Capillary Pressure From Centrifuge —A New, Direct Method

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Abstract

Centrifuge data from short core samples can be interpreted by a direct, non-iterative method. For larger samples, the increase in centrifugal force along the core leads to a Volterra integral equation. Direct numerical solution methods are scarce, unless the data are forced into an assumed functional form of the capillary pressure curve.

This paper presents a method for solving the Volterra equation directly by a modified midpoint procedure, without iterations, numerical differentiation, curve fitting, or any assumed functional form. The method has been verified by comparison with other procedures, interpretation of artificially generated data, and by measuring the capillary pressure curves of the same nine cores with the three standard procedures.

The method is demonstrated on data from forced imbibition experiments. It is not overly sensitive to the number of datapoints, but a lower limit of 5-6 is suggested.

Introduction

Hassler and Brunner [1] formulated the problem of deriving the capillary pressure curve from centrifuge data. They presented a method for interpretation of measured data from short cores and suggested an iteration procedure for long cores. Their method has been widely used, but is not quite satisfactory. For short cores, the differentiation of experimental data may lead to errors and for long cores, the iteration procedure entails numerical integration and differentiation for each step.

Many authors have suggested improvements and tried to formulate a more direct solution of the centrifuge equation, Ayappa *et al.* [2], Glotin *et al.* [3], and references therein.

Hermansen [4] recognized the problem as a Volterra integral equation of the first kind and examined published solution procedures to solve the centrifuge equation. Several sets of experimental data were interpreted to investigate the practicality of the methods. The main results of his work are included in the present paper.

Several authors have questioned the validity of centrifuge equation itself, for example the boundary condition of zero capillary pressure at the outlet end. We do

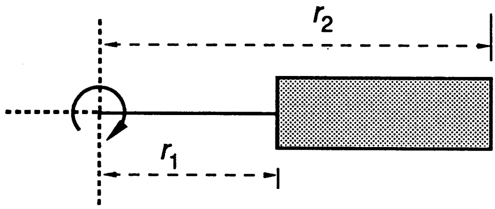

Figure 1: Centrifuge schematic

not address these problems and limit our scope to solve the classic Hassler-Brunner centrifuge equation for long cores.

Theory

Centrifuge Equation

Referring to Fig. 1, the capillary pressure at a radius r: $r_1 \le r \le r_2$, is given by [2]

$$p_c = \tfrac{1}{2}\Delta\rho\omega^2(r_2^2 - r^2), \tag{1}$$

and the capillary pressure at the inlet end at r_1 is

$$p_{c1} = \tfrac{1}{2}\Delta\rho\omega^2(r_2^2 - r_1^2). \tag{2}$$

The average saturation is defined by

$$\overline{S} = \frac{1}{r_2 - r_1}\int_{r_1}^{r_2} S(r)dr. \tag{3}$$

Substituting p_c from Eq. 1 and simplifying gives the centrifuge equation that has to be inverted based on measured data:

$$\overline{S}(p_{c1}) = \frac{1+f}{2p_{c1}}\int_0^{p_{c1}} \frac{S(p_c)\,dp_c}{\sqrt{1 - \frac{p_c}{p_{c1}}(1-f^2)}}, \tag{4}$$

where $f = r_1/r_2$, $p_{c1} = p_c(r_1)$, and $p_{c2} = 0$.

Eq. 4 is a Volterra integral equation of the first kind. It is well known as an ill-posed problem, causing numerical instabilities, and higher order numerical methods will diverge, Linz [5]. The equation may be reformulated into a more stable Volterra equation of the second kind. This procedure is not recommended, however, since the measured data then have to be differentiated, with the possibility of introducing large errors.

The centrifuge is used to measure negative capillary pressure during forced imbibition, as in the USBM-method [6] for wettability determination. A waterwet, oil saturated core, after water has spontaneously imbibed, is placed in an inverted bucket, and more water is forced into the core by centrifugation. It is easily shown that the form of Eq. 4 is still the same, provided the following substitutions are made: $f \to r_2/r_1$, and $p_{c1} \to p_{c2}$. Oil is produced at r_1 where $p_{c1} = 0$, and the capillary pressure p_{c2} is now a negative number.

Modified Hassler and Brunner Procedure

In this paper is given a direct numerical method for solving the Volterra equation, Eq. 4. Several authors have tried to modify the equation, as discussed by Ayappa *et al.* [2]. We have modified the Hassler and Brunner [1] procedure, in order to have alternate ways of interpreting the data and to validate the direct Volterra method.

