The Impact of Using Composite Cores on Core Analysis Results

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Abstract

We present a theoretical study of how the properties of the individual cores in a composite core influence the measured relative permeabilities of a waterflood. The assumptions are that relative permeabilities and porosities are equal for all individuals in the composite core, and also that the individual capillary pressures are scaled by a Leverett-J relationship. Realistic data from a North Sea sandstone reservoir are used to select all core and fluid properties.

A new criterion for ordering individuals in composite cores is proposed. The best ordering is after decreasing permeability in the flow direction, i.e., such that the core with the lowest permeability is placed at the outlet. This ordering will produce a residual oil saturation closest to the true one. Our new criterion is in conflict with what we believe is common practice, as well as with the Huppler criterion (Huppler,1969).

Introduction

Relative permeabilities are important parameters for characterizing fluid flow in an oil and gas reservoir. Since reservoir cores are relatively small (5-7 cm), several such cores are butted together, to form a composite core for relative permeability experiments. This increase of length of the sample reduces the influence of the capillary end effects and increases the relative accuracy of volume measurements. The underlying assumption is that the individual cores are similar both with respect to capillary pressure and relative permeability functions. Also, one assumes capillary continuity between individual cores. Hence, the composite is regarded as one single core, and the interpretation of experimental results is done accordingly.

In reality, the different components often have different properties. The ordering of the composite can then be crucial for the correspondence between measured and true relative permeabilities.

Huppler (1969) presented an ordering criterion, based on the permeability of the individual cores. He neglected capillary effects, assumed equal individual relative permeability curves, and considered a waterflood unsteady state displacement evaluated with the JBN-method (Johnson, Bossler and Naumann, 1959). The criterion is to order the individual cores such that the harmonic-average-permeability of all sections (a section is a part of the total composite core, built up of one or several individual cores) is as close as possible to the over-all average permeability for the composite. Also, those sections having average permeabilities that are closest to the over-all average permeability should be located near the end of the composite. The last argument is based on the claim that the pressure gradient has the largest value at the end, which is true only for unfavourable mobility ratios. (In his example he uses a water to oil viscosity ratio of 1:20). With four rocks of identical lengths and porosities, and individual absolute permeabilities 400, 600, 800 and 1000 mD, the harmonic average is 623 mD. Therefore the 600 mD core is placed at the outlet. The 800 mD rock is next, since this gives a harmonic average of this two-core-section closer to the harmonic average of the total composite, than using either the 400 mD or the 1000 mD rocks. After one more calculation, the Huppler criterion gives the following ordering along the flow direction: 1000, 400, 800 and 600 mD.

However, a common practice, both in steady state and unsteady state displacement experiments, appears to be ordering after increasing permeability, i.e., having the one with highest permeability at the outlet. The argument is that the capillary forces are lower in a high permeability medium, and hence the capillary end effects should become minimal. Below, we show that this is a wrong assumption.

To our knowledge, the problem of how to best order a composite core in the presence of capillary effects, has not previously been addressed.

Theory

One dimensional two-phase flow in porous media without gravity effects is assumed to follow the generalized Darcy law,

$$u_j = -\frac{kk_{rj}(S_w)}{\mu_j}\frac{\partial p_j}{\partial x}. \tag{1}$$

Here, u_j is the Darcy velocity, k absolute permeability, k_{rj} relative permeability, μ_j fluid viscosity, and p_j pressure of phase $j(o, w)$.

Oil and water pressures are related through the capillary pressure,

$$P_c(S_w) = p_o - p_w. \tag{2}$$

Leverett (1941) suggested a relation between the capillary pressure of different porous media,

$$J(S_w) = \frac{P_c(Sw)}{\sigma_{ow}\cos\theta}\sqrt{\frac{k}{\phi}}, \tag{3}$$

where σ_{ow} is the interfacial tension between oil and water, θ the wettability angle, and ϕ the porosity. J is what has become known as the Leverett-J function, common to all media in question. Brown (1951) found that this relationship was able to correlate a number of samples of limestone from the same lithology type.

Marle (1981, p. 23) doubted the general validity of Eq. 3, but said that it is a reasonable assumption that $P_c \propto \sqrt{\phi/k}$, as long as the cores are of same lithology type. This should be true for a composite core, where the individual cores are drilled close-by from one seal peel. We therefore assume that

$$P_c(Sw) = \tilde{J}(S_w)\sqrt{\frac{\phi}{k}}, \tag{4}$$

throughout the study.

Steady state saturation distribution

If one knows the steady state (SS) saturation distribution in a composite core, one is able to calculate the effective relative permeabilities (Dale et.al., 1994). These relative permeabilities are exactly the same as those measured in a standard SS experiment (given that the theory of two-phase flow is correct).

Knowing the outlet boundary condition, S_w^1, the SS saturation distribution for a homogeneous (single) core of length l is given by,

$$x(S_w) = l + \frac{k}{u}\int_{S_w^1}^{S_w} \frac{k_{rw}k_{ro}dP_c/dS_w}{f_w(\mu_o k_{rw} + \mu_w k_{ro}) - \mu_o k_{rw}} dS_w, \tag{5}$$

where f_w is the fractional flow of water. Inserting Eq. 4 into Eq. 5 gives

$$x(S_w) = l + \frac{\sqrt{k\phi}}{u}\int_{S_w^1}^{S_w} \frac{k_{rw}k_{ro}d\tilde{J}/dS_w}{f_w(\mu_o k_{rw} + \mu_w k_{ro}) - \mu_o k_{rw}} dS_w. \tag{6}$$

At $f_w = 1$ this simplifies to

$$x(S_w) = l + \frac{\sqrt{k\phi}}{u\mu_w}\int_{S_w^1}^{S_w} k_{rw}\frac{d\tilde{J}}{dS_w} dS_w. \tag{7}$$

Single core

The standard outlet boundary condition for a core experiment is zero capillary pressure in the outlet chamber. We assume that both phase pressures, and thereby the capillary pressure, are continuous across the outlet-interface, and that the capillary pressure takes the value zero for some water saturation S_w^0. Hence, for a single core, we have $S_w^1 = S_w^0$.

