Estimation of Relative Permeability and Capillary Pressure Functions Using Transient and Equilibrium Data From Steady-State Experiments

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Abstract

A method for simultaneous determination of relative permeability and capillary pressure from transient and equilibrium data gathered during steady state experiments is developed and tested. The method uses B-spline representation of the unknown properties and determines the parameters within that representation through a regression-based approach. A linearized covariance analysis is utilized to calculate the confidence intervals for the estimated functions. Two sets of production and pressure drop data are successfully analyzed; one simulated and one experimental case. The results show that it is possible to determine the functions quite accurately over a relatively large saturation range.

Introduction

Relative permeability and capillary pressure functions are important properties of porous media, and they are essential for understanding multiphase reservoir flow behavior. Estimates of these functions are required as inputs to reservoir simulators used to evaluate reservoir performance and to make operational decisions. Since the reservoir itself is quite inaccessible for measurements of the relative permeability and capillary pressure functions, samples from some few wells with estimates obtained through laboratory investigations are typically used.

Determination of the relative permeability functions is a problem which has received considerably attention over the past fifty years. It is a particularly challenging problem as the relative permeabilities can not be determined in any direct manner, but have to be inferred through analyses of data from some process imposed on a sample of the porous media. The reason for the necessity of such an approach is that the relative permeability functions are defined through the system of equations that describes the flow through porous media. In particular, the relative permeabilities enter into the Darcy's laws (or the flow equations) which relate the superficial velocities of each individual phase to the corresponding pressure gradient and viscosity ($v_i = -(kk_{ri}/\mu_i)\partial P_i/\partial x$). The relative permeabilities are empirical properties in these equations. This simple linear relationship between the velocity, the pressure gradient, and the reciprocal of the viscosity, is generally assumed to be adequate for description of capillary dominated flow through porous media, i.e., flow for which the capillary number (i.e., ratio of viscous-to-capillary forces, often given as $N_{cap,i} = \mu_i v_i/\sigma$) is relatively low (see, e.g.,[4]). In the capillary dominated regime, i.e., for a range of sufficiently low capillary numbers, the relative permeability is assumed to be independent of the capillary number, depending only on saturation and its history. Outside of this regime, the relative permeabilities may be functions of the velocity and possibly other variables. As the reservoir flow generally will be capillary dominated (with the possible exception of the near-wellbore flow), the relative permeabilities should be determined in the corresponding capillary number region.

However, the traditional determination procedures are generally not capable of doing such determinations. The most commonly used unsteady-state method[10] neglects the capillary pressure altogether. This has some computational advantages, as the system of equation describing flow through porous

media then can be solved analytically. Experimentally, this requires high injection rate with correspondingly large capillary numbers which may be outside the range of interest. In the steady-state technique (see, e.g., [4, 7]), the injection rates (and thus the capillary numbers) may be kept low by using special experimental setups; however, troublesome experimental procedures, as well as undesirably long experimental time, make this alternative less attractive.

A better alternative is to account for the capillary pressure effects in determining the relative permeability. In principle, this can be done by first determining the capillary pressure on the sample of interest through an independent experiment (see, e.g., [8, 9, 15]). However, there can be considerable difficulties in running sequential multiphase experiments on a sample, including reestablishing initial states and wetting conditions and matching saturation ranges. Also, the experimental time required can be inordinately long. The use of properties determined from a neighboring, but not identical, sample may lead to errors of unknown magnitude. It would be much more desirable to simultaneously determine the capillary pressure and relative permeability functions from a single experiment.

Simultaneous estimates can be obtained by conducting an experiment so that the measured data are sensitive to both relative permeability and capillary pressure effects, and then estimating the properties through solution of an appropriate inverse problem. In such an approach, we choose porous media properties so that the solution of a mathematical model for the process (i.e., a simulation) "matches" the measured data. The methodology for solving inverse problems in core analysis has evolved rapidly in the past decade, and a sound, regression-based technique is available. It utilizes a flexible B-spline representation of the relative permeability and capillary pressure functions, and solves the associated parameter estimation problem in a robust manner[20]. This methodology has been demonstrated by analyzing both unsteady-state (pressure drop and production data) as well as centrifuge displacement experiments (production data)[16, 18].