Eq. 4 may be reformulated as an iteration procedure, following Hassler and Brunner[1],

$$S = S_1 + S_2 + S_3 + \cdots + S_i + S_{i+1} + \cdots, \tag{5a}$$

$$S_1(p_{c1}) = \frac{d}{dp_{c1}}\left[p_{c1}\cdot\overline{S}(p_{c1})\right], \tag{5b}$$

$$S_{i+1}(p_{c1}) = \frac{d}{dp_{c1}}\int_0^{p_{c1}}\left[1-\frac{1+f}{2\sqrt{1-\frac{p_c}{p_{c1}}(1-f^2)}}\right]S_i(p_c)\,dp_c, \quad \text{for } i=1,2\cdots. \tag{5c}$$

Differentiation of the integral in Eq. 5c, gives

$$S_{i+1}(p_{c1}) = (1-\frac{1+f}{2f})\cdot S_i(p_{c1}) + \frac{(1+f)(1-f^2)}{4p_{c1}^2}\int_0^{p_{c1}}\frac{p_c\,S_i(p_c)\,dp_c}{\left[1-\frac{p_c}{p_{c1}}(1-f^2)\right]^{3/2}}. \tag{5d}$$

This version requires differentiation only during the first iteration, while in the standard Hassler and Brunner procedure, integration and differentiation have to be performed during each iteration. Since numerical differentiation is a well known source of error, the modified procedure is presumably more stable.

We have tested the modified procedure extensively on the datasets reported in this work. It is almost as accurate as the Volterra procedure, but slightly more sensitive to the number of datapoints.

Direct Numerical Solution

Eq. 4 may be solved directly by the *modified midpoint method*, as suggested by Anderssen and White [7]. Let n: $1 \le n \le N$ enumerate the centrifuge frequencies in a measurement series of N points. Also, let i: $0 \le i \le n$ denote an interval counter, for a given n. The integral in Eq. 4 may now be represented by a sum of integrals over the measured intervals,

$$\overline{S}(p_{c1,n}) = \frac{1+f}{2p_{c1,n}}\cdot\sum_{i=1}^{n}\int_{p_{c1,i-1}}^{p_{c1,i}}\frac{S(p_c)dp_c}{\sqrt{1-\frac{p_c}{p_{c1,n}}(1-f^2)}} \tag{6}$$

$$\approx \frac{1+f}{2p_{c1,n}}\cdot\sum_{i=1}^{n}S(p_{c1,i-\frac{1}{2}})\int_{p_{c1,i-1}}^{p_{c1,i}}\frac{dp_c}{\sqrt{1-\frac{p_c}{p_{c1,n}}(1-f^2)}}, \tag{7}$$

where $p_{c1,n}$ is the capillary pressure at the inlet end for the n'th centrifuge frequency; $S(p_{c1,i-\frac{1}{2}})$ is the saturation at $\frac{1}{2}(p_{c1,i-1}+p_{c1,i})$; and $p_{c1,0} \equiv 0$.

The integral in Eq. 7 is now evaluated. Introducing the notation $S(p_{c1,k}) = S_k$ and

$$u_{n,i} = \sqrt{1 - \frac{p_{c1,i}}{p_{c1,n}}(1-f^2)}, \tag{8a}$$

Eqs. 6 and 7 give

$$S_{n-\frac{1}{2}} = \frac{\overline{S}_n \cdot (1-f) - \sum_{i=1}^{n-1} S_{i-\frac{1}{2}} \cdot (u_{n,i-1} - u_{n,i})}{u_{n,n-1} - u_{n,n}}. \tag{8b}$$

The solution given by Eq. 8b often exhibits oscillations, as observed from numerical experimentation. The amplitude of the oscillations tends to increase with the stepsize and the level of experimental errors. This seems to be a common trait of direct numerical schemes for solving Volterra integral equations of the first kind. Small steps in centrifuge frequency and a smooth experimental curve, $\overline{S}$ *versus* p_{c1}, inhibit oscillations.

Jones [8] suggested a smoothing scheme for oscillating solutions obtained by the trapezoidal method, and Linz [5] rigorously justified the procedure.

We have experimented with both the modified midpoint and the trapezoidal methods, Anderssen and White [7]. Both methods give (small) oscillations. Applying the smoothing scheme of Jones [8], the oscillations from the modified midpoint method usually disappear. From the first set of solutions $[S_1^*, S_2^*, \cdots, S_k^*, \cdots, S_n^*]$, Eq. 8b, the smoothed saturation at datapoint k, S_k, is formed by

$$S_k = \tfrac{1}{4}(S_{k-1}^* + 2S_k^* + S_{k+1}^*). \tag{9}$$

No oscillations are observed, and smoothing is superfluous, if the data are artificially generated from Eq. 4, with high precision numbers.

Validation

Artificial Data

Bentsen and Anli's [9] expression

$$S(p_c) = S_{iw} + (1 - S_{iw}) \exp\left[-\frac{p_c - p_d}{\sigma}\right], \tag{10}$$

was used to generate artificial centrifuge data from Eq. 4. In Fig. 2 is a plot of the interpreted results by the direct Volterra method, together with input curve given by Eq. 10 with the following parameters: $S_{iw} = 0.1$, $\sigma = 0.6$ bar, $p_d = 0.05$ bar. The match is excellent, as also experienced with other sets of parameters.