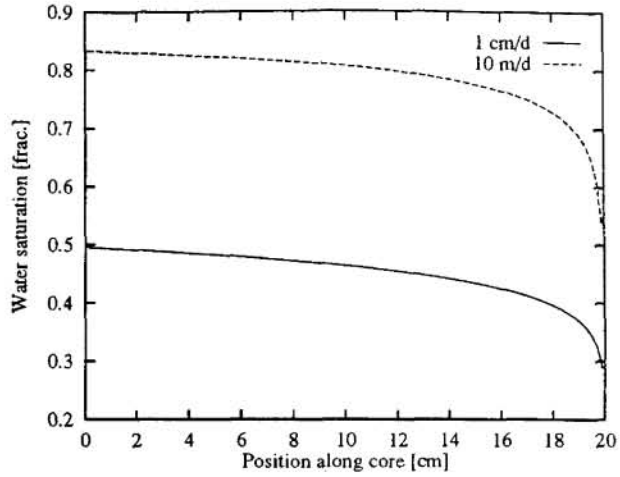


Figure 1: Saturation profiles ($f_w = 1$) in a homogeneous 1000 mD core at different rates.

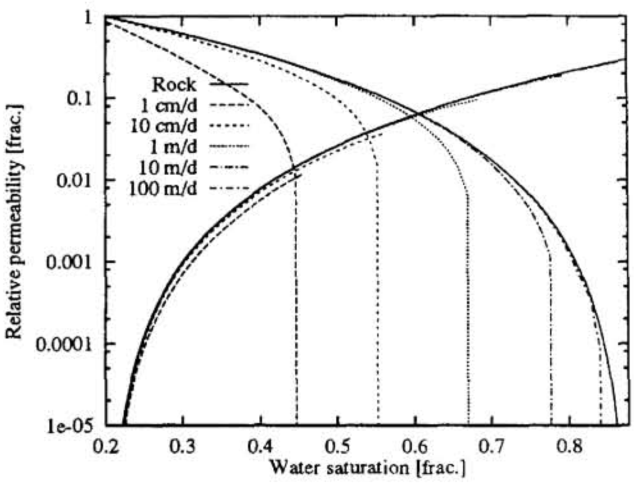


Figure 2: Steady state relative permeabilities for different Darcy velocities. Homogeneous 1000 mD rock.

From Eq. 6, a small $\sqrt{k\phi}/u$ corresponds to a large x (close to l), and in turn, to a large saturation gradient. Fig. 1 illustrates this for a homogeneous 1000 mD rock of length 20 cm with otherwise the base data set (see below). Increasing the rate is seen to lower the end-effect. Here S_w^0 is equal 0.2. Fig. 2 shows the SS relative permeabilities calculated for different Darcy velocities. As the rate increases, they approach the true (rock) relative permeabilities.

Similarly, a low permeability will give a small end-effect, see Eq. 6. This is because the viscous forces increase as $1/k$, while the capillary forces increase only as $1/\sqrt{k}$.

In the limit $f_w = 1$, Eq. 7 shows that even the water viscosity goes outside the integral, and a high water viscosity will give a small end-effect and an estimated residual oil

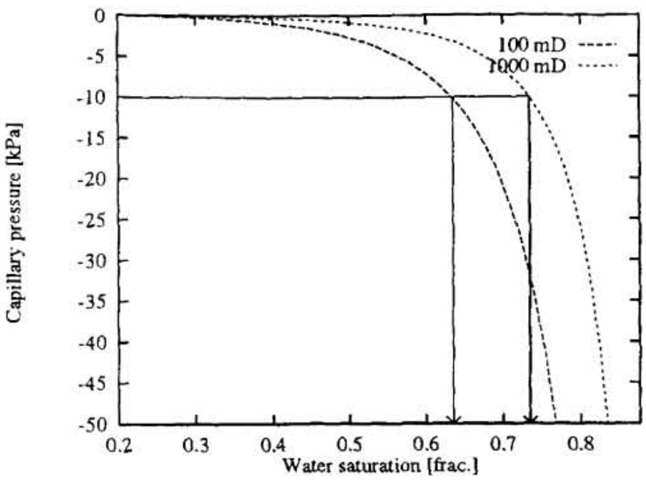


Figure 3: Capillary continuity causes a saturation jump between two neighbour cores.

saturation close to the true one.

The shape of the capillary pressure will influence through the value of $d\tilde{J}/dS_w$ in Eq. 6. An increase in magnitude of $d\tilde{J}/dS_w$ will increase the capillary end-effect.

Dividing by l in Eq. 6 shows that a large l decreases the relative end-effect.

Composite core

Throughout the study, it is assumed that a composite core is comprised of individually homogeneous cores. For a composite core, we can calculate the saturation distribution in the core closest to the outlet (the outlet-core), where S_w^1 is known (typically S_w^0). At the interface between the outlet-core and its neighbour, we again assume capillary continuity, such that the capillary pressure is continuous across the interface. This will often cause a discontinuity in the saturation. Fig. 3 gives an example of this, assuming that the 100 mD rock is the outlet-core, and that its saturation at the interface is 0.63, which corresponds to -10 kPa capillary pressure. Capillary continuity then implies that the capillary pressure of the neighbouring rock (1000 mD) also is -10 kPa at this interface, which corresponds to a water saturation of 0.73. The result is an upward jump in the water saturation at the interface, as seen from the outlet core. The boundary (outlet) condition for the new rock is then known, and it is possible to calculate its saturation distribution through Eq. 6. Continuing in this way, we are able to calculate the saturation distribution of the whole composite core, and through it, the SS relative permeabilities. This routine is implemented in the software EFFECTIVE (Ekrann, 1995).

