

DUALITY PRINCIPLE FOR INTEGRAL AND DISTRIBUTED METHODS OF CAPILLARY PRESSURE AND RELATIVE PERMEABILITY DETERMINATION FROM STEADY-STATE FLOW

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Abstract. The general analysis for various methods of the interpretation of two-phase steady-state flow experiments is offered. The experimental technique is especially organized in the way, when the convective flow is equilibrated by the capillary counter-flow. We differ integral and distributed methods. The fact that two classes of methods are based on the same differential equation is proved. This allows to develop a similar technique for both classes. We offer three new versions of methods leading to the full determination of both relative permeability curves, as well as the capillary pressure. The first version corresponds to integral class and is based on the techniques offered by Ramakrishnan and Capiello. The second version is a combination of distributed and integral methods; the third one is a half-parametric purely distributed version, making use of some theoretical laws for singularly behavior of the saturation distribution nearly the sample end.

The case is especially examined, when relative permeability curves depend on flow velocity. This situation is appropriated to the gas-condensate mixture flow, in particular. The half-parametric distributed method is enable to solve this problem.

1. INTRODUCTION

Capillary pressure functions are generally obtained by the static methods, such as centrifuge or porous plate. The dynamic methods (Brown, 1951) are applied to the relative permeability determination. The new type of methods for the determination both the capillary pressure and the relative phase permeabilities has been offered in the paper of Ramakrishnan and Capiello (1991), and was after developed by Lenormand, Eisenzimmer and Zarcone (1993) as a "semi-dynamic" method. It was based on the two-phase steady-state flow organized in such a way, that the convective flow has been equilibrated be the counter-current capillary imbibition. This method has defects. Firstly, it disables the determination of the water permeability function. Secondly, it requires a big volume of experiments. Some modifications have been examined by Virnovski and al. (1995), but they require the measurement of phase pressure in the sample.

In the present paper we build the complete classification of steady-state methods, where we distinguish the group of integral methods and the group of distributed methods. We show the analogy between these two groups (duality principle), and we develop several versions of methods, which use only the data easily measured.

We study the linear porous core sample having a cylindrical form with a radius R and a length L . The sample is initially water saturated. The experiment consists in injections of the water-oil mixture, when the flow rates of each fluid are done. The outlet face of the sample is constantly washed by water; therefore the outlet saturation is always equal to the oil percolation threshold s^* . The porosity and the permeability, as well as the initial saturation are assumed to be known.

The aim of works consists in the determination of the capillary pressure curve and relative permeabilities.

2. THEORETICAL BASIS OF THE STEADY-STATE FLOW

The two-phase incompressible flow through the medium with a cross-section A and a length L may be described by the following problem for the water saturation s (C. Marle, 1981):

$$\begin{aligned} \frac{\partial s}{\partial \tau} + w(\tau)\varepsilon \frac{\partial F}{\partial x} &= \frac{\partial}{\partial x} \left(\varphi(s) \frac{\partial s}{\partial x} \right), \quad x \in (0,1), \quad \tau > 0 \\ s(x,0) &= s^0(x), \quad s(1,\tau) = s^*, \quad \left[F(s) - \frac{\varphi(s)}{\varepsilon \cdot w(\tau)} \frac{\partial s}{\partial x} \right]_{x=0} = \lambda(\tau) \end{aligned} \quad (2.1-a)$$

and for the water pressure p :

$$-\frac{\partial p}{\partial x} = \frac{1}{k_w(s)} \left[\varepsilon \cdot w(\tau) \cdot F(s) - \varphi(s) \frac{\partial s}{\partial x} \right], \quad p|_{x=1} = p^*(\tau) \quad (2.1-b)$$

