature (Jones, 1972), Figure 9. It would be prudent to use the field-wide correlation for slip factor until more data is available on a facies basis.

The study has therefore yielded correlations on a facies-grouped basis. However, the field-wide equations are:

$$\beta = 1.56 \times 10^{11}\ k_l^{-1.84} \tag{15}$$

$$b = 8.4\ k_l^{-0.347} \tag{16}$$

Inserting the constants 'a' and 'c' from equation (15) into equation (10) gives:

$$Q_L = 8.28\ k_l^{0.844} \tag{17}$$

which is the limiting flowrate/permeability relationship for 1" diameter

TABLE 2 Morecambe Bay Experimental Results

FACIES	ILLITE?	PLUG No.	ϕ	k (mD)	b (psi)	β (ft^{-1})
A	Y	794	0.134	1.45	6.75	3.81E+11
		795	0.110	4.41	4.04	2.86E+10
		798	0.075	0.019	34.1	5.76E+13
		799	0.056	0.474	7.28	3.41E+11
		800	0.099	0.013	63.3	1.21E+15
		857	0.124	3.39	5.78	3.16E+10
		864	0.129	0.37	7.14	2.21E+12
		867	0.097	22.6	1.09	2.76E+09
		872	0.152	0.11	13.1	1.98E+13
		873	0.159	0.57	10.5	1.79E+11
	N	227	0.096	30.8	3.35	3.01E+08
		232	0.095	1.56	5.54	9.58E+10
		235	0.107	100	1.30	5.61E+07
		241	0.104	197	2.04	9.02E+07
		291	0.082	32.9	1.54	8.52E+08
		299	0.094	28.2	6.15	3.30E+08
		304	0.077	7.87	3.28	2.11E+10
		305	0.094	56.4	3.16	4.60E+08
		314	0.083	10.8	3.23	1.97E+09
C	Y	683	0.166	10.6	5.12	9.17E+09
		684	0.124	0.053	11.3	8.62E+13
		685	0.148	0.36	6.61	2.12E+12
		686	0.062	0.39	8.25	1.31E+13
		691	0.198	3.56	8.06	2.50E+10
		698	0.144	0.11	11.5	1.08E+13
	N	65	0.220	1000	1.66	1.40E+06
		69	0.126	630	0.51	6.92E+06
		150	0.085	0.083	9.98	2.28E+12
		152	0.118	24.6	1.83	2.28E+07
		196	0.068	0.146	52.0	7.86E+12
		198	0.119	11.6	3.69	1.47E+07
		262	0.105	54.9	0.69	1.35E+06
F	Y	640	0.257	13.2	15.3	4.29E+08
		643	0.161	0.60	12.9	2.81E+11
		646	0.171	5.80	8.29	1.68E+09
		647	0.124	0.30	17.6	1.19E+12
		648	0.166	1.42	15.4	1.06E+11
		1046	0.177	78.5	1.87	6.15E+08
		1047	0.220	648	0.91	2.87E+05
		1059	0.156	29.0	0.92	6.39E+09
	N	60	0.114	28.3	3.85	3.19E+07
		62	0.161	276	0.21	5.38E+05
		64	0.112	4.82	13.5	4.99E+08
		140	0.102	10.0	5.98	3.43E+08
		220	0.167	92.6	4.33	4.23E+07
		266	0.222	945	0.73	6.85E+05

