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A New Concept for the Correlation of Relative Permeability Data and Capillary Pressure for Microfissured Rocks

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1. Introduction

Multiphase flow in porous media is generally explained on the basis of the modified Darcy's law. The modification requires the introduction of a fractional permeability, depending on phase saturation. The physical models for the description of the geometry of flow path and flow behaviour is consistently derived from simple pore structures, as for example capillary bundles and for piston like displacement or parallel flow of phases. The simplicity of this approaches is never compatible with the complexity of flow paths in natural rocks and therefore always needs empirical corrections. However, even the most simple models can help to understand measured data to adopt them for fluid flow modeling and to upscale processes.

The flow geometry in porous models is mainly controlled by pore size distribution and interconnectivity. There are two petrophysical parameters, which are based on pore size distribution and therefore can be intercorrelated, capillary pressure and permeability. Purcell /1/ has introduced a correlation between capillary pressure and rock permeability as a reliable tool to check the consistency of measured data. His model is based on cylindrical, parallel capillaries and his correlation requires an empirical correction for the influence of irregular shapes of capillaries and tortuosity of flow path. Fatt and Dykstra /2/ have extended this concept on two-phase flow of immiscible fluids assuming that the pore entrance pressure defines which part of the capillaries of different diameter under a constant pressure flow regime is flushed by the displacing phase. They have introduced the effect of tortuosity of flow paths depending on pore diameter. Burdine /3/ could improve this concept by expressing the tortuosity effect through a normalized saturation term, which distinguishes between the mobile and immobile part of flowing phases. Corey /4/ has replaced the capillary pressure functions through an empirical saturation dependent function which enables the engineer to correlate the measured relative permeability data with saturation.

A progress in the development of these deterministic models of two-phase flow was achieved by Stone /5/ in his 'multiphase flow-theory'. Assuming that the flow rate of the wetting and non-wetting phases are mainly dependent on the saturation of its mobile saturation part, the mobility of the intermediate phase depends on the saturation distribution of the accompanying phases. Using this concept he could treat three phase flow as a parallel process of 2 two-phase flow regimes. Additionally Stone has introduced a probabilistic concept, that flow channels of wetting and non-wetting phases are interconnected. The number and distribution of crossing channels retards the flow of the displacing phase, but phase properties like viscosity do not play any role.

Yuster /6/ and Odeh /7/ have pointed out the influence of the viscosity ratio on the non-wetting phase relative permeabilities, remarkably for low permeability rocks. If rock permeability exceeds 1 Darcy this effect has no importance anymore.

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Parallel to the development in reservoir mechanics the concept of relative permeabilities was also introduced in soil mechanics, using identical, empirical flow concepts as those used by Corey et al., Luckner and van Genuchten /8/ have reported on this concepts. The sealing efficiency of cap rocks or barrier rocks of underground repositories depends on the flow resistance for the migrating non-wetting or invading wetting phases. This type of rock usually exhibits permeabilities in range of nanodarcies up to microdarcies. Relative permeability functions are very difficult to measure. Steady-state two phase injection measurements cannot be applied. Alternatively constant pressure desaturation and resaturation methods are often used, which exhibit relatively low values for the non-wetting phase permeabilities. Following the observations and discussions on Odeh's relative permeability data on core samples of the millidarcy range a mathematical correlation based on a simple displacement concept was adopted to match the evidenced experimental results. A macroscopic two-phase flow model which takes into account the channel flow theory and the mobilities of the different phases has been published by Dykstra and Parsons /9/ for stratified layers. Their idea, that under a constant pressure regime the mobilities of the phases influence the rate of displacement in a staggered network of flow path has been adopted for our new concept of a microscopic two- phase flow model. Since our porous media of investigation are mainly from the type of low permeability rocks, as for example crystalline rocks, we have derived our theory on the basis of a network of parallel cracks, already published by Torsaeter et al. /10/.

In this paper the theory will be explained and applied to match relative permeabilities for non-wetting phase flow in less permeable porous media.

2. Theoretical Model

Crystalline rocks are composed from large crystal aggregates with a dimension of length in the order of millimeters up to a centimeter. Compared to the partical size of sedimentary rocks of some micrometers up to millimeters. Porosity of granites therefore is mainly of the intercrystalline type of voids and can be better represented by a network of parallel cracks than by network of cylindrical capillaries. The geometric model adopted for our theory is shown in figure 1 and is characterized by a 2-dimensional network of individual cracks of varying widths.