Although the relative permeability and capillary pressure curves can, in principle, be identified in those conventional experiments[16, 18], the accuracy may not be adequate. This situation can be improved by measuring additional data, such as *in situ* saturations or pressures[2, 13, 14], this approach has not been sufficiently demonstrated with actual data. Also, this approach can require substantial investments in equipment and training. We present a much simpler approach that can provide accurate simultaneous estimates with conventional equipment and measurements. In this method, we utilize data which are normally available, but generally not used, in steady-state experiments.

Methodology

Experiment

Traditionally, steady-state experiments are performed by injecting two (or more) fluid phases simultaneously through a porous medium sample. The average core saturation and the pressure drop across the core are measured at "equilibrium" (i.e., when the average saturation profile and the pressure drop are stable). The average core saturation along with the relative permeabilities values calculated directly with Darcy's law then specifies one point on each of the relative permeability curves. The calculations are based on the assumption that the saturation is uniform throughout the sample. By varying the rate fractions, a sequence of relative permeability points may be determined. For example, with a sample that is initially saturated with the wetting phase, a steady-state experiment may be conducted by increasing the nonwetting phase rate fraction, in discrete steps, from zero to unity. Typically, less than 10 such rate fractions are used to construct the relative permeability curves.

A significant source of error in the steady-state approach is the assumption of uniform saturation at equilibrium. Several different experimental procedures have been developed to try to mitigate that error. The two most frequently used techniques today are (1) to place a test core between two similar cores (in which the capillary end-effects take place), and measure the saturation in the test sample by some *in situ* technique (this method is often referred to as the Penn-State technique[5]); and (2) to measure the volumes produced from the core, and use material balance calculations to obtain the

average saturation[7]. In the latter method, the flow rate should be kept high in order for the capillary end-effects to be negligible. This is undesirable as the flow rates then often may differ quite substantially from typical reservoir flow rates. In (1) the flow rate could be kept low; however, in the case of large capillary pressure, long samples, with correspondingly long, and impractical, equilibration time would be necessary to eliminate significant error. In addition, lack of, or only partial, capillary continuity between the test samples and the two end cores may cause the saturation profile in the test core to become nonuniform during the tests, resulting in poor estimates of the relative permeabilities. This will also be the case if the samples have different wettabilities[3].

In this paper, a different experimental design for the steady-state experiment is used. The production and pressure data are collected throughout the equilibration at each rate fraction and are used to determine simultaneous estimates for the capillary pressure and relative permeability curves. In addition to providing accurate estimates of both the relative permeability and capillary pressure curves from a single experiment, this method provides a number of experimental advantages over the steady-state method. The experiment may be performed at low rates without having any end cores (as in the Penn-State technique), and it is not necessary to wait for equilibrium at each rate fraction. Thus, the experimental time can be controlled while representative flow conditions are used.

Parameter Estimation

The method used to determine estimates of the relative permeability and capillary presure functions from the measured data is described in this section. In the regression-based approach (for details, see[20]), we seek a solution to the least-squares problem defined by

$$J(\vec{\beta}) = [\vec{Y}_m - \vec{Y}_s(\vec{\beta})]^T \boldsymbol{W} [\vec{Y}_m - \vec{Y}_s(\vec{\beta})] \tag{1}$$

subject to the constraints

$$\boldsymbol{G}\vec{\beta} \geq \vec{b}. \tag{2}$$

In Eq. 1, $\vec{Y}_m$ is a vector containing the measured data, $\vec{Y}_s(\vec{\beta})$ contains the corresponding values calculated by using a mathematical model for the process (in this work, the fully implicit core flood simulator CENDRA[6] has been utilized to generate $\vec{Y}_s(\vec{\beta})$), the parameter vector $\vec{\beta}$ characterizes the desired properties, and $\boldsymbol{G}$ and $\vec{b}$ are a constraint matrix and vector, respectively. If we assume that the measurement errors associated with the data can be represented by a normal distribution with zero mean and covariance matrix $\boldsymbol{C}$, maximum-likelihood estimates of the parameters are obtained when the weighting matrix, $\boldsymbol{W}$, is taken to be a scalar multiple of $\boldsymbol{C}^{-1}$[1]. A trust-region implementation of the Levenberg-Marquardt algorithm including the linear inequality constraints has been used to solve the nonlinear least-squares problem defined by Eqs. 1 and 2[17].