Ayappa et al. [2] have used a different capillary pressure curve given by

$$S = 1, \quad 0 < p_c \le 2, \tag{11a}$$

$$S = \frac{1.5}{p_c} + 0.25, \quad p_c > 2, \tag{11b}$$

and have demonstrated the applicability of the different interpretation methods for several ratios of radii. In Fig. 2 is a plot of the interpreted results, together with

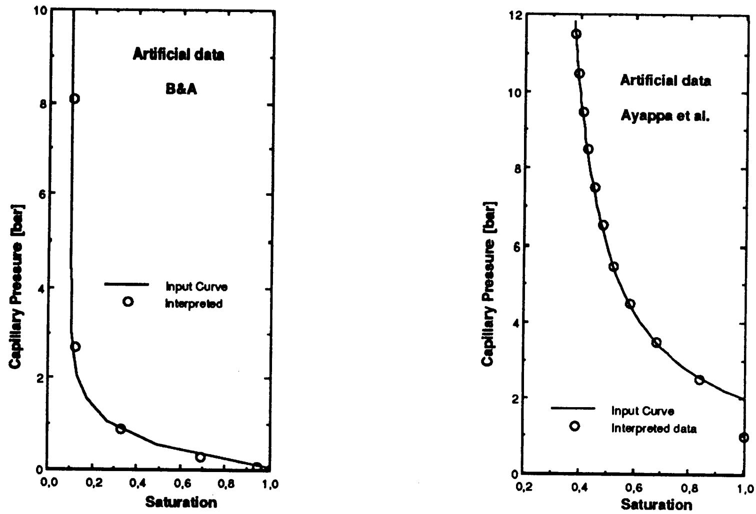

Figure 2: Interpretation of synthetic data from Bentsen and Anli [9], [B&A], and Ayappa *et al.* [2]

the input curve, for $r_1/r_2 = 0.2$. The same pressure intervals have been used as in Ref. [2]. The match is excellent.

From these examples, where no smoothing has been performed, it is concluded that the direct Volterra method accurately reproduces the given input curve.

The practical usefulness of the method, however, can only be determined by interpretation of real data. Since the problem is ill-conditioned, even a high experimental accuracy may cause instability problems.

Experiments

Drainage

The data are displayed in Appendix A, Tables A-1, A-2, A-3, and A-4. Drainage experiments were performed on six Berea cores and three from a North Sea reservoir, with properties displayed in Table A-1. The same cores, or subsamples thereof, were subjected to centrifuge expulsion, porous diaphragm displacement, and mercury injection.

The nine samples cover a wide range of permeabilities and two groups of r_1/r_2-ratios. Two porosities are given, one for the core and one for the subsample, both measured by helium injection to give grain volume and mercury displacement to give bulk volume. They are in good agreement except for Core 10. From comparison with the other two methods, it is obvious that the correct porosity of Core 10 should be 16.5 %. It is possible that the core and the subsample are different in this case. To be consistent, the core porosities have been used in the analysis, and the mercury injection saturations in Table A-3 have been adjusted accordingly, prior to comparison with the porous diaphragm and centrifuge methods.

Capillary pressure for the nine cores were determined by the porous diaphragm method, and the results are shown in Table A-2. The capillary pressure was actually increased to 4 bars for all the cores. Before this pressure was reached, air intruded into the capillary contact powder between the core and the porous disk, and proper readings were unfortunately lost.

Three centrifuge runs were made with the following grouping of cores,

Run 1 Cores 1D, 2.5E, and 5E,

Run 2 Cores 3, 8, and 9,

Run 3 Cores 5, 6, 10.

The results are shown in Table A-4. Note that the capillary pressures in each subtable correspond to the saturations in the first column. To find the capillary pressure for the other two columns, adjustment for the factor $(r_2^2 - r_1^2)$ has to be made according to Eq. 2. In the tables, $\overline{S_w}$ is the average equilibrium saturation in the core at each centrifuge frequency. These are actual, unprocessed data determined from the produced volume of water, without smoothing. We have measured extraordinarily many datapoints in order to check the sensitivity of the Volterra method to the frequency stepsize.

Forced Imbition

The data from the three cores 4-1.6wa, 7-1.6wa, and 8-1.na given in Appendix B, in Table B-1, originate from forced imbibition in a centrifuge, following a sequence of drainage and spontaneous imbibition. The cores are from a North Sea sandstone reservoir, and the study was part of an effort to determine the wettability preference.

The table contain actual, unprocessed data determined from the produced volumes of oil in an inverted bucket in the centrifuge.

Analysis

Interpretation and analysis of the data were made with the computer program *Mathematica* [10]. About 50 seconds are required to process one of datasets in Table A-4 on a Macintosh SE/30, including smoothing, averaging, and plotting of input and output.

Experimental errors in centrifuge readings may cause oscillation in the interpreted results. The data are therefore smoothed before interpretation. A least-square parabola is fit through five neighboring points, and the quintuplet is pointwise moved through the dataset.

After interpretation, the results are routinely averaged according to Eq. 9. For smooth experimental curves and artificial data, the averaging is superfluous.