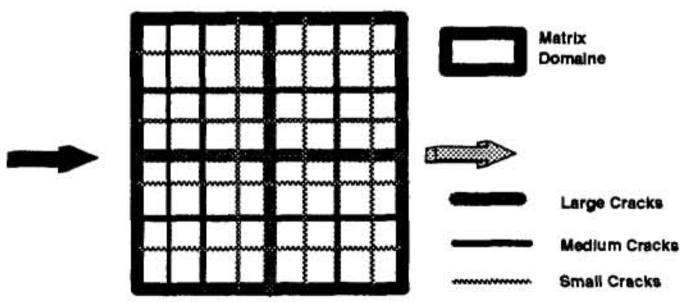

Fig. 1 2-Dimensional Crack Network

One dimensional flow in the x-direction is assumed for the sake of simplicity. But the theory can be extended to three-dimensional flow as well. The flow concept of existing models is mainly based on drainage conditions, where a non-wetting phase desaturates capillary after capillary in a sequence of decreasing diameter. Desaturation is only possible if the existing inflow pressure exceeds the entrance pressure of the pore.

If the pore diameter or crack width decreases below a critical value, the friction pressure loss may exceed the entrance pressure at it is shown in figure 2 for laboratory conditions.

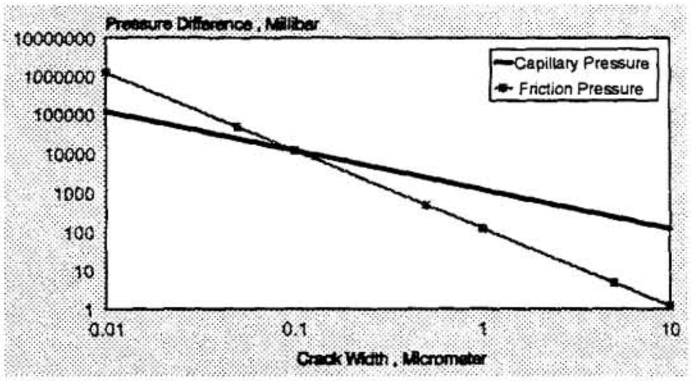

Fig. 2 Pressure Difference for Fluid Flow in a Crack Network (v = 0.86 m/d, L = 0.1 m)

This effect is enhanced when imbibition conditions are considered, since the wetting phase is spontaneously imbibed until a critical saturation level is approached and further on capillary displacement pressure is smaller than for drainage conditions. The following assumptions have been used to derive the theory:

- compressibility of fluids is neglected
- displacing and displaced fluids are immiscible
- the flow resistance of the rock voids determines the penetration of a displacing phase
- residual phase saturations in the flooded cracks are neglected, however they were considered in a fraction of inaccessible cracks
- pessure gradients which control the phase migration depend on phase viscosity
- the crack width distribution is characterized by a drainage capillary pressure curve
- tortuosity of flow path is neglected.

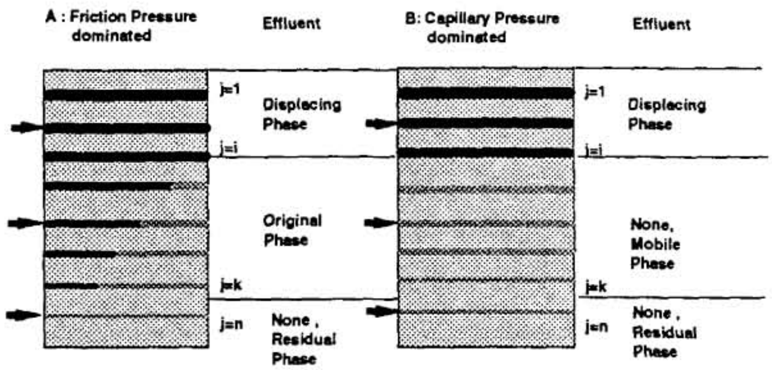

Fig. 3 Displacement Concepts for Channel Flow (Imbibition Mode)