An important parameter in this calculation is the estimated residual oil saturation,

$$S_{or}^{\text{est}} = \frac{\int_0^L (1 - S_w(x)_{f_w=1})\phi(x)dx}{\int_0^L \phi(x)dx}. \tag{8}$$

Table 1: Base data set

Permeabilities [mD]	400, 600, 800, 1000
Irreducible water sat. [frac.]	0.2
Residual oil sat. [frac.]	0.123
Porosity [frac.]	0.3
Core length [cm]	4*5
Viscosity of water [cP]	0.4
Viscosity of oil [cP]	2.4
Darcy velocity [m/d]	2.53
Rate (3.81 cm diam.) [ml/min]	2

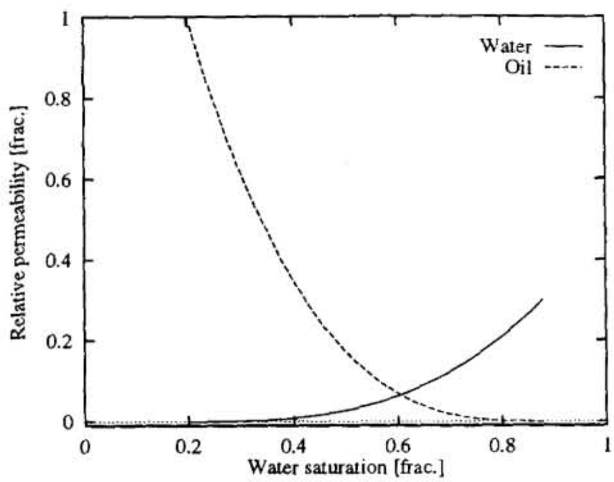


Figure 4: Relative permeabilities used throughout the analysis.

Base data set

A base data set was chosen to match realistic data from a North Sea sandstone reservoir. Some of these properties are given in Table 1. The flow rate is 2 ml/min for a 3.81 cm in diameter core, corresponding to a Darcy velocity of 2.53 m/d. In this study, the porosity and relative permeability are assumed equal for all the individual rocks. Equal porosities should be a reasonable assumption, since this parameter varies little compared to the permeability in a typical composite core. With equal relative permeabilities, we know the "true" average relative permeabilities of the composite core, which are the individual relative permeabilities. We also assume full capillary continuity between the segments.

The core-flooding scenario to be studied was that of water displacing oil. Relative permeabilities were Corey-type of degree 3, $k_{rw} = 0.3\tilde{S}^3, k_{ro} = (1-\tilde{S})^3$ where $\tilde{S} = (S_w - S_{wi})/(S_w - S_{wi} - S_{or})$, see Fig. 4. The capillary pressure shows no spontaneous imbibition of water, $P_c =$

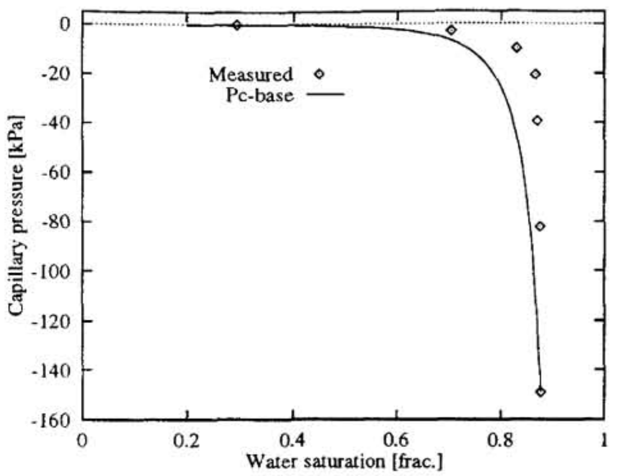


Figure 5: The capillary pressure used as a basis in this analysis. Also shown are measured data from a typical North Sea reservoir.

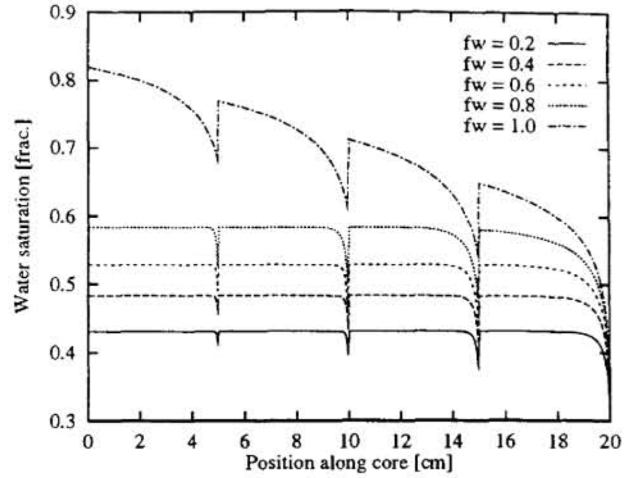


Figure 6: Steady state saturation profiles for the extreme permeability variation case. Ordering 1234.

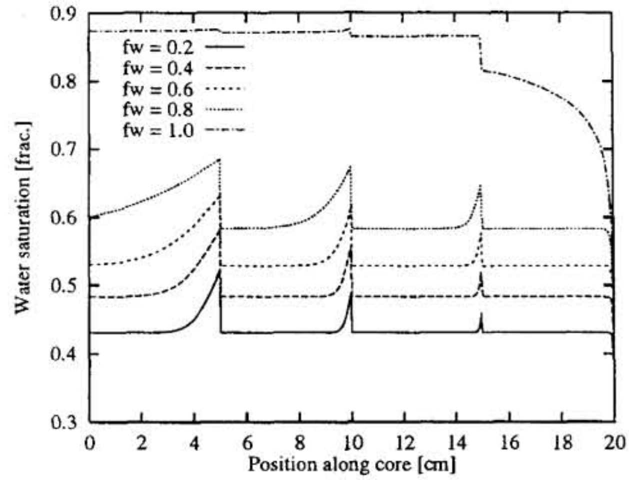


Figure 7: Steady state saturation profiles for the extreme permeability variation case. Ordering 4321.