Drainage

In Appendix C, Figs. C-1, C-2, and C-3 are shown the results of the interpretation of the 9 datasets in Table A-4. For each core sample, three types of data are plotted (Note that the value axis start at -0.5 bar, to increase readability).

The fully drawn curve shows the mercury injection data, Table A-3, adjusted for the difference in surface tensions between air-mercury and air-water, Table A-1. The conversion factor was found by least-square fitting an expression of the form $S = a/p_c + b$ to both the mercury and the porous diaphragm data over the same saturation interval, starting at the first saturation below 1.0 and limited by the range of the porous diaphragm saturations. The conversion factor in Table A-2 is the ratio between the a-values in the two cases. It is applied to all the mercury data to convert them to the equivalent air-water system. In the plots, the o's represent the centrifuge results, and the Δ's the porous diaphragm data directly.

The nine cores represent a variety of capillary pressure curves, and the general agreement between the centrifuge results and the converted mercury data is very satisfactory, for all cases.

For Core 3, a separate plot is shown in Fig. C-2 to demonstrate the effect of averaging the results according to Eq. 9. The fully drawn curve represents the averaged saturations, while the circles mark results with no averaging. The points oscillate slightly, especially at low saturations. The effect is minor due to smooth input and dense sampling.

The effect of reducing the number of datapoints is shown in the plots for Cores 5 and 8, where the squares represent interpreted results after reducing the number of datapoints to six, for both cores; Table A-4, table entries (5,8,11,14,17) for Core 5; and (6,9,12,15,18) for Core 8. The reduced dataset fairly well traces the converted mercury curve for Core 5. For Core 8, the bend is not properly represented by the reduced number of points.

Forced Imbibition

In Appendix D, Fig. D-1 is shown the interpretation of the data in Table B-1, with straight lines drawn between the interpreted results after averaging. Each case has six datapoints, and a fairly smooth curve is produced by the same *Mathematica* computer program as for drainage.

Only the final result is shown for Core 8-1.na. For the other two, the interpretation is presented with and without averaging. The averaging procedure is clearly superfluous for Core 7-1.na, but necessary for Core 4-1.6wa, where oscillations take place, probably caused by less accuracy in measured data.

Discussion

From the results in Appendices A and B, and similar experiments, we recommend that the number of datapoints from the centrifuge should not be less than 5-6. If the capillary pressure curve has marked characteristics, higher density of datapoints is necessary to catch the fine structure of the curve. It is difficult to give general advice regarding the data sampling procedure. We have experienced, however, that increasing the frequency by a constant factor seems to give reasonable results.

Inaccuracy in the reading of produced volumes from the centrifuge may create oscillations in the interpreted results. The accuracy in a visual reading is typically ± 0.125 cm^3, and may be about 10 % of the first measurement. An automated centrifuge, as described by Torsæter and Munkvold [11], may increase the accuracy to ± 0.03 cm^3, and reduce the need for smoothing the data and the results. The problem with the smoothing is, of course, that it may eliminate any fine structure in the capillary pressure curve, e.g. two-porosity systems.

All the drainage curves reported here were interpreted with the modified Hassler and Brunner method and gave results in very good agreement with those of the Volterra method, Hermansen [4].

Conclusions

A direct, numerical method is developed to extract the capillary pressure curve from centrifuge data. The method is validated by interpretation of artificial data and by comparison with the porous diaphragm and mercury injection methods.

The applicability of the method is demonstrated by interpretation of a series of measured drainage and forced imbibition data.

Nomenclature

a	=	arbitrary constant, bar
b	=	arbitrary constant, dimensionless
f	=	r_1/r_2 for drainage
	=	r_2/r_1 for forced imbibition
N	=	number of frequencies in a centrifuge run
n	=	counter for readings in a centrifuge run
r	=	radius from center of centrifuge, m
r_1	=	radius to inner boundary of core, m or cm
r_2	=	radius to outer radius of core, m or cm
p	=	pressure, bar, mb, or Pa
S	=	saturation, dimensionless
$\overline{S}$	=	average saturation, dimensionless
u	=	defined by Eq. 8a
Δ	=	difference operator
ϕ	=	porosity, fraction
ω	=	angular frequency, rad/s
ρ	=	density, kg/m^3
σ	=	parameter in Eq. 10, bar
	=	surface tension, mN/m

Subscripts

c	=	capillary
d	=	displacement
i	=	counter of iterations, or intervals
	=	irreducible
k	=	counter of intervals
w	=	water

Superscripts

$*$ = result from interpretation, not averaged

Acknowledgement

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References

[1] Hassler, G.L., and Brunner, E.: "Measurements of Capillary Pressure in Small Core Samples," *Trans.*, AIME (1945) **160**, 114-123.

[2] Ayappa, K.G., *et al.*: "Capillary Pressure: Centrifuge Method Revisited," *AIChE Journal* (1989) **35**, No. 4, 365-372.

[3] Glotin, J., Genet, J., and Klein, P.: "Computation of Drainage and Imbibition Capillary Pressure Curves From Centrifuge Experiments," paper SPE 20502 presented at the 1990 SPE Annual Technical Conference and Exhibition, New Orleans, Sept. 23-26.