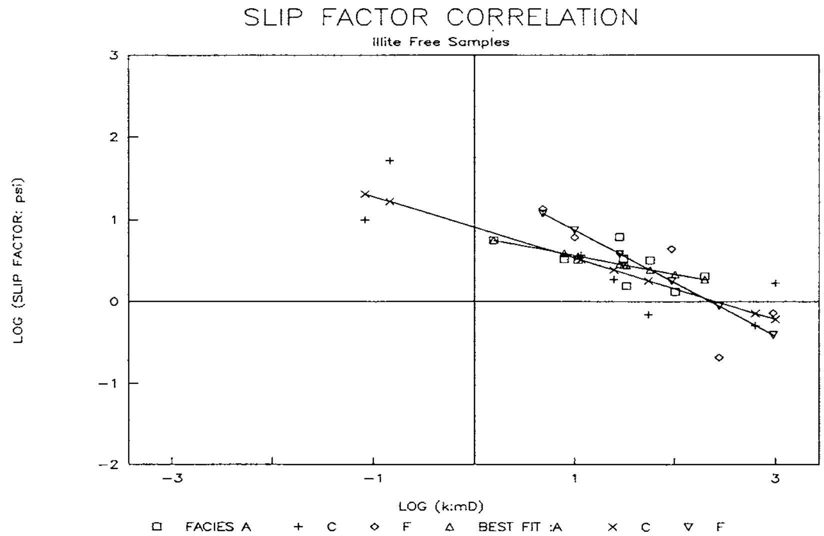

FIGURE 6 Slip correlation, facies basis (illite-free samples)

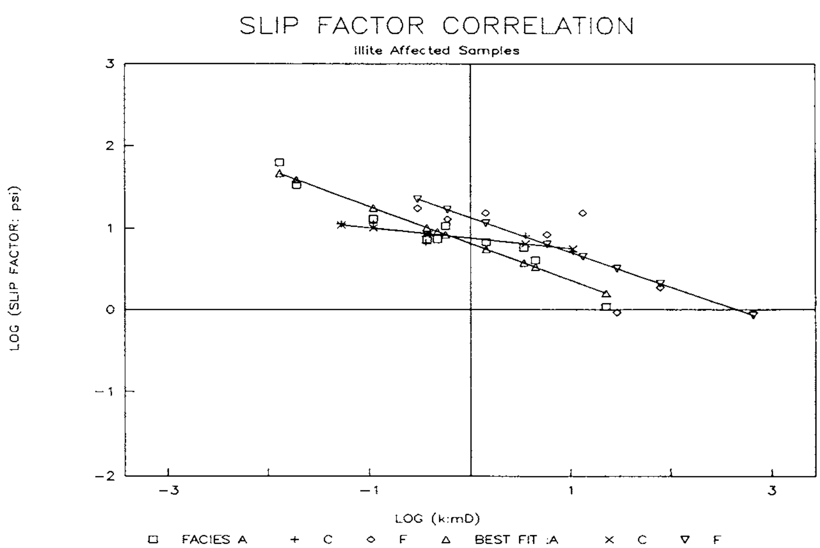

FIGURE 7 Slip correlation, facies basis (illite-affected samples)