$a((1+\epsilon-\tilde{S})/(\epsilon+\tilde{S}))^{\gamma}$, $P_c(S_{wi}) = -0.01$ kPa, $P_c(1-S_{or}) = -149$ kPa, $\gamma = 2$, see Fig. 5. This capillary pressure is always associated with a 1000 mD rock. The capillary pressures of the other rocks, are given through Eq. 4.

When nothing else is specified, we always use this set of data.

The best ordering of a composite core

The following short notation is used to describe the composite cores: The individual core with smallest permeability is called 1, the one with the second smallest permeability 2 etc.. With this notation, a composite of four individual cores, ordered after increasing permeability in the flow direction, is called 1234. Ordering after decreasing permeability becomes 4321. With four different individual cores there are 24 combinations. The two most important ones are 1234 and 4321, but the one proposed by Huppler will also be presented for comparison.

Initially an extreme case of four 5 cm cores of permeabilities 10, 100, 1000 and 10000 mD were used. The 1000 mD rock keep the base capillary pressure in Fig. 5, and the others were scaled by Eq. 4. Fig. 6 and Fig. 7 show the steady state saturation profiles for the orderings 1234 and 4321. Several important observations can be made from them:

The outlet rock has the same outlet boundary condition in the two combinations ($S_w^0 = S_{wi}$). As explained by Eq. 6, the 10000 mD rock (4) has a saturation profile which increases much more slowly than that of the 10 mD rock (1), going from right to left. Focusing at the $f_w = 1$-curve, we see that 4321 has a much better starting point to get a good S_{or}^{est}. Continuing to the left across the interface gives saturation upward jumps for 4321 while downward jumps for 1234, as discussed previously.

The $f_w = 1$ saturation profile for 1234 shows huge capillary effects, and a poor S_{or}^{est} is therefore expected. The 4321-combination is seen to estimate this parameter much better.

For fractional flows 0.8 and below, it is seen that the internal end-effects are a bit larger in 4321 than in 1234. This will affect the form of the estimated relative permeability curves.

Fig. 8 shows the saturation distribution with the ordering after Huppler (2341 for these permeabilities). This ordering causes a dramatic internal end-effect at low f_w's between two rocks, resulting in poor relative permeabilities at low water saturations. The S_{or}^{est} for 2341 is much better than for 1234, but not as good as for 4321.

The relative permeabilities for these three composite cores are given in Fig. 9, and it is easy to see the correspondence to the saturation profiles. The 1234-

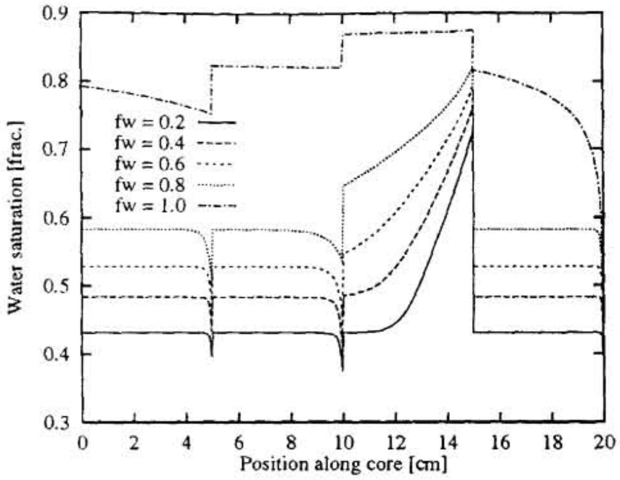


Figure 8: Steady state saturation profiles for the extreme permeability variation case. Ordering 2341.

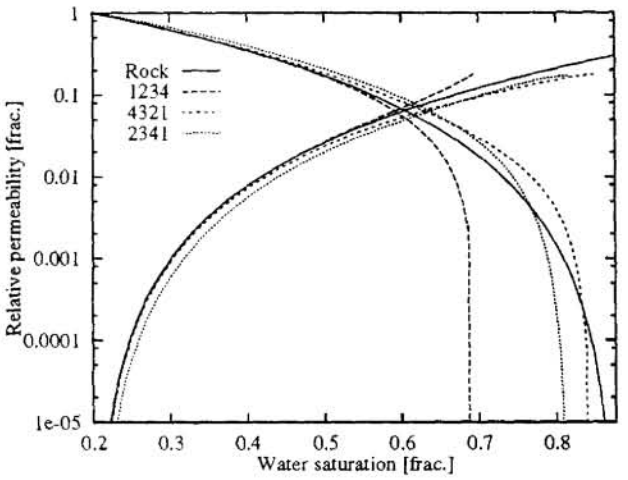


Figure 9: Steady state relative permeabilities for different compositions of the extreme permeability variation case.

combination gives a bad S_{or}^{est}, and also a bad estimate for water and oil relative permeabilities for water saturation above 0.6. However for water saturations below this value, 1234 is seen to give the best estimation. Huppler's composite, 2341, gives the worst relative permeabilities for water saturations below about 0.6. The 4321-combination is clearly the best, both with respect to estimated form of the relative permeabilities and S_{or}^{est}.

The extreme case above is not realistic. Therefore, these results were quantified for the base permeability distribution (400, 600, 800 and 1000 mD). Fig. 10 shows the relative permeabilities for 1234, 4321 and 4132 (Huppler's ordering). The qualitative difference in S_{or}^{est} between the compositions is the same as for the extreme case. The 4321-combination does the best job. On the other hand, ordering 1234 now has the best match to the water relative permeability in its entire interval (below a water saturation of about 0.73). Ordering 4321 underestimates the

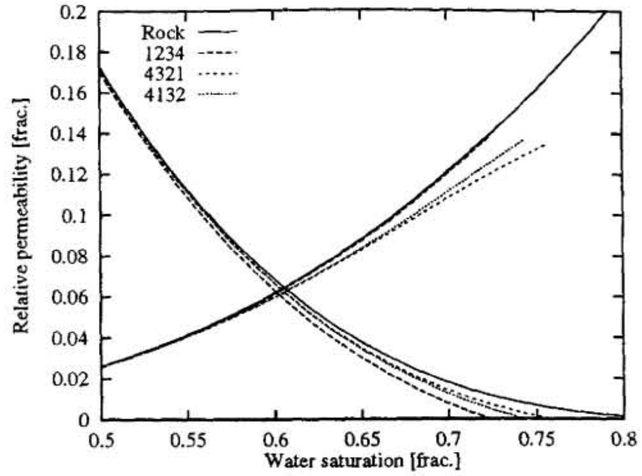


Figure 10: Steady state relative permeabilities for different compositions. Base data set.

water relative permeability endpoint, because at $f_w = 1$ the outlet-core, which has the smallest permeability, also has a low water saturation and a low water relative permeability, and it will therefore make a large contribution to the total relative permeability.