[4] Hermansen, H.: "Using Volterra Integral Equation to Convert Centrifuge Data Into a Capillary Pressure Curve," MS Thesis, Rogaland U., Stavanger (1987).

[5] Linz, P.: "The Solution of Volterra Equations of the First Kind in the Presence of Large Uncertainties," *Comp. J.* (1969), **12**, 393-397.

[6] Donaldson, E.C., Thomas, R.D., and Lorenz, P.B.: "Wettability Determination and Its Effect on Recovery Efficiency," *SPEJ* (March 1969) 13-20.

[7] Anderssen, A.S., and White, E.T.: "Improved Numerical Methods for Volterra Integral Equations of the First Kind," *The Computer J.* (1971) **14**, No. 4, 442-443.

[8] Jones, J.G.: "On the Numerical Solution of Convolution Integral Equations and Systems of such Equations," *Math. Comp.*, (1961) **15**, 131-142.

[9] Bentsen, R.G., and Anli, J.: "Using Parameter Estimation Techniques to Convert Centrifuge Data Into a Capillary-Pressure Curve," *SPEJ* (Feb. 1977) 57-64.

[10] Wolfram, S.: *Mathematica™ —A System for Doing Mathematics by Computer,* Addison-Wesley Publishing Company, Inc., New York (1988).

[11] Torsæter, O., and Munkvold, F.R.: "Automated Centrifuge for Measurement of Capillary Pressures and Relative Permeability," *Proc.*, 4th European Symposium on Enhanced Oil Recovery, Hamburg, 27th-29th October 1987, 999-1006.

A Drainage Data

Table A-1: Physical properties of the nine Berea cores used for drainage experiments with centrifuge, porous diaphragm, and mercury injection

Physical Properties of Cores							
Core id	r_1 cm	r_2 cm	ϕ^a %	ϕ^b %	k md	ρ_g kg/m^3	$\frac{\sigma_{hg-air}}{\sigma_{w-air}}$ [c]
1D	4.53	9.38	19.4	19.4	217.0	2650	7.75
2.5E	4.52	9.38	21.7	21.6	715.0	2650	9.22
3	6.06	8.60	21.0	21.0	609.0	2650	8.98
5	4.54	9.38	17.8	17.6	76.5	2650	7.74
5E	4.52	9.38	22.9	22.7	855.0	2650	9.05
6	4.39	9.38	16.9	17.8	53.9	2630	4.68
8	6.05	8.60	20.0	20.1	259.0	2650	7.43
9	6.06	8.60	23.0	23.2	996.0	2650	7.39
10	4.46	9.38	16.5	19.9	8.5	2660	8.02

[a]cores
[b]subsamples
[c]curvefit

Table A-2: Air-water capillary pressure measured on the nine Berea cores by the porous diaphragm method

p_c	Water Saturation								
bar	1D	3	2.5E	5	5E	6	8	9	10
0.05	1.000	0.803	0.884	1.000	0.640	0.965	1.000	0.686	1.000
0.10	0.813	0.455	0.508	0.936	0.425	0.904	0.746	0.499	1.000
0.30	0.345	0.230	0.225	0.759	0.236	0.638	0.330	0.236	0.948
0.60	0.263	0.178	0.196	0.360	0.183	0.455	0.253	0.188	0.707
1.0	0.216	0.151	0.157	0.304	0.154	0.364	0.199	0.147	0.528

Table A-3: Air-mercury capillary pressure measurements for the nine Berea cores. Saturations to be adjusted to core porosity, Table A-1

p_c	Mercury Saturation [%]								
bar	1D	3	2.5E	5	5E	6	8	9	10
0.28	—	0.5	0.3	—	0.3	0.4	—	0.3	—
0.38	0.3	1.7	1.4	0.2	2.1	2.1	0.5	3.8	—
0.48	0.6	10.1	14.4	—	34.2	5.6	1.1	38.5	—
0.58	1.2	31.0	42.5	1.1	52.0	9.5	2.1	52.5	—
0.68	3.1	—	—	2.7	—	13.5	6.7	—	—
0.78	13.4	54.2	61.7	8.2	63.1	17.7	22.7	62.3	0.2
0.88	30.9	—	—	18.9	—	21.9	37.1	—	—
1.08	47.0	63.9	68.4	34.2	68.4	28.0	50.3	67.5	0.5
1.28	—	—	—	42.8	—	33.0	—	—	—
1.58	60.1	70.1	73.4	49.5	72.8	38.8	61.8	71.7	1.6
1.78	—	—	—	—	—	—	—	—	2.4
2.08	66.0	72.9	76.1	54.9	76.5	47.5	66.1	74.1	4.2
2.58	68.1	—	—	58.7	—	52.7	68.9	—	10.6
3.08	—	76.7	79.4	—	78.9	55.4	—	77.4	17.9
3.58	71.7	—	—	62.7	—	—	72.4	—	24.0
4.08	—	—	—	—	—	60.4	—	—	28.8
5.08	75.1	80.5	82.9	66.8	82.4	63.7	75.6	81.3	35.7
6.08	—	—	—	—	—	—	—	—	39.1
8.08	78.9	83.4	85.4	71.4	85.1	69.4	79.4	83.6	44.2
12.1	82.2	85.5	87.6	74.7	87.3	73.9	82.6	85.7	49.3
17.1	84.6	87.0	88.8	77.3	89.0	77.4	84.5	86.9	53.4
25.1	87.1	88.3	90.4	80.2	90.5	80.9	86.4	88.3	57.7
35.1	89.0	89.6	91.5	82.7	91.4	83.7	88.6	89.6	61.1
50.1	90.6	90.8	92.5	85.6	92.5	86.2	90.1	90.5	64.4
70.1	92.0	91.7	93.3	87.8	93.3	88.3	91.5	91.6	67.4
100.0	93.6	92.9	94.3	90.2	94.2	90.7	92.9	92.8	70.8
140.0	94.7	94.9	94.9	91.8	94.8	92.1	93.8	93.7	73.3