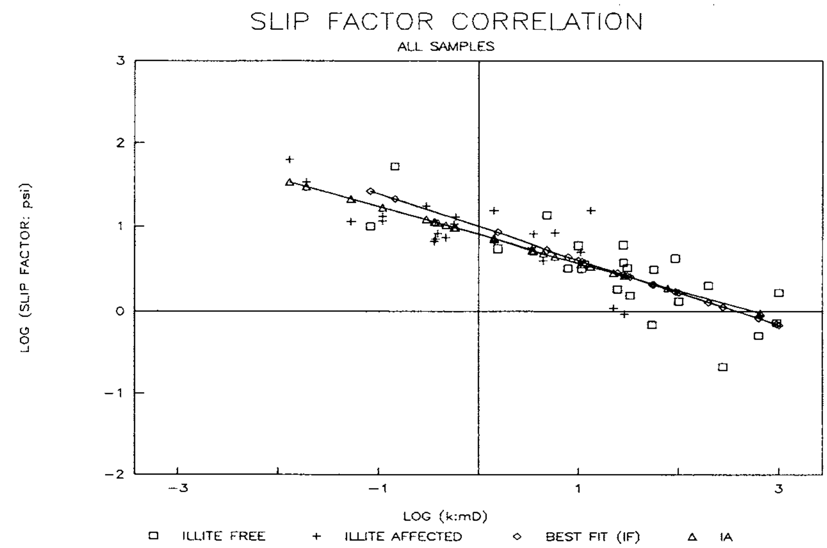

FIGURE 8 Comparison of illite-free and affected samples

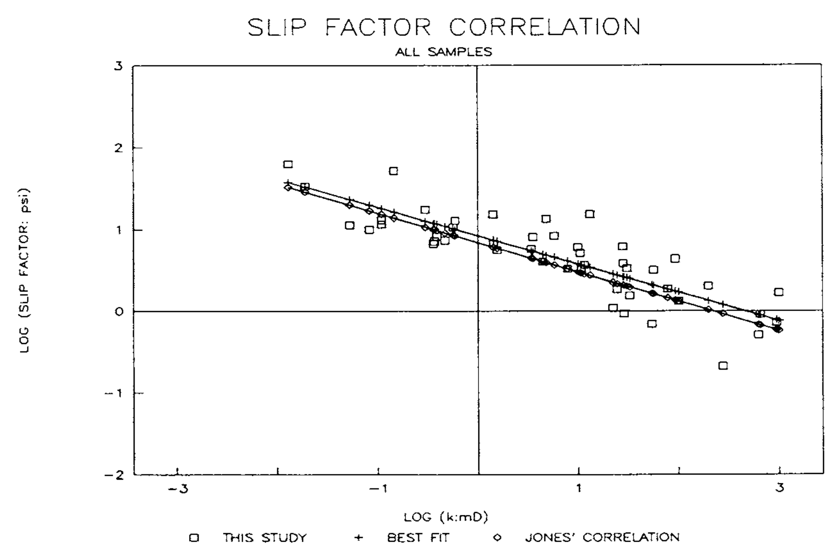

FIGURE 9 Comparison of this study with Jones correlation

plugs with a 1% non-Darcy tolerance for gas permeability measurement using nitrogen gas. This correlation is illustrated in Figures 10 to 12, for high, medium and low permeability ranges.

GENERAL APPLICATIONS

Routine Permeability Measurement

The correlation can be used in the following way:

(i) Measure gas permeability kg at some mean pore pressure, P_m, and flowrate, Q.

(ii) Calculate Q_L from equation 17 and compare to Q. If it is greater, then remeasure k_g using a lower flowrate and repeat.

(iii) Having measured a k_g with insignificant non-Darcy effects, convert to k_l.

(a) Estimate slip factor from equation 16 using k_g as starting point.

(b) Estimate Klinkenberg permeability from:

$$k_l = \frac{k_g}{(1+b/P_m)} \tag{18}$$

(c) Re-estimate b using k_l from equation 18. If it is close to that in (a) then stop. If not, repeat from step (b) until convergence is reached.

When k_l has converged satisfactorily, then we have determined the value to be used in the correction factor procedure described earlier.

Klinkenberg Permeability Measurement

Although the application of equation 9 has been used to illustrate the specific situations of Morecambe Bay routine k_g and correction factor measurements, it can also be used to ensure that each point in a multiple point Klinkenberg permeability test is not significantly affected by a non-Darcy contribution to the pressure gradient.

CONCLUSIONS

1. The Forchheimer equation is a continuous function. The point at which non-Darcy effects become significant is a function of the