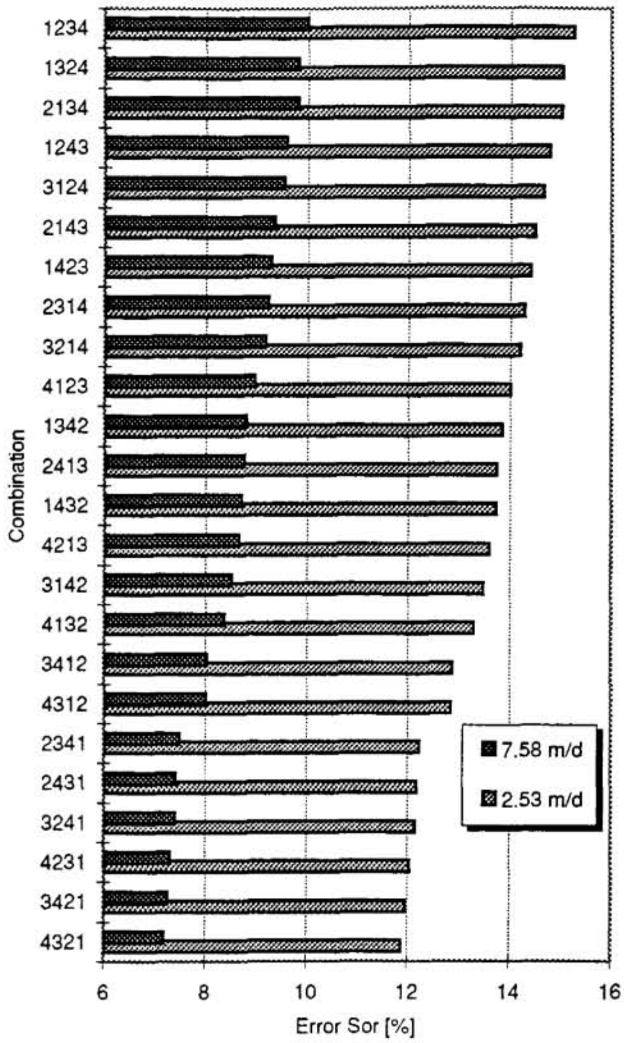
Which is the best combination? In the high water saturation part, the oil relative permeability is the most important, and S_{or} is a very important parameter. We therefore use S_{or}^{est} as a quality-parameter, and by this parameter, 4321 is the best combination. The 4321-combination has an error of 11.9 % in the S_{or}^{est}, while for 1234 the error is 15.3 %. Huppler's combination, 4132, comes in between. Fig. 13 shows the error in S_{or}^{est} for all the 24 possible orderings, for two different flow rates, 2.53 m/d (2 ml/min) and 7.58 m/d (6 ml/min). It is seen that 4321 is the best, and 1234 the worst to estimate S_{or}. For these two flow rates the same ranking is observed.

In the Appendix, we give a proof that 4321 always estimates S_{or} better than 1234.

Shape of Leverett-J function

In the theory section, we discussed the impact of the shape of the capillary pressure curve. To quantify this for our base data, three different Leverett-J functions, were used. Fig. 12 shows the 1000 mD capillary pressure curves based on these J-functions. All capillary pressures are of the same functional type, without any spontaneous part. Pc1 is the original base. Two others, which are closer to the measured data points, are also constructed. For each of these bases, we have calculated the SS relative permeabilities for the composites 1234, 4321 and 4132 using the 400, 600, 800 and 1000 mD rocks. Table 2 shows that all three Leverett-J functions give the same qualitatively picture regarding S_{or}^{est} for the different combinations.

When the capillary pressure has both a spontaneous and a forced part, spontaneous imbibition will result in



igure 11: Error in S_{or}^{est} for all 24 orderings of permeabil-
y.

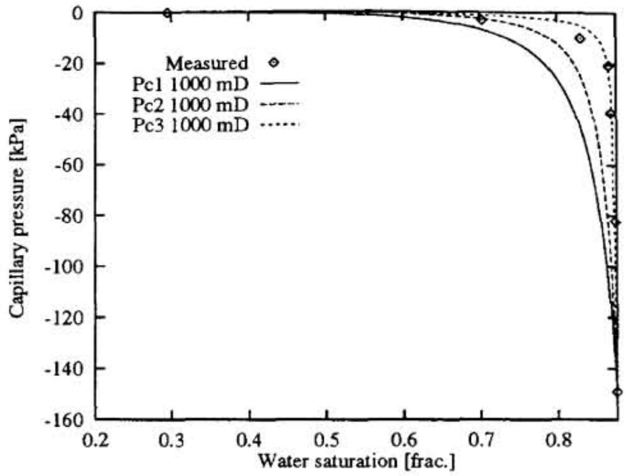


Figure 12: 1000 mD capillary pressure curves for three different Leverett-J functions.

Table 2: Error in S_{or}^{est} [%] for three Leverett-J functions

Leverett-J	1234	4321	4132
Pc1	15.27	11.90	13.31
Pc2	7.94	6.11	6.87
Pc3	5.49	3.76	4.47

fferent SS relative permeabilities at low water satura- ons (in the spontaneous region). However, if the spon- .neous part does not dominate the capillary pressure, will not alter the picture regarding S_{or}^{est} and estimated lative permeabilities at high water saturations (in the rced region).