Table A-4: Data from centrifuge experiments

p_c 3^a	$\overline{S_w}$ 3	$\overline{S_w}$ 8	$\overline{S_w}$ 9
0.011	1.000	1.000	1.000
0.022	1.000	1.000	1.000
0.031	1.000	1.000	1.000
0.060	0.981	0.988	0.983
0.074	0.977	0.979	0.957
0.093	0.938	0.950	0.885
0.124	0.840	0.843	0.784
0.188	0.719	0.706	0.630
0.292	0.571	0.566	0.497
0.433	0.454	0.462	0.392
0.643	0.375	0.396	0.320
0.895	0.329	0.343	0.281
1.277	0.285	0.298	0.245
1.764	0.231	0.260	0.209
2.432	0.211	0.228	0.190
4.069	0.168	0.210	0.173
5.140	0.160	0.190	0.162
6.820	0.133	0.174	0.140
8.530	0.129	0.166	0.133

[a]Adjust 8 and 9 for $(r_2^2 - r_1^2)$

p_c 5^a	$\overline{S_w}$ 5	$\overline{S_w}$ 6	$\overline{S_w}$ 10
0.069	1.000	1.000	1.000
0.101	1.000	1.000	1.000
0.129	1.000	1.000	1.000
0.147	0.961	0.940	1.000
0.189	0.895	0.896	1.000
0.200	0.879	—	1.000
0.236	0.804	0.846	1.000
0.291	0.724	0.791	0.988
0.337	0.669	0.753	0.982
0.407	0.619	0.730	0.964
0.496	0.562	0.670	0.939
0.686	0.512	0.591	0.866
0.968	0.432	0.516	0.733
1.356	0.395	0.451	0.648
1.950	0.348	0.386	0.561
2.839	0.315	0.348	0.488
3.959	0.277	0.308	0.426
6.190	0.253	0.269	0.363

[a]Adjust 6 and 10 for $(r_2^2 - r_1^2)$

p_c $1D^a$	$\overline{S_w}$ 1D	$\overline{S_w}$ 2.5E	$\overline{S_w}$ 5E
0.024	1.000	1.000	1.000
0.035	1.000	1.000	1.000
0.051	1.000	1.000	1.000
0.069	1.000	0.974	0.950
0.099	1.000	0.829	0.784
0.126	0.962	0.742	0.676
0.173	0.851	0.611	0.556
0.233	0.649	0.532	0.465
0.330	0.587	0.427	0.373
0.497	0.486	0.344	0.303
0.741	0.404	0.265	0.257
1.052	0.342	0.226	0.212
1.456	0.328	0.213	0.199
2.088	0.260	0.178	0.170
3.173	0.236	0.151	0.141
4.704	0.203	0.134	0.124
6.151	0.183	0.125	0.116

[a]Adjust 2.5E and 5E for $(r_2^2 - r_1^2)$

B Forced Imbibition Data

Table B-1: Forced imbibition data from a North Sea sandstone reservoir

Forced Imbibition Data					
4-1.6wa		7-1.6wa		8-1.na	
r_2 0.1663	r_1 0.115	r_2 0.1663	r_1 0.1213	r_2 0.1663	r_1 0.1214
p_c [mb]	$\overline{S_w}$	p_c [mb]	$\overline{S_w}$	p_c [mb]	$\overline{S_w}$
-56.3	0.6434	-35	0.5682	-35	0.4776
-121.1	0.7430	-70.7	0.5903	-70.6	0.5239
-256.2	0.7928	-143.0	0.7233	-142.8	0.6269
-546.7	0.8346	-290.8	0.8013	-290.2	0.6918
-1173.8	0.8512	-589.3	0.8234	-588.2	0.7477
-2501.6	0.8758	-1187.4	0.8456	-1185.2	0.7761