Figure 3 explains the differences in the existing capillary displacement theory and the new concept of friction controlled displacement. Introducing this concept, the two-phase flow theory can be applied for imbibition and drainage conditions as well. For the characterization of crack flow the Hagen-Poiseuille law has been used under a constant pressure drop regime. Inflow and outflow of phases are considered as rate balanced. From this assumption the phase dependent pressure gradient can be derived :

$$\left(\frac{\Delta p}{\Delta l}\right)_w = V\left(\frac{\Delta p}{\Delta l}\right)_{nw} \quad V = \text{Viscosity Ratio } \frac{\mu_w}{\mu_{nw}}$$

$$\Delta p = x\left(\frac{\Delta p}{\Delta l}\right)_w + (L - x)\left(\frac{\Delta p}{\Delta l}\right)_{nw} \tag{1}$$

Using this phase dependent pressure gradients the front velocity of the displacing phase in a discrete crack with a length L and a width b_j can be calculated

$$v_{dj} = \frac{-b_j^2 \Delta p\, V}{12\mu_w (L + x_j(V-1))} \tag{2}$$

This equation 2 implies that for a constant pressure drop the front velocity depends on the viscosity ratio and on the front position x_j. If the front propagates to the end of the porous medium, the breakthrough-time of the displacing phase in an individual crack can be calculated, assuming that x_j equals L

$$\left[L + x_j(V-1)\right]dx_j = -b_j^2 \frac{\Delta p dt}{12\mu_{nw}}$$

Integration over the length x and the time elapsed t yields:

$$\frac{x_j^2(V-1)}{2} + Lx_j + \frac{b_j^2 \Delta pt}{12\mu_{nw}} = 0 \tag{3}$$

From this equation the break-through time t_i in the individual crack b_i can be derived setting x = L

$$t_i = -\frac{6L^2(V+1)\mu_{nw}}{b_i^2\ \Delta p}$$

The quadratic equation (3) can be solved with regard to x

$$x_j = \frac{1}{2(V-1)}\left[-2L + \left(4L^2 - 4\frac{(V-1)\, b_j^2\, \Delta p t_i}{6\mu_{nw}}\right)^{0.5}\right]$$

With ongoing time the displacing phase removes the non-wetting phase from the various cracks, ranked in a sequence of decreasing width b_j.
Introducing the term for the break-through time t_i into the equation for x_j, the saturation profile in the various cracks can be determined relative to the position i of the temporarily swept cracks

$$\frac{x_j}{L} = \frac{1}{(V-1)}\left[\left(1 + (V^2 - 1)\frac{b_j^2}{b_i^2}\right)^{0.5} - 1\right] \tag{4}$$

Following this concept of simultaneous influx of the displacing phase into the network of cracks, three zones have to be differentiated with respect to the form of fluid flow. For practical purposes it is assumed, that the rock is perfectly water wet.

Flow domaines:

1. Minor cracks where the wetting phase can partially imbibe . The remaining non-wetting phase and the immobile wetting phase volumes form the irreducible saturations.
2. Medium cracks where the wetting phase is displacing the non wetting in a piston like manner
3. Major cracks where the wetting phase has already broken through and is flowing as a single phase.

The flow rates of the recovered fluids can be calculated as follows

$$q_w = \sum_1^i \frac{2L\, b_i^3\, \Delta p}{12\, \mu_w L} \tag{5}$$

$$q_{nw} = \sum_{i+1}^k \frac{2L\, b_i^3\, \Delta p}{12\, \mu_{nw} L\left[1 + \frac{x_i}{L}(V-1)\right]} \tag{6}$$

The term $(b_j)^3$ can be replaced by the drainage capillary pressure function and by the partial water saturation ΔS_{wj} of the crack network.

$$b_j = \frac{2\sigma \cos\theta}{p_{cj}} \qquad \Delta S_{wj} = \frac{2 b_j L^2}{L^3 \phi} \frac{x_j}{L}$$

σ = interfacial tension $\qquad p_{cj}$ = capillary pressure

θ = wetting angle $\qquad \phi$ = porosity

Balancing the Hagen-Poiseuille Law for water and gas flow with the corresponding Darcy flow equations, yields an expression for the effective phase permeabilities

$$k_w = \frac{\sigma^2 (\cos\theta)^2 \phi}{3} \sum_1^i \frac{\Delta S_{wj}}{p_{cj}^2} \tag{7}$$

$$k_{nw} = \frac{\sigma^2 (\cos\theta)^2 \phi}{3} \sum_{i+1}^k \frac{\Delta S_{wj}}{p_{cj}^2} \left[\frac{x_j}{L}\left(1 + \frac{x_j}{L}(V-1)\right)\right]^{-1} \tag{8}$$