ıdividual core length

ı the Appendix, we prove that 4321 always estimates S_{or} ɜtter than 1234, independent of the individual lengths. 'e demonstrate this by looking at a realistic problem. ɿy, the available core material is three 6 cm long cores ˙permeability 1000 mD, and one 2 cm long core of per- eability 400 mD, all drilled close-by from one seal peel. it worthwhile to take this short core into the compos- e? Fig. 13 shows the SS relative permeabilities of three ımposite cores; 444 represents the composite core of the ıree 1000 mD rocks, 1444 further includes the short 400 mD rock at the inlet, and 4441 has the short core at the outlet. Placing the short core at the inlet gives hardly any effect, but having it at the outlet reduces the error in S_{or}^{est} with almost 3 %. We recommend therefore to add the short 400 mD core to the composite and place it at the end.

Viscosity ratio

The effect of water viscosity on S_{or}^{est} was discussed previously. For intermediate fractional flows, the effect is harder to see. To quantify this, the oil-viscosity was kept constant at 2.4 cP while the water viscosity was varied. Fig. 14 shows the resulting relative permeabilities. As expected, increasing water viscosity improves S_{or}^{est}. No dramatically change in the form of the estimated relative permeabilities is observed.

Unsteady state

The industrial standard of relative permeability measurements is the unsteady state method. There are two good arguments for performing USS experiments: First of all this process takes less time than a SS experiment. Secondly, the flow process is believed to be more similar to that in the actual reservoir with a moving front. The problem with USS is that the evaluation method, the JBN-method (Johnson, Bossler and Naumann, 1959), is

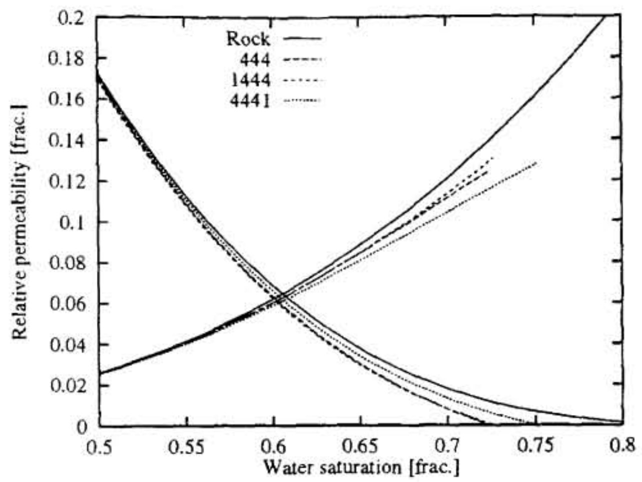


Figure 13: SS relative permeabilities for different composite cores made of three 6 cm, 1000 mD cores and one 2 cm, 400 mD core.

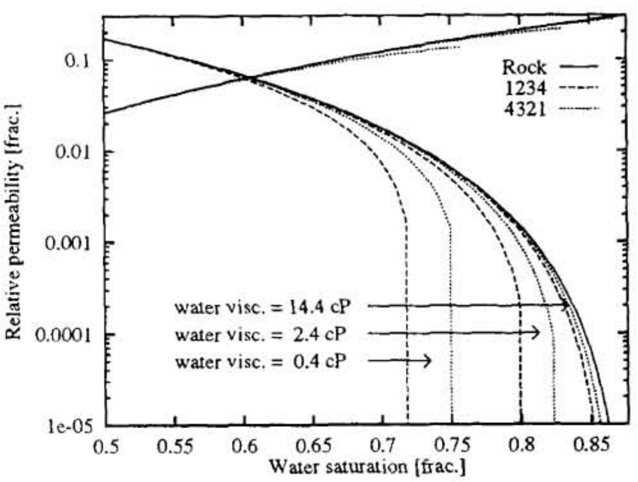


Figure 14: SS relative permeabilities for different water viscosities. Oil viscosity is always 2.4 cP.

based on the Buckley-Leverett (1942) solution with zero capillarity. This is not always a good approximation.

Both the endpoint relative permeabilities and endpoint saturation are equal for USS and SS, if only the USS-experiment is run long enough. Since S_{or}^{est} is equal for the two processes, our compositional criterion found for SS is valid also for USS, as measured by this parameter. The purpose of this section is to investigate how the USS evaluation method affects the form of the estimated relative permeability curves.

Artificial production data were produced using the core-flood simulator CENDRA (Guo,1993), and USS relative permeabilities were calculated with the JBN-method. To validate our JBN-implementation and to control numerical dispersion in CENDRA, we have used an extension of the analytical Buckley-Leverett solution (Buckley and Leverett, 1942) for a composite core.

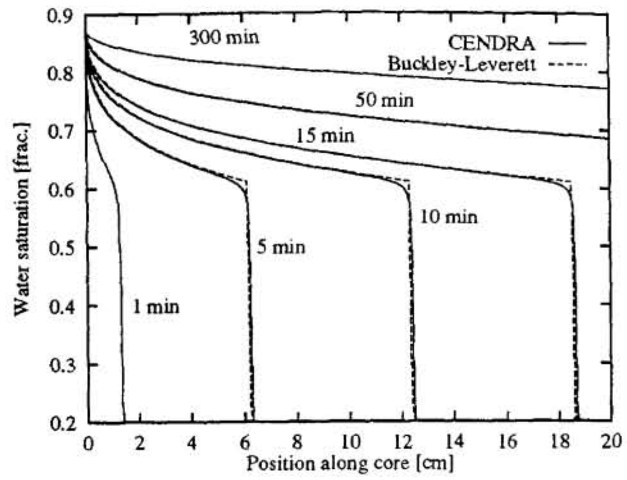


Figure 15: USS saturation profiles without capillary pressure. Other properties as the base.