A key element for determining the relative permeability and capillary pressure functions through solution of an inverse problem is the selection of function representation[11]. It has been shown that B-splines can provide the necessary "flexibility" in order to adequately represent the true (although unknown) properties[19]. The B-spline representations are given by

$$k_{ri}(S_w) = \sum_{j=1}^{N_i} C_j^i B_j^m(S_w, \vec{y}^{\,i}), i = w, nw \tag{3}$$

$$P_c(S_w) = \sum_{j=1}^{N_c} C_j^c B_j^m(S_w, \vec{y}^{\,c}) \tag{4}$$

The functions are defined on $0 \leq S_w \leq 1$ and are specified by the order m, the spline coefficients C, and the extended partition $\vec{y}$. The vector of unknown parameters then becomes

$$\vec{\beta} = [C_1^{nw}, \ldots, C_{N_{nw}}^{nw}, C_1^w, \ldots, C_{N_w}^w, C_1^c, \ldots, C_{N_c}^c]. \tag{5}$$

While the parameters for a specific representation (i.e, for specified extended partition vectors, $\vec{y}^{\,i}$) are given by the solution of the parameter estimation problem in Eqs. 1 and 2, the solution of the inverse

problem consists of solving a series of such problems for different extended partitions in order to find adequate representations for the true functions. One can always provide excess flexibility by using a large spline dimension; however, the accuracy of the determination of the parameters may then suffer[11]. Therefore, we seek the minimum dimension that provides for adequate function representation. The regression-based approach has been designed in order to provide for a systematic procedure for selecting the extended partition[20].

In the case considered here, particular characteristics of the steady state process are used to help select the extended partition. Consider, for simplicity, a primary drainage process (similar arguments are valid for secondary as well as imbibition processes). After injection at the initial rate fraction has begun, the wetting phase saturation will decrease, eventually approaching equilibrium. The data collected during that period reflect the relative permeability and capillary pressure curves for a range of saturations from unity down to the minimum wetting saturation in the sample, say $S_{w,1}$. As the rate fraction of the nonwetting phase is increased, the measured data will reflect a greater range of saturations. Data corresponding to the nth rate fraction will reflect the relative permeability and capillary pressure curves corresponding to a range of saturations from unity down to $S_{w,n}$, where $S_{w,n}$ represents the lowest saturation value experienced in the nth rate fraction. All the $S_{w,i}$ values will form a descending sequence. This feature can be utilized beneficially in selecting the extended partition. In this work, we investigate the residuals (i.e., $\vec{Y}_m - \vec{Y}_s(\vec{\beta})$), corresponding to the sequence of rate fractions, in order to locate the first rate fraction for which the data are not well predicted (i.e, has large residuals). By inspecting the saturation profiles (calculated by the simulator) corresponding to that rate fraction, an estimate of the saturation region for which the properties are not well determined may be obtained. Flexibility is then added by inserting a spline knot in the saturation region that is least well determined and to the function which we anticipate will have the largest impact in that region. Generally, this means that for lower saturations, a spline knot is added to the wetting phase relative permeability or the capillary pressure function, while for higher saturations, the nonwetting phase will initially be selected. Good initial estimates of the relative permeability functions may be obtained by fitting a spline function to the relative permeability points obtained from the equilibrium (or near equilibrium) data. Of course, how close this initial estimate is to the true (and unknown) curve will depend upon the degree of capillary effects.