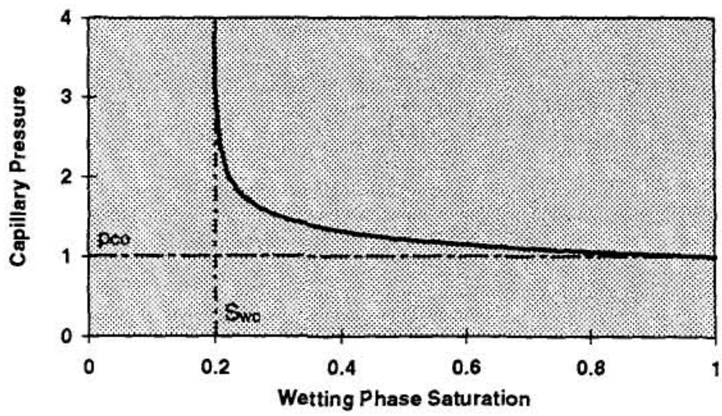

Fig. 4 Drainage Capillary Pressure Function

Figure 4 shows the drainage capillary pressure function from which the crack width distribution has been derived. This function can be normalized by a three parametric equation

$$p_{cj} = p_{co}\left(\frac{1-S_{wc}}{S_w-S_{wc}}\right)^m \tag{9}$$

S_{wc} = irreducible wetting phase saturation
p_{co} = capillary threshold pressure
m = curvature exponent

Relative permeabilities are expressed as the ratio of effective to a standard permeability. It is common use to adopt the water permeability at 100 % water saturation as the standard permeability. But it is also possible to use absolute permeability or non-wetting phase permeability at irreducible water saturation as a reference value.

$$k_{r_w} = \frac{k_w}{k_{w100}} \qquad k_{r_{nw}} = \frac{k_{nw}}{k_{w100}}$$

The material and geometric constants in front of the integral, which are identical for the effective phase permeability and for the standard permeability can be cancelled. Integral equations must be solved according to the defined boundary values

$$k_{r_w} = \frac{\int_{S_{wi}}^{1} \frac{dS_w}{p_{cj}^2}}{\int_{S_{wc}}^{1} \frac{dS_w}{p_{cj}^2}} \quad (10) \qquad k_{r_{nw}} = \frac{\int_{S_{wc}}^{S_{wi}} \frac{dS_w}{p_{cj}^2\left[\frac{x_j}{L}\left(1+\frac{x_j}{L}(v-1)\right)\right]}}{\int_{S_{wc}}^{1} \frac{dS_w}{p_{cj}^2}} \quad (11)$$

For the partially flooded cracks in the network, the equation for the relative position of the flood front (4) is introduced into the expression of the relative non-wetting phase permeability (11) and the crack width replaced by the capillary pressure function.

$$k_{r_{nw}} = \frac{\int_{S_{wc}}^{S_{wi}} \frac{(V-1)dS_w}{p_{cj}^2\left\{\left[1+(V^2-1)\left(\frac{p_{ci}^2}{p_{cj}^2}\right)\right]-\left[1+(V^2-1)\left(\frac{p_{ci}^2}{p_{cj}^2}\right)\right]^{0.5}\right\}}}{\int_{S_{wc}}^{1} \frac{dS_w}{p_{cj}^2}}$$

Finally the capillary pressure is replaced by the normalized function of the mobile wetting phase saturation (9).

$$k_{r_w} = \frac{\int_{S_{wi}}^{1} \left(\frac{S_w - S_{wc}}{1 - S_{wc}} \right)^{2m} dS_w}{\int_{S_{wc}}^{1} \left(\frac{S_w - S_{wc}}{1 - S_{wc}} \right)^{2m} dS_w} \tag{12}$$

$$k_{r_{nw}} = \frac{\int_{s_{wc}}^{s_{wi}} \frac{(v-1)\left(\frac{s_w - s_{wc}}{1 - s_{wc}} \right)^{2m} ds_w}{\left[1 + (v^2 - 1)\left(\frac{s_w - s_{wc}}{s_{wi} - s_{wc}} \right)^{2m} \right] - \left[1 + (v^2 - 1)\left(\frac{s_w - s_{wc}}{s_{wi} - s_{wc}} \right)^{2m} \right]^{0.5}}}{\int_{s_{wc}}^{1} \left(\frac{s_w - s_{wc}}{1 - s_{wc}} \right)^{2m} ds_w} \tag{13}$$