Without capillary pressure

For the base composite core without capillarity we compared CENDRA with the generalized Buckley-Leverett solution. 400 equally distributed grid cells were used in the simulation, and Fig. 15 shows that this was sufficient to produce almost as sharp fronts as the analytical solution.

With the JBN-method, only data after breakthrough are used, i.e., after the front arrives at the outlet. Therefore the estimated relative permeabilities are determined in the interval from the front-saturation (about 0.62), and upwards to 1-S_{or} (0.877).

Huppler (1969) derived his compositional criterion for USS water-floods with no capillarity. Since his conclusion is based on an unfavourable mobility ratio, the criterion should apply to our data when capillary pressures are zero. Without capillary pressure, the production curves and the saturation profiles (Fig. 15) are equal for all orderings, while the pressure distributions are different. Fig. 16 shows the USS relative permeabilities for Huppler's composite, 4132, together with those of 1234 and 4321. The 4132-ordering is clearly the best composition. These simulations are all run to 3000 minutes (87.7 PV) to ensure the agreement between SS and USS relative permeabilities.

It is important to note that the USS relative permeabilities of a composite core of equal individual relative permeabilities and no capillarity, not become the true (rock) relative permeabilities, as is the case for SS relative permeabilities. (In this case, with no capillarity, the SS relative permeabilities for all orderings are the rock relative permeabilities.)

With capillary pressures

With capillary pressure present, the displacement front will be smeared out and an end-effect will be built up.

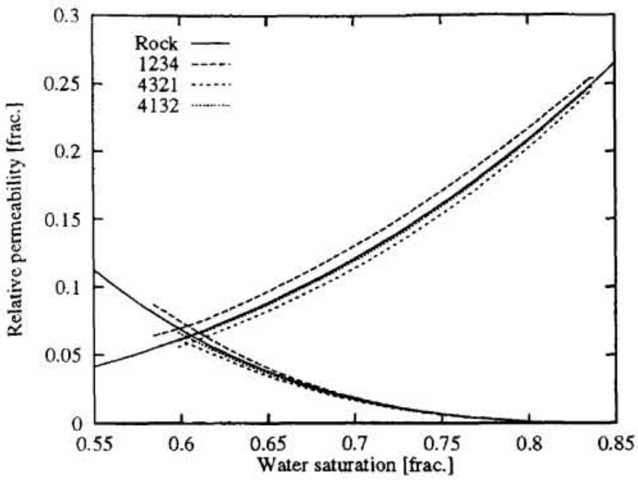


Figure 16: USS relative permeabilities without capillary pressure.

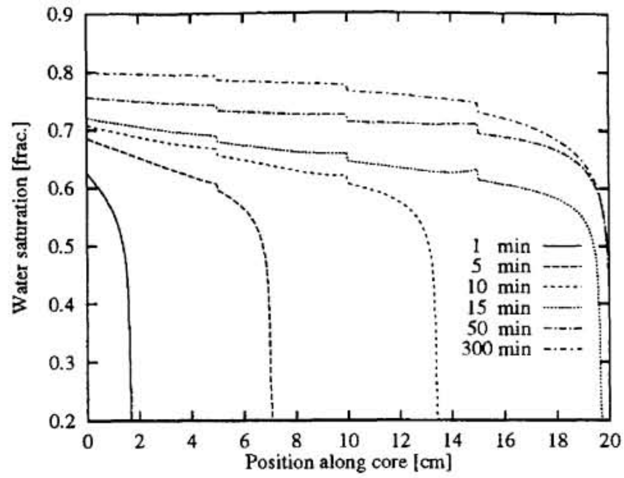


Figure 18: USS saturation profiles with capillary pressure. Ordering 4321.

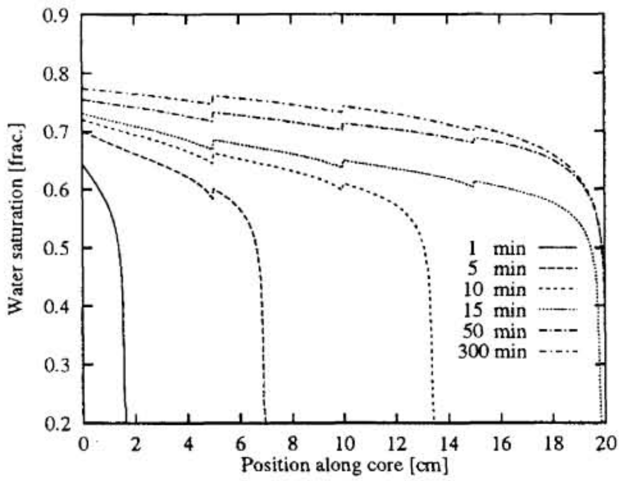


Figure 17: USS saturation profiles with capillary pressure. Ordering 1234.

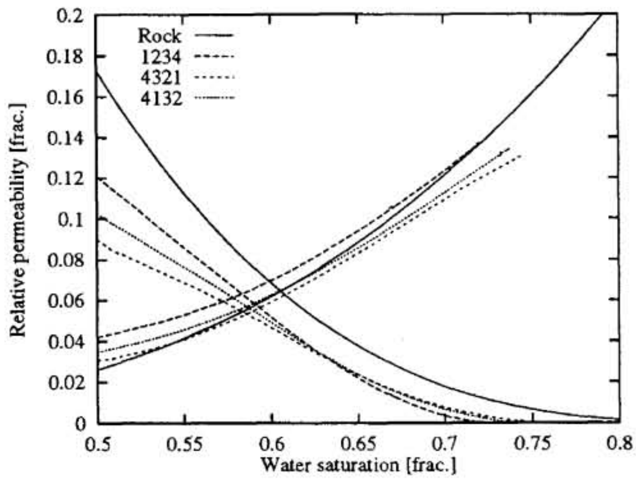


Figure 19: USS relative permeabilities with capillary pressure.

The JBN-method is no longer ideal, but is applied anyway, as it is used in practice.