where $x = X/L$, $\tau = t/t_*$, $\varphi(s) = -k_{oil} F p'_{cs}$, $p_c = P_c/P_c^0$, $w(\tau) = Q(\tau)/Q^0$; X , t are the space coordinate and the time, $t_* = m\mu_{oil} L^2 / KP_c^0$, $Q = Q_{oil} + Q_w$ is the total flow rate, K is the permeability, m is the porosity; $F(s) = k_w / (k_w + \bar{\mu} k_{oil})$; $P_c(s) = P_{oil} - P_w$ is the capillary pressure function; $\bar{\mu} = \mu_w / \mu_{oil}$; V_w, V_{oil} are flow velocities; k_w, k_{oil} are relative permeabilities; $p \equiv P/P_c^0$, $p^*(\tau) \equiv P^*(t)/P_c^0$, $P^*(t)$ is the outlet face pressure; $\varepsilon = Q^0 \mu_w L / AKP_c^0$. The indexes "w" and "oil" correspond to the water and oil. The values of P_c^0, Q^0 correspond to the characteristic capillary pressure and the total flow rate. The values s_*, s^* correspond to the percolation threshold for both liquids.

The last condition in (2.1) shows that the inlet fractional flow is known and equal to $\lambda(\tau)$, because we have for the fractional flow: $\frac{Q_w}{Q_w + Q_{oil}} = F(s) - \frac{1}{\varepsilon w} \varphi(s) \frac{\partial s}{\partial x}$.

We are going to use the stabilised solutions, which correspond to the steady-state flow reaching at large time magnitudes. Let us assume the functions $\lambda(\tau), w(\tau)$ have finite limits: $\lambda(\tau) \rightarrow \lambda \equiv const$, $w(\tau) \rightarrow 1$, when $\tau \rightarrow \infty$. Then the problem (2.1) is reduced to the following steady-state formulation:

$$\varepsilon \frac{dF}{dx} = \frac{d}{dx} \left(\varphi(s) \frac{ds}{dx} \right), \quad x \in (0,1); \quad \left[F(s) - \frac{\varphi(s)}{\varepsilon} \frac{ds}{dx} \right]_{x=0} = \lambda, \quad s(1) = s^* \quad (2.2-a)$$

$$-\frac{dp}{dx} = \frac{1}{k_w(s)} \left[\varepsilon F(s) - \varphi(s) \frac{ds}{dx} \right], \quad p|_{x=1} = p^*(\tau) \quad (2.2-b)$$

It is clear that this problem has a unique solution, which is not trivial. This fact leads to the existence of the non-uniform saturation distribution in the steady-state case. This distribution is supported by the counter-flow consisting of the convection flow directed from the inlet to the outlet, and the capillary imbibition directed in the inverse side.

The last problem has an integral:

$$\frac{\varphi(s)}{\varepsilon} \frac{ds}{dx} - F(s) + \lambda = 0, \quad \forall x \in [0,1]; \quad s(1) = s^* \quad (2.3-a)$$

$$-\frac{dp}{dx} = \frac{\varepsilon \lambda}{k_w(s)} \quad p|_{x=1} = p^* \quad (2.3-b)$$

The solution of (2.3-a) may be expressed through an explicit form:

$$x = 1 + \frac{1}{\varepsilon} \int_s^s \frac{\varphi(s')}{F(s') - \lambda} ds' = 1 - \frac{1}{\varepsilon} \int_s^s \frac{k_{oil}(\bar{s})F(\bar{s})p'_{cs}(\bar{s})}{F(\bar{s}) - \lambda} d\bar{s} \quad (2.4)$$

where $p'_{cs}(s) \equiv \frac{dp_c}{ds}$.

3. INTEGRAL METHODS: GENERAL CLASSIFICATION

3.1. Measured parameters

The integral methods use only the averaged information on the flow process, and the data on the boundaries of the porous medium. These methods apply a simple measurement techniques. Instead of this it requires a big amount of experiments, each of them is much longer in the time.

Really it is possible to measure the following averaged or boundary parameters (all values are dimensionless): the total flow rate, ε ; the fractional flow at the inlet on the medium, λ ; the averaged water saturation in the medium, \bar{S} ; the pressure at the outlet, which is the same in both phases, p^* . Anytime it is possible to measure:

- the capillary pressure at the inlet p_c ; if only the oil is injected in the medium, the measured inlet pressure is equal to $p_c - p^*$, because of the water pressure is constant along the sample (this will be shown later);
- the water saturation at the inlet σ , if one can measure the saturation within the medium. In this case we have also the full curve of the saturation distribution along the core;
- the total pressure drop in the water Δp ; if we can measure the pressure in each phase.