Equation 13 explains that the relative permeability of the non-wetting phase depends on the viscosity ratio between the wetting and non-wetting phase V, the irreducible wetting phase saturation S_{wc} and on the curvature exponent m of the capillary pressure function. It is relatively easy to extend this equations for tortuosity effects, by multiplying the integral equations for k_{r_w} with the term $\left(\frac{s_w - s_{wc}}{1 - s_{wc}} \right)^2$ and for $k_{r_{nw}}$ with the term $\left(\frac{1 - s_w}{1 - s_{wc}} \right)^2$ after Burdine. Regardless that these formulas describe in principle an unsteady-state flow regime, for each small saturation change a temporary steady-state condition can be assumed comparable to the concept of Buckley-Leverett. From the first experimental calibrations on steady-state or quasi-steady-state experiments it can be empirically stated to neglect the partial saturation term x/L in equation 8, to come closer to the steady-state experimental conditions, without loosing the acceleration or retardation effect of the bracket term x/L in equation 8.

$$k_{r_{nw}} = \frac{\int_{S_{wc}}^{S_{wi}} \left(\frac{S_w - S_{wc}}{1 - S_{wc}} \right)^{2m} \left[1 + (V^2 - 1)\left(\frac{S_w - S_{wc}}{S_{wi} - S_{wc}} \right)^{2m} \right]^{-0.5} dS_w}{\int_{S_{wc}}^{1} \left(\frac{S_w - S_{wc}}{1 - S_{wc}} \right)^{2m} dS_w} \tag{14}$$

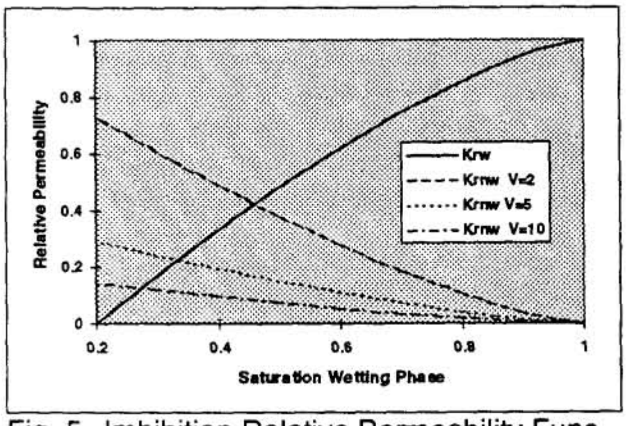

Fig. 5 Imbibition Relative Permeability Functions: Influence of Viscosity Ratio, S_{wc}=0.2, m=0.2

Figure 5 shows, that the shape of the imbibition relative permeability curve for the non-wetting phase is sensitive for the magnitude of the viscosity ratio between displacing and recovered phase. Lower viscosity ratios than 1, which are representative for water/oil-systems, may exceed the reference value of the effective water permeability at 100 % saturation. In this case the reference value should be replaced by the effective permeability for the non-wetting phase at irreducible wetting phase saturation.

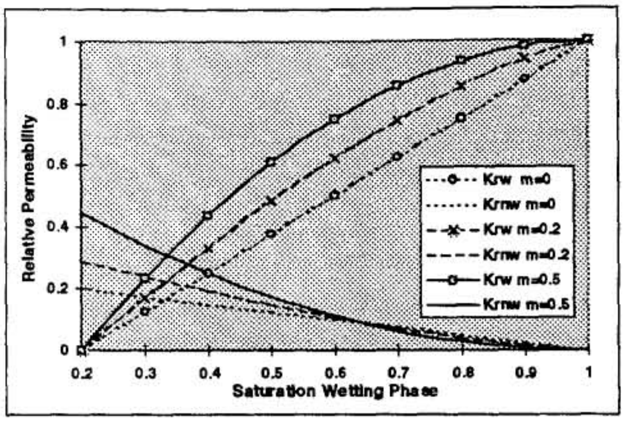

Fig. 6 Imbibition Relative Permeability Functions:Influence of Curvature Exponent S_{wc}=0.2, V=5