A linearized covariance analysis is utilized to obtain an estimate of the accuracies of the estimated functions[11]. In this analysis it is assumed that the selected representations adequately represent the true (although unknown) functions (and hence avoid the so-called bias error[11]). It is further assumed that the simulator adequately represents the physics of flow through porous media and that the measurements errors are additive. The analysis is based upon the assumption that the objective function (Eq. 1) may adequately be approximated by a linear function in the parameters in a sufficiently large parameter region near the true parameter values. By Taylor expansion of $J(\vec{\beta})$, it can be shown[11] that a linear relationship exists between the parameter errors and the errors in the measurements. By assuming a normal distribution for the measurement errors, pointwise confidence intervals for the relative permeability and capillary pressure functions may be calculated using the sensitivity matrix ($\boldsymbol{A}$= $[\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_M]$, $\vec{a}_j = \partial \vec{Y}_s(\vec{\beta})/\partial \beta_j$ for $j = 1, \ldots, M$, are the sensitivity coefficients) and the covariance matrix for the measurements errors, $\boldsymbol{C}$[11].

Results and Discussion

This section demonstrates the feasibility of the proposed approach by analyzing two test cases; one using simulated data, and one using experimental data.

Demonstration of Methodology

To demonstrate the methodology, a simulated case is considered. "True" relative permeability and capillary pressure functions, along with core and fluid properties (see **Table 1**), are selected, and a set

of "true" data (i.e., water production and pressure drop as a function of time) for a water-wet primary drainage process are generated with the simulator. Quadratic splines were used for the water and oil relative permeability curves and cubic splines for the capillary pressure function. Three interior knots were used for each curve. (In **Table 1**, the rate fractions are denoted by the percent of total flow rate (wetting phase first), e.g., 99%/1% is the rate fraction of 99% wetting phase and 1% nonwetting phase.) Random errors, with zero mean and a given standard deviation, are drawn from a normal distribution and added to these true data, giving a set of "experimental" data. Note that the initial total flow rate is quite low, giving a low capillary number. The simulated data therefore depend on a relatively narrow range of saturations. To resolve the functions over a broader range, the oil rate is increased three times after a near equilibrium condition is reached at the last rate fraction of the steady state experiment.

Table 1: Core and fluid properties in both simulated and experimental test case.

Water viscosity [cP]	1.097
Oil viscosity [cP]	1.19
Core length [cm]	6.81
Core'diameter [cm]	3.73
Porosity [%]	28.4
Absolute permeability (water) [mD]	4.17
Initial water saturation [frac]	1.0
Injection rate [cc/min]	0.1
Rate fractions [%]	q_w/q_o = 99/1, 88/12, 40/60, 0/100
Step-up rate [cc/min]	0.25, 1.0, 3.0
Standard deviation ΔP (*) [kPa]	1.2, 7.0
Standard deviation $\bar{S}_w$ (*) [cc]	0.03, 0.125

(*): Standard deviation increases with pressure drop and flow rate.

The procedure described in the previous section was used to estimate the multiphase functions. A good correspondence between the predicted and experimental pressure drop and water production is obtained (see **Fig.1a**). The estimated relative permeability and capillary pressure functions are shown in **Fig.1b**, along with the true functions. The "equilibrium" relative permeability values are those points that would correspond to the traditional interpretation of the data from this experiment. In this case, these are consistent (close to the true curves) due to relatively small capillary pressures.

The presented procedure makes it possible to determine the relative permeability as well as the capillary pressure functions quite accurately. The confidence intervals are calculated around the estimated curves showing that the true curve is within this interval for most of the saturation region.

Experimental Results

In the experimental case, a reservoir condition steady-state equipment was utilized. This apparatus consists of four major parts: a pump system, a core holder, a differential pressure transmitter, and a separator. These parts are assembled together in order to recycle two phases through a core sample at various constant rates (for more detailed apparatus description, see [7]). The pump system consists of five cylinders, two cylinders for each phase and the one cylinder acting as a back pressure regulator to keep a constant line pressure in the set-up. The cylinders are able to deliver continuous and virtually pulse-free fluid flow at a constant rate with high accuracy. The differential pressure across the core is measured using a high resolution transmitter connected to the core faces through distribution plugs placed at each of the faces. An acoustic separator is used for fluid separation and production monitoring at the downstream end of the core. The top of the separator is connected to the return line for the oil phase and the bottom part is connected to the water return line. The core is placed in a triaxial core holder between two distribution plugs at each core face. The inlet distribution plug has two separate spiral grooves for oil and water, ensuring distribution of both fluids across the core inlet face. The outlet

distribution plug is of standard type with a screen placed between the plug and the core face to minimize particle wash-out and to ensure fluid flow through the entire core cross-sectional area. A non-flexible stainless steel support screen with no flow possibilities in the vertical direction is placed against each core face to prevent the sample to be forced into the grooves located on the distribution plugs. Several rubber washers are placed behind the distribution plugs to transmit axial stress proportional to the confining pressure. The core is completely covered by teflon tape, tin foil, and a viton sleeve.