C Interpreted Data —Drainage

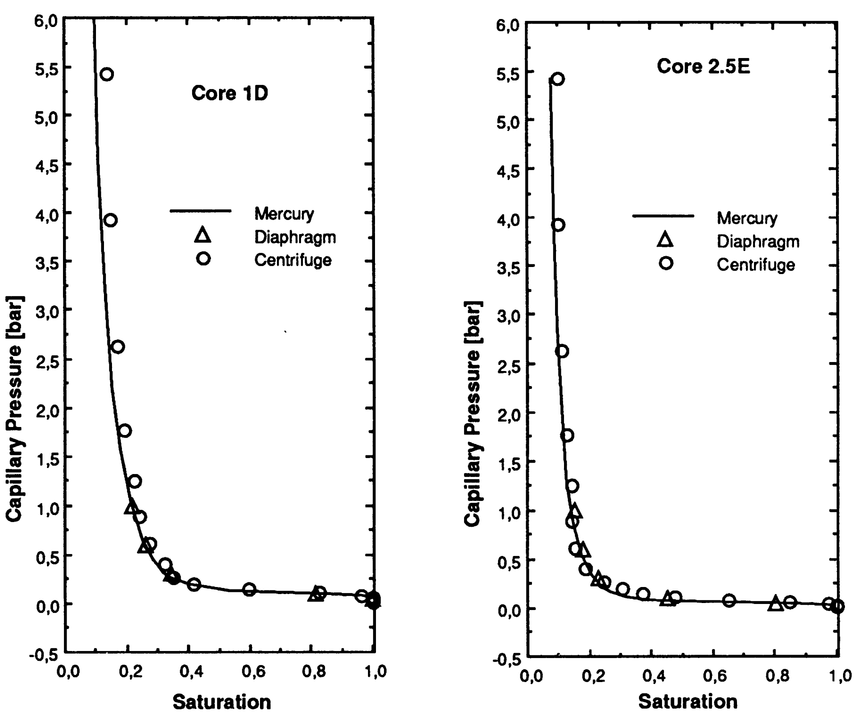

Figure C-1: Interpreted results for drainage of Core 1D and 2.5E

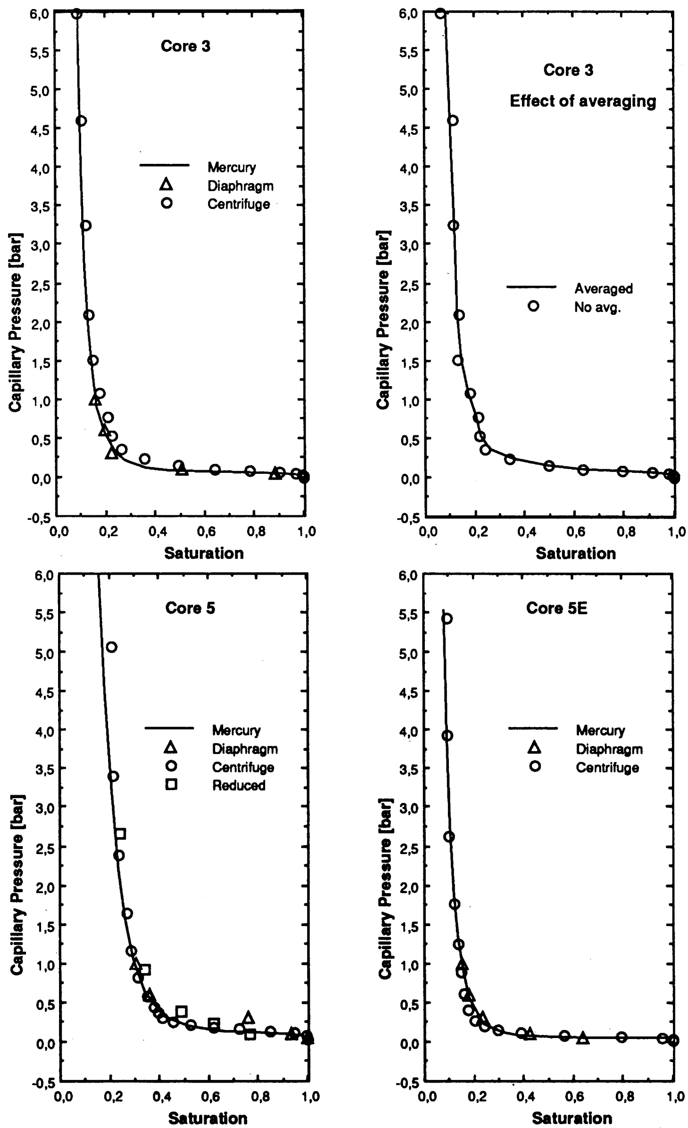

Figure C-2: Interpreted results for drainage of Core 3, 5, and 5E; Core 3 with and without averaging