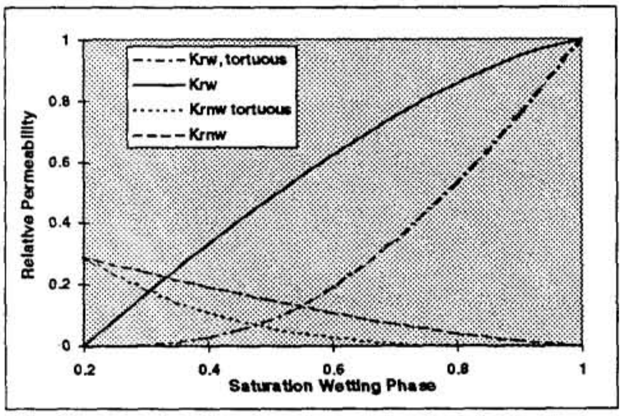

Fig. 7 Imbibition Relative Permeability Functions: Influence of Tortuosity S_{wc}=0.2, V=5, m=0.2

The new concept does not held for the case V = 1, since this implies the steady state flow concept (compare equations 5 and 6 for V = 1) of references /2-4/, which should be preferably used in that case. It should be further not concealed that compressible flow of gas cannot be treated accurately in the new concept of unsteady-state flow. However it is possible to match measured relative permeability data of gas/oil and gas/water systems using V as a fitting parameter. In figure 6 the influence of the curvature exponent m on the shape of relative permeability curves for the non-wetting phase and the wetting phase is shown. The higher the exponent m, the greater is the influence of pore geometry and interfacial tension of phases. If the exponent m approaches zero, the relative permeability curves are converted into linear functions of saturation, which is normally assumed to be the case in macrofractures. The irreducible saturation for the wetting phase is introduced in the correlation via the drainage capillary pressure function end point. The irreducible saturation of the non-wetting phase can be introduced by the adaptation of the saturation scale on measured valules. Figure 7 explains how the shape of the derived relative permeability functions can be converted into the more common form by using the tortuosity correction published by Burdine.

3. Calibration of the New Correlation on Published Relative Permeability Data

Odeh /7/ has published steady-state relative permeability data on a consolidated low permeability rock. The Penn-State method was applied for measurement and end pieces of equally permeable rock were used to minimize end effects. The following characteristic core and fluid data are cited from the original paper

Diameter [cm]	Length [cm]	Porosity	Corrected Air Permeability [mD]	Water Permeability [mD]
2.4	3.4	0.162	15.7	15.2

Table 1: Core Data: Odeh's Experiment

Sample	Density [g/cm³]	Viscosity [cp]	Viscosity Ratio
Oil 1	0.72	0.42	2
Oil 4	0.88	71.30	0.012
Brine	1.03	0.86	---

Table 2: Fluid Data: Odeh's Experiment

Since the unsteady-state displacement concept does not fit to the experimental conditions of Odeh, the influence of the partial saturation in equation (8) was cancelled by setting the first term x_j/L equal to one. The retarding or vice versa accelerating effect of the viscosity ratio on the non-wetting phase flow however was taken into account.

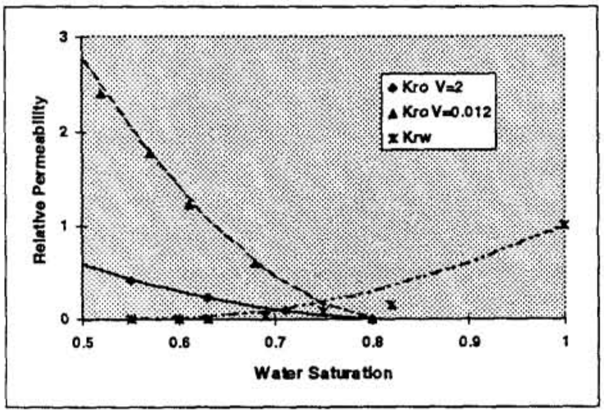

Fig.8 Imbibition Relative Permeability Data after Odeh /7/, Matched Curves use m=0.38 for Equation (14)

From Odeh's measurements no information was available concerning the pore radii distribution of the core sample. Therefore the curvature exponent m was used as a matching parameter.