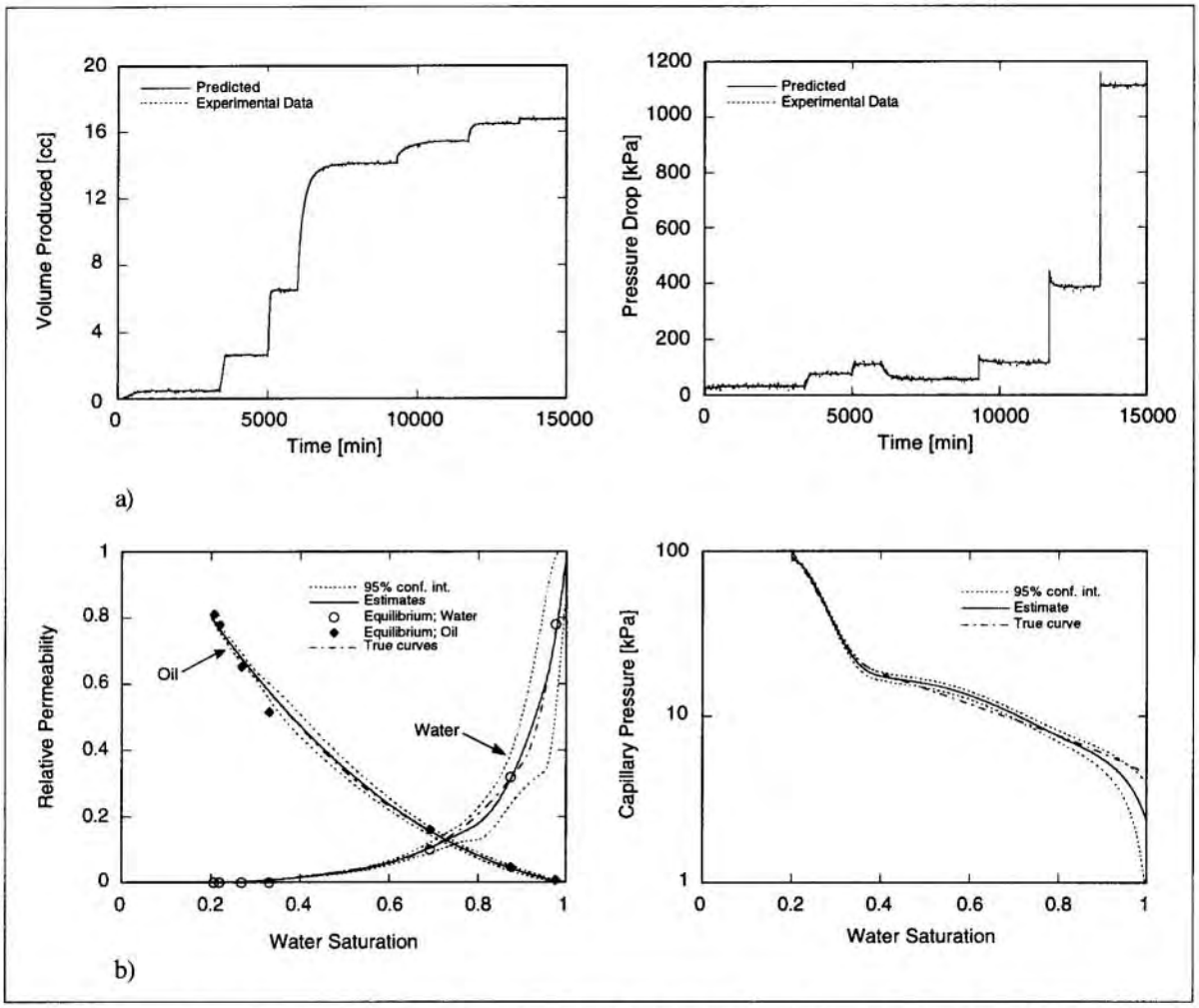

Figure 1: Simulated case. a) Measured and predicted production (left) and pressure drop (right); b) Estimated relative permeability (left) and capillary pressure (right) functions with 95% confidence intervals and true curves shown.