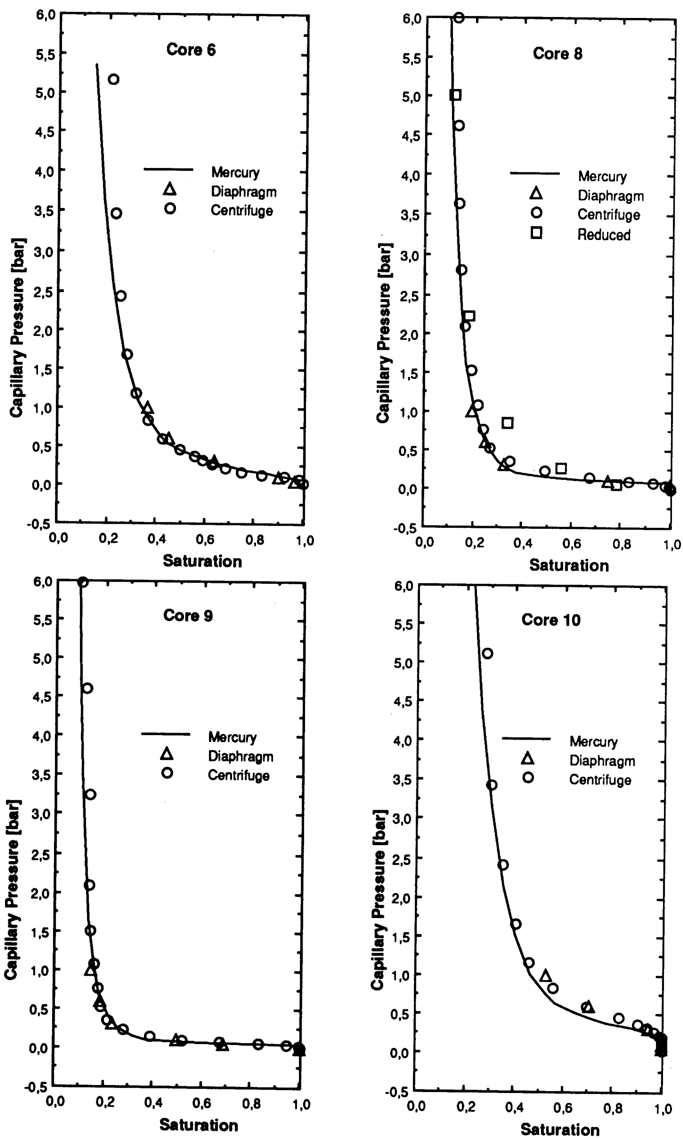

Figure C-3: Interpreted results for for drainage of Core 6, 8, 9, and 10

D Interpreted Data —Forced Imbibition

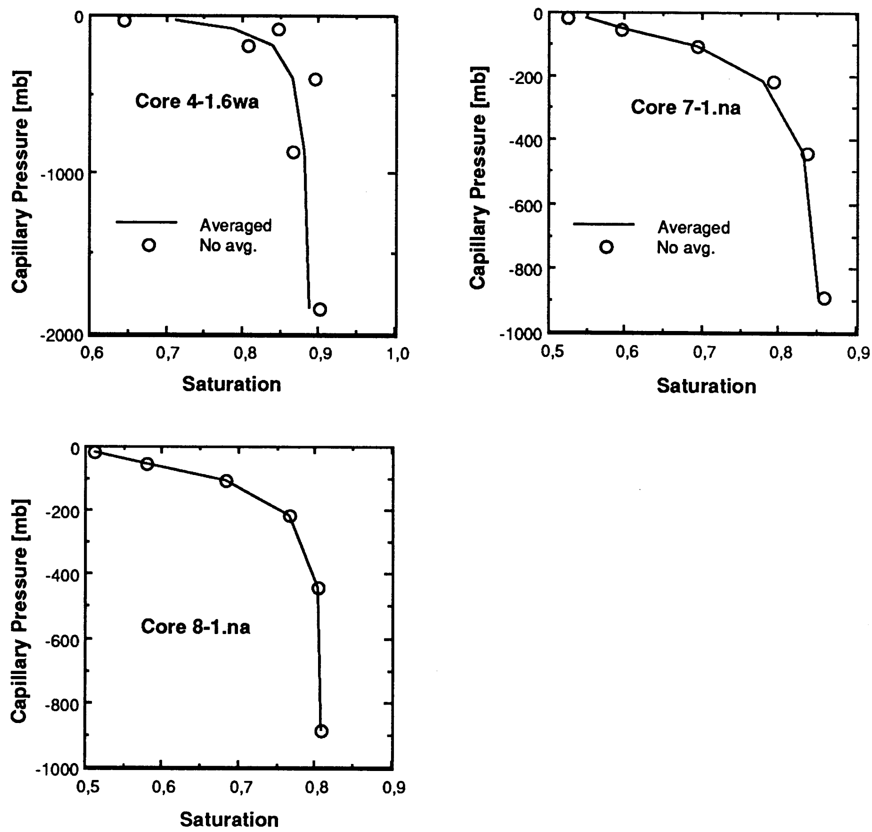

Figure D-1: Interpreted results for forced imbibition of Core 4-1.6wa, 7-1.na, and 8-1.na