TABLE 3 Correlation parameters

FACIES	ILLITE ?	a1	a2	a3	a4	a5	a6	a7	a8	a9	a10	R1	R2	R3	R4	n
A	Y	11.45	-1.622	12.36	-1.644	0.941	0.812	-0.453	1.066	-0.495	0.265	0.973	0.976	0.962	0.965	10
	N	11.23	-1.601	8.04	-1.484	-2.927	0.789	-0.230	1.922	-0.272	1.037	0.945	0.952	0.634	0.661	9
C	Y	11.69	-1.762	9.67	-1.573	-2.349	0.879	-0.128	0.976	-0.137	0.113	0.970	0.998	0.833	0.844	6
	N	10.26	-1.675	10.29	-1.678	0.031	0.905	-0.375	2.484	-0.503	1.511	0.906	0.906	0.862	0.881	7
F	Y	11.18	-1.740	6.51	-1.429	-5.783	1.128	-0.427	3.928	-0.613	3.471	0.938	0.961	0.847	0.963	8
	N	9.75	-1.386	15.81	-2.162	5.514	1.519	-0.647	7.234	-1.379	5.204	0.943	0.963	0.849	0.920	6
A	ALL	11.40	-1.679	12.21	-1.672	0.821	0.890	-0.338	0.961	-0.337	0.071	0.984	0.985	0.911	0.911	19
C	ALL	11.06	-1.944	12.35	-2.013	1.363	0.838	-0.322	1.076	-0.325	0.253	0.930	0.932	0.857	0.859	13
F	ALL	10.91	-1.801	12.88	-1.915	2.287	1.201	-0.482	1.856	-0.519	0.761	0.933	0.940	0.845	0.855	14
ALL	N	10.63	-1.625	6.43	-1.270	-3.858	0.998	-0.388	1.339	-0.417	0.314	0.882	0.900	0.799	0.801	22
ALL	Y	11.42	-1.778	10.40	-1.696	-1.150	0.906	-0.330	1.643	-0.389	0.835	0.966	0.969	0.835	0.873	24
ALL	ALL	11.19	-1.844	11.08	-1.839	-0.119	0.923	-0.347	1.269	-0.363	0.367	0.947	0.947	0.845	0.851	46

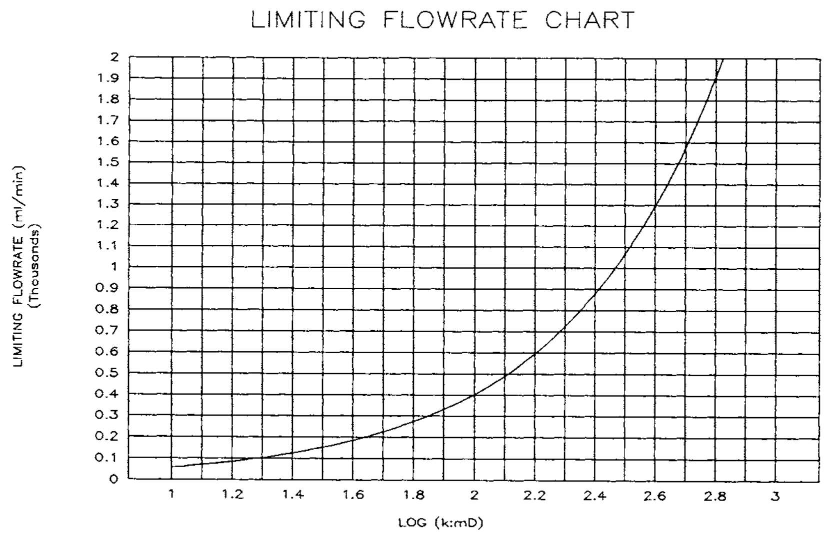

FIGURE 10 Limiting flow rate chart (high permeability)