Figure 8 shows that the relative permeability data of the oil phase could be perfectly matched for the viscosity ratios V=2 and V=0.012 setting m equal to 0.38.

Constant pressure desaturation/resaturation measurements for gas/water on Grimsel granite have been described in a GRS-report /12/.

Diameter [cm]	Length [cm]	Porosity	Corrected Air Permeability [µD]	Water Permeability [µD]	Gas Viscosity [cp]	Water Viscosity [cp]
6	10	0.005	6.5	5.4	0.0176	1

Table 3: Core and Fluid Data: Grimsel Granite

The pressure differences required for the desaturation of the water saturated core with nitrogen ranged from 0.8 to 6 MPa and for the resaturation from 0.05 to 1.5 MPa.

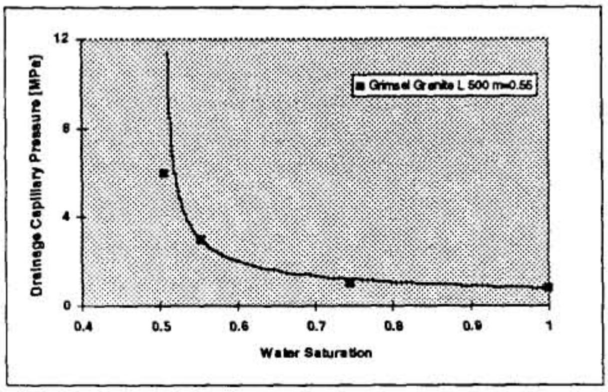

Fig. 9 Capillary Pressure Function for Granite, Matched by m=0.55

The capillary pressure curve for a drainage experiment with this core is presented in figure 9. The curvature exponent was determined with m=0.55, the irreducible water saturation equals S_{wc}=0.5. Using these parameters from equations (12) and (13) the relative permeability functions have been calculated. If the actual viscosity ratio of V=56.8 is adopted the measurd data in figure 10 cannot be matched. But a reduction of this figure to V=10 gives an acceptable match. One reason for the deviation from the theoretical value could be the compressible flow of the gas phase which was not taken into account.

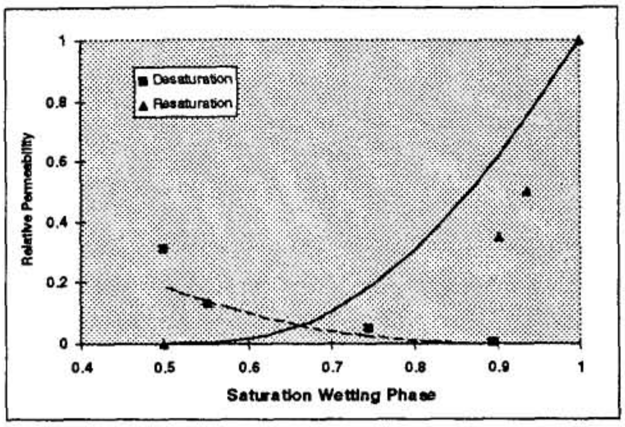

Fig. 10 Relative Permeability Function for Grimsel Granite Calculated for Imbibition Conditions, m=0.55, V=5

4 Conclusions

A controverse discussion about viscosity dependent non-wetting phase relative permeability functions in literature has not yielded an unequivocal result, at least not for low permeable rocks /13/.

Constant pressure desaturation and resaturation methods are applied for the determination of relative permeabilities for rock specimens in the range of nano- to microdarcies. If channel flow is assumed, the relative permeability of the non-wetting phase could depend on friction effects. A simple piston like displacement concept was derived which explains the viscosity effect for the non-wetting phase displacement. The new correlation was used to match published data. However measurement conditions and boundary conditions of the derived theory are not compatible in all respects. Therefore the resulting matches are no proof for the validity of the new concept. We need more imbibition type measurements to test its applicability. Nevertheless the new correlation can be used to match measured data as it is routineously done with the Brooks-Corey correlation. The advantage of our correlation is that it contains one additional parameter V for the fitting procedure.

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