The mean difficulties are caused by the needs of the phase pressure measurement. Now it is recognized that the half-permeable membrane techniques leads to big errors in phase pressure definition.

3.2. Theory of integral methods

The integral methods use only the relations, averaged over the sample length:

$$\varepsilon = \int_0^p \frac{k_{oi}(p_c)F(p_c)}{F(p_c) - \lambda} dp_c, \quad \Delta p = \lambda \int_0^p \frac{k_{oi}(p_c)F(p_c)}{k_w(p_c)[F(p_c) - \lambda]} dp_c, \quad \bar{S} = \frac{1}{\varepsilon} \int_0^p \frac{s(p_c)k_{oi}(p_c)F(p_c)}{F(p_c) - \lambda} dp_c \quad (3.1-a,b,c)$$

The equation (3.1-a) results from (2.4), if one takes $x=1$. The equation (3.2-b) may be deduced by integrating the relation (2.3-b), and by change the integration variable $x \rightarrow s \rightarrow p_c$ using the relation (2.3-a):

$$dx = - \frac{k_{oil}(s)F(s)p'_{cs}(s)}{\varepsilon(F(s) - \lambda)} ds = - \frac{k_{oil}(p_c)F(p_c)}{\varepsilon(F(p_c) - \lambda)} dp_c \quad (3.2)$$

The equation (3.1-c) results from the definition of an averaged value $\bar{S} = \int_0^1 s(x)dx$ and the change of variables (3.2) after this.

Anytime one also uses another form of these equations, which may be obtained by the inverse change of variables $p_c \rightarrow s$:

$$\varepsilon = \int_s^\sigma \frac{k_{oil}(s)F(s)p'_{cs}(s)}{F(s) - \lambda} ds, \quad \Delta p = \lambda \int_s^\sigma \frac{k_{oi}(s)F(s)p'_{cs}(s)}{k_w(s)[F(s) - \lambda]} ds, \quad \bar{S} = \frac{1}{\varepsilon} \int_s^\sigma \frac{sk_{oi}(s)F(s)p'_{cs}(s)}{F(s) - \lambda} ds \quad (3.3-a,b,c)$$

The system of equations (3.1), or (3.3), describes the sequence of steady-state regimes, which has been established in the series of experiments, and which differ by the parameters $\{\varepsilon, \lambda, \rho_c, \bar{S}, \Delta p\}$ in the first case, or $\{\varepsilon, \lambda, \bar{S}, \sigma, \Delta p\}$ in the second case.

Three equations (3.1), or (3.3) contain five parameters. Consequently, only two of them are independent. Let us chose the parameters ε and λ as independent. Thus, the parameters $\bar{S}, \rho_c, \Delta p, \sigma$ are the functions of ε, λ :

$$\rho_c = \rho_c(\varepsilon, \lambda), \quad \bar{S} = \bar{S}(\varepsilon, \lambda), \quad \Delta p = \Delta p(\varepsilon, \lambda) \quad (3.4)$$

or

$$\sigma = \sigma(\varepsilon, \lambda) \quad \text{instead of } \rho_c$$

The different methods of the experiment interpretation may be proposed by reducing the number of parameters which can be varied.

4. ε -METHOD

4.1. The general form of the ε -method

Let assume that the fractional flow λ is constant for all the series of experiments. We only change the total flow rate ε . Then we have from (3.4), that: $\rho_c = \rho_c(\varepsilon), \bar{S} = \bar{S}(\varepsilon), \Delta p = \Delta p(\varepsilon), \sigma = \sigma(\varepsilon)$. We can make a differentiation in (3.1), that leads to the following relations:

$$\begin{cases} \frac{d\rho_c}{d\varepsilon} = -\frac{F(\rho_c) - \lambda}{k_{oil}(\rho_c)F(\rho_c)} \\ \rho_c|_{\varepsilon=0} = 0 \end{cases} \quad \begin{cases} \frac