The base rock and fluid properties are used. Now different ordering also causes different production/saturation data, see Fig. 17 and Fig. 18. The effect of the capillarity can be quantified by comparing the saturation profiles in Fig. 17 and Fig. 18 with those of Fig. 15, which is for the no capillarity case. More importantly, the same saturation-jumps are observed as for steady state. This is again caused by capillary pressure continuity across the interfaces between individual rocks.

Fig. 19 shows the USS relative permeabilities from the different orderings. Now also data for water saturations below the no-capillary-front-saturation (0.62) are obtained. Focusing on the oil relative permeability, it is seen that none of the orderings give a particularly good estimate. This is caused by the level of the capillary pressure used. Different ordering of the components is seen to give minor effects, at least for water saturations above 0.6.

It is of interest to compare USS and SS relative permeabilities. Fig. 10 shows the SS relative permeabilities that corresponds to the USS relative permeabilities in Fig. 19, and it is seen that the SS relative permeabilities are much better. The endpoint saturations and endpoint relative permeability values are almost the same for the two cases, which is expected a priori. In SS, S_{or}^{est} is seen to describe the quality of the oil relative permeability. In USS, this is only the case for water saturation above 0.63, which is approximately the no-capillary front saturation.

Summary and conclusions

We have studied how the individual core properties of a composite core impact on the measured relative permeabilities. The calculated relative permeabilities in this study correspond exactly to those measured in standard SS and USS experiments. Special attention has been given finding the best ordering of cores when capillarity is present. Core and fluid properties were selected to match realistic waterflood data from a North Sea sandstone reservoir.

The assumptions made are that all individual cores have equal relative permeability curves and porosities, and also that their capillary pressures follow a simplified Leverett-J relationship.

We have these conclusions:

1. Given a constant injection rate, a (single) core with low permeability will have a smaller end effect than a (single) core with a high permeability.

2. All our computations suggests that the best ordering of a composite core, as measured by the estimated residual oil saturation, is after decreasing permeability along the flow direction, i.e., such that the core with lowest permeability is placed at the outlet.

3. We have proved mathematically that the suggested ordering allays is better than the opposite ordering, of increasing permeability, again as measured by estimated S_{or}. This result is independent of the individual lengths and mobility ratio.

4. In USS with zero capillarity, Huppler's criterion holds for our example. With capillary pressure however, our proposed criterion is better, since estimated S_{or} are equal for USS and SS.

5. The SS method gives a better estimate of the relative permeability curves than do the USS method.

Nomenclature

Symbols

f	fractional occurrence
J	Leverett-J function
$\tilde{J}$	extended Leverett-J function
k	absolute permeability
k_r	relative permeability
l	length of singel core
L	total length of composite core
N	number of individual cores
p	pressure
P_c	capillary pressure
S	saturation
$\tilde{S}$	normalized water saturation
u	Darcy velocity
x	coordinate along flow direction
μ	viscosity
ϕ	porosity
σ	interfacial tension
θ	wettability angle
a, ϵ, γ	unimportant coefficients

Subscripts

i	initial, component
j	phase, component
o	oil
r	residual
w	water

Superscripts

d	decreasing ordering
est	estimated
i	increasing ordering
$*$	arbitrary ordering
0	evaluated at $P_c = 0$
1	evaluated at the outlet

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Appendix

Consider a composite core made up of N individual rocks, with absolute permeabilities $k_i, i = 1, N$. Individual rocks may have different lengths. Porosity and relative permeabilities are assumed to be the same in all rocks. Furthermore, all rocks share a common Leverett-J function, making capillary pressure inversely proportional to the square root of permeability, see Eq. 4. The Leverett-J function is assumed to take the value zero for some water saturation $S_w^0 < 1 - S_{or}$. Relative permeabilities and Leverett-J curves are supposed to satisfy their usual monotonicity requirements, see Figs. 4 and 5. Rocks are labelled after increasing permeability, i.e., such that $k_j \geq k_i$ whenever $j > i$. We shall compare the two monotonic orderings of the composite core, i.e., with permeabilities increasing $(1, 2, .., N)$ or decreasing $(N, N-1, .., 1)$ from inlet to outlet. With the assumptions listed, and with a finite injection rate, we claim that

$$S_{or}^{\text{est}}(N, N-1, .., 1) \leq S_{or}^{\text{est}}(1, 2, .., N)$$

where equality prevails only in the trivial case of equal permeability of all individuals.

Outline of proof

Let superscript d indicate the decreasing ordering $(N, N-1, .., 1)$, i the increasing ordering, and $*$ an arbitrary ordering of rocks in the composite core. The discussion to follow pertains to $f_w = 1$ (Corresponding to S_{or}^{est}).

- Neglect first the saturation discontinuities at the intersections between rocks, and show that $S_w^d(x) \geq S_w^*(x), 0 \leq x \leq L$:
  - If two adjacent rocks i, j in the arbitrary sequence have the higher permeability rock (j) closest to the outlet, interchange the two rocks and use Eq. 7 to show that the two cases have identical saturations at the endpoints of the subinterval, and $S_w^{j,i} \geq S_w^{i,j}$ inside. Equality prevails throughout the subinterval only if $S_w = 1 - S_{or}$ at its outlet end.
  - Interchanging pairs of rocks does not change the saturation profile outside the subinterval, due to the non-influence on endpoint saturations. Thus, one can continue switching pairs until the decreasing sequence has been obtained, never decreasing the water saturation at any point in the process.
- In particular, $S_w^d(x) \geq S_w^i(x), 0 \leq x \leq L$. Equality cannot prevail throughout, since $S_w(L) = S_w^0 < 1 - S_{or}$.
- Include next the saturation discontinuities. In moving from outlet to inlet, the saturation always increase across rock intersections for the decreasing ordering, and decrease for the increasing ordering. This is examplified in Figs. 6 and 7. Thus, we have shown $S_w^d(x) \geq S_w^i(x), 0 \leq x \leq L$ for the full problem. Equality cannot prevail throughout. Since porosity is constant, the original statement follows.