The core used is an outcrop chalk sample from the Beer Stone, collected in Beer Stone Quarries, East Devon, England [21]. It was selected on the basis of having relatively homogeneous CT-scan images. The core was cleaned with solvents and dried before testing, and is water-wet. Mineral oil (Exxon Isopar H) has been used as the oil phase and a simulated North-Sea chalk formation water as the water phase. Calcium is added to the formation water to ensure chemical stability of the chalk matrix. Basic core data and fluid properties are given in **Table 1**. A rate step-up scheme similar to the synthetic case has been utilized to obtain a large saturation region.

Fig.2 shows the results of the analyses for a primary drainage process; very good agreement between the measured and predicted pressure drop and water production is obtained, although we are not able to obtain a close "match" to the data corresponding to the last rate step-up. These data correspond to very low water saturation, and the estimate in this region may consequently suffer. The estimated curves were

all quadratic with 6, 5, and 5 interior knots for k_{rw}, k_{ro}, and P_c, respectively. In this case, the relatively high capillary pressure causes the equilibrium relative permeability points to be far from the estimated curves. The estimated confidence regions are narrow in most of the saturation range, indicating that the curves are accurately determined. For low saturations (below approximately $S_w = 0.15$, see **Fig.2b**), however, the confidence intervals are large. The reason for this is that saturations in this region may evidently not be obtained in the experiment. Nevertheless, estimates of the functions are obtained. This has been done to avoid making *a priori* assumptions with respect to irreducible water saturation in the analyses (i.e.,the irreducible water saturation has been kept equal to zero throughout the analyses). This is desirable as there is generally no *a priori* information available about the irreducible water saturation.

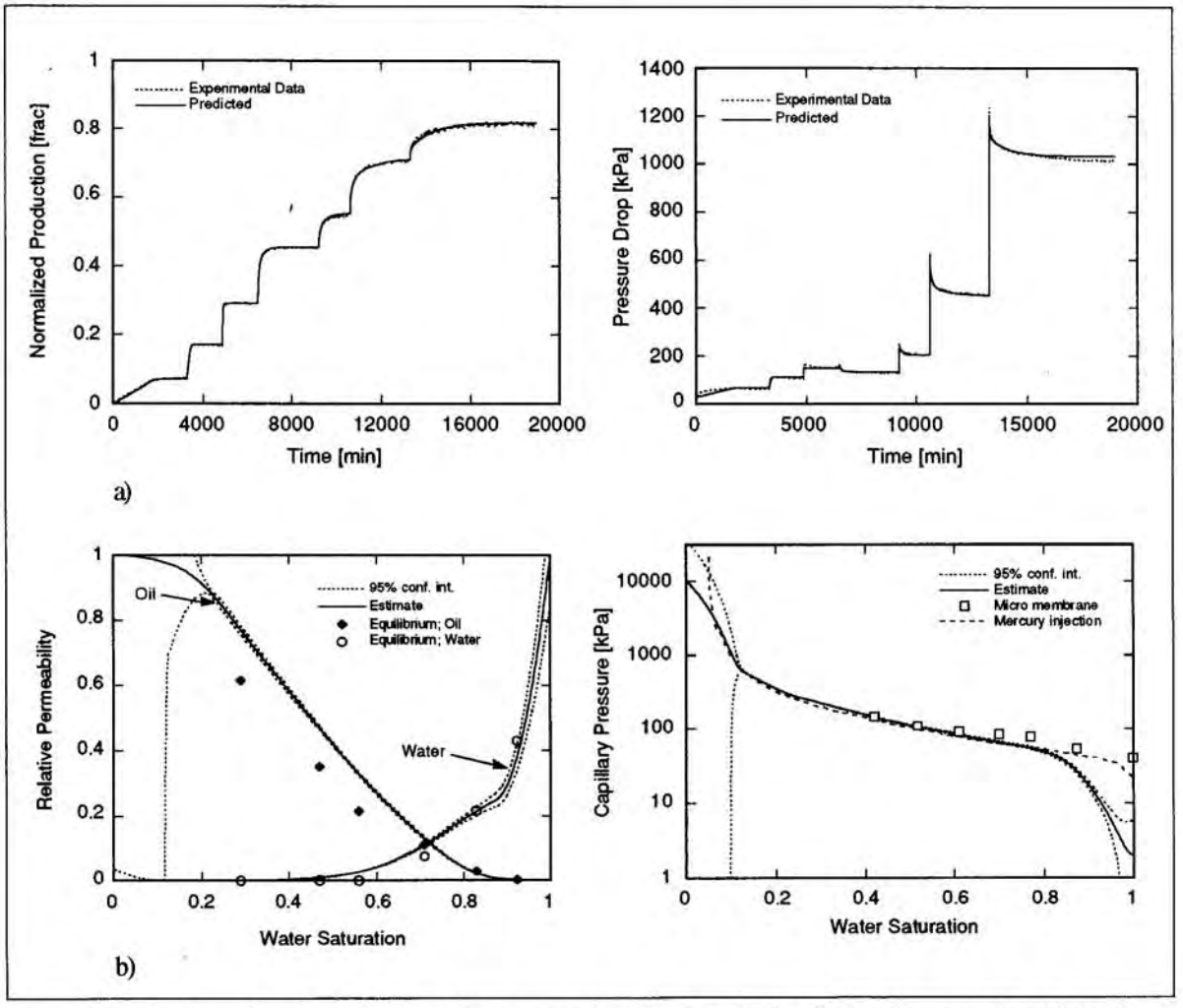

Figure 2: Experimental case. a) Measured and predicted production (left) and pressure drop (right); b) Estimated relative permeability (left) and capillary pressure (right) functions with 95% confidence intervals. Independent measurements of the capillary pressure is shown.

The capillary pressure estimate has been compared to two other experimental techniques; both the micro-membrane[9] as well as the mercury injection techniques have been utilized to obtain independent measurements, see **Fig.2b** (right). In both these cases, the capillary pressure curves are obtained on a neighboring sample. This sample had a somewhat different permeability and porosity from the one used in the steady-state test. The mercury injection and micro membrane curves in **Fig.2b** are therefore scaled so that the capillary pressure curves should correspond to the capillary pressure function for the test sample considered here. This scaling is done using the so called Leverett J-function[12]. As can be seen from **Fig.2b** (right), the three techniques provide very similar results except for high saturation values.

Conclusions

1. A method for simultaneous determination of relative permeability and capillary pressure functions from equilibrium and transient data measured during steady state experiments has been developed and tested.
2. The accuracy of the estimates has been calculated using a linearized covariance analysis. The analysis shows that all the curves can be determined accurately over a large saturation range.
3. The feasibility of the method has been demonstrated by analyzing simulated and actual experimental data. For the experimental case, the estimated capillary pressure function corresponds well with independent estimates using the micro membrane and mercury injection techniques.

Nomenclature

$\vec{a}$	Sensitivity coefficient
$\boldsymbol{A}$	Sensitivity matrix
$\vec{b}$	Constraint vector
B	Spline basis functions
C	Spline coefficients
$\boldsymbol{C}$	Covariance matrix
$\boldsymbol{G}$	Constraint matrix
J	Objective function
k	Permeability
M	Number of measured data
m	Spline order
N	Dimension of spline
N_{cap}	Capillary number
S	Saturation
v	Superficial velocity
x	Length coordinate
$\vec{y}$	Spline partition
$\vec{Y}$	Vector of measured or simulated data
$\boldsymbol{W}$	Weighting matrix

Greek letters

$\vec{\beta}$	Vector of unknown parameters
μ	Viscosity
σ	Interfacial tension

Subscripts

c	Capillary
m	Measured
nw	Nonwetting phase
r	Relative
s	Simulated
w	Wetting phase

Superscripts

c	Capillary
nw	Nonwetting phase
w	Wetting phase

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