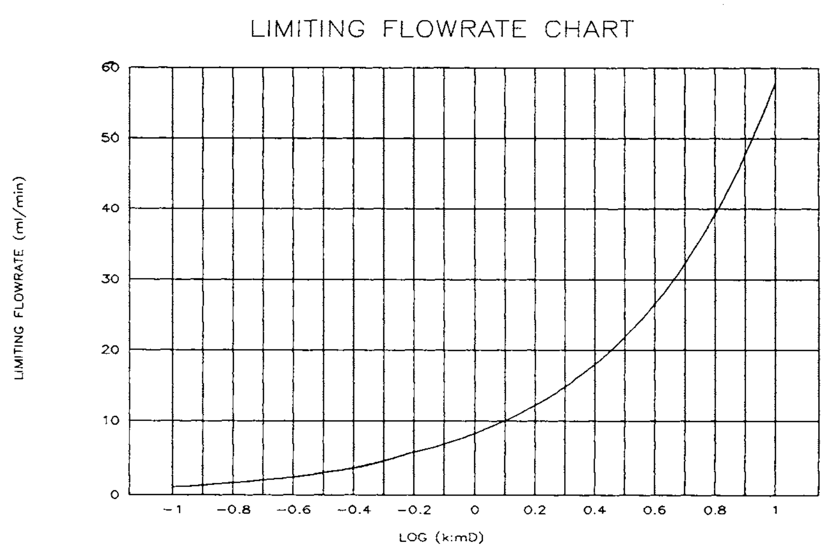

FIGURE 11 Limiting flow rate chart (medium permeability)

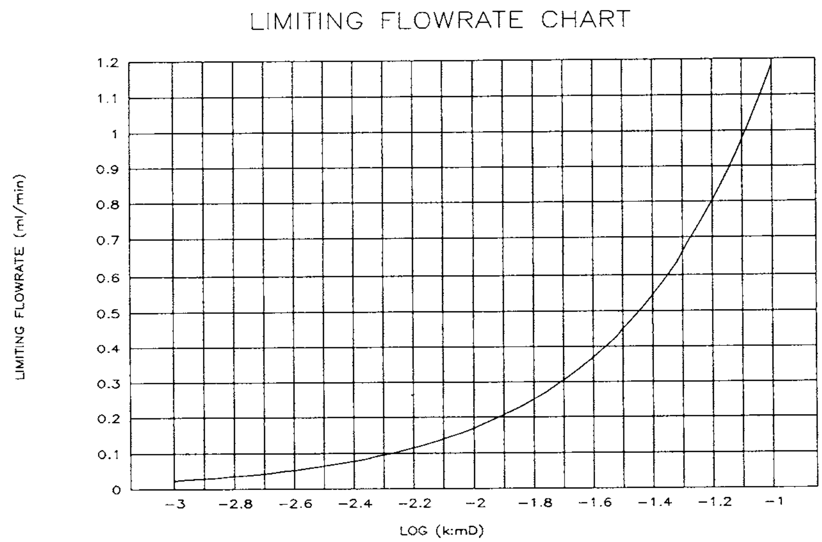

FIGURE 12 Limiting flow rate chart (low permeability)

accuracy of the measurement and the tolerance of such effects. There is some pressure drop due to non-Darcy effects across the whole range of flow rates.

2. A procedure has been presented which ensures valid single-point gas permeabilities that are not significantly affected by non-Darcy effects.

3. A correlation has been formulated and represented graphically which relates plug permeability to limiting flowrate for Morecambe Bay samples.

4. For the Morecambe Bay samples, pore geometry variations - represented by facies/diagenetic grouping - showed surprisingly little influence over the relationships obtained. Porosity was also shown to be of secondary importance for this field.

5. The technique demonstrated here can be applied to conventional core measurements for other fields, ideally at an early stage in field appraisal.

6. Backpressure should be used during Klinkenberg permeability measurements to obtain a range of mean pore pressures, rather than using increasingly high flowrates.

ACKNOWLEDGEMENTS

The authors would like to thank the management of British Gas for their help and permission to publish this paper.

NOMENCLATURE

A	Cross-sectional area, cm^2
b	Slip factor, psi
CF	Correction Factor, dimensionless
k	Permeability, mD
k_g	Single point gas permeability, mD
k_l	Equivalent liquid (Klinkenberg) permeability, mD
l	Length, cm
P	Pressure, psia
P_m	Mean pore pressure, psi
Q_L	Limiting flowrate, $cm^3\ min^{-1}$
Re'	Modified Reynolds number, dimensionless
S_w	Water saturation, fractional
T	Tolerance of non-Darcy effects, fractional
v	Darcy velocity, cm min^{-1}

Greek Letters

β	Non-Darcy coefficient, ft^{-1}
ϕ	Porosity, fractional
μ	Viscosity, cP
ρ	Density, kg m^{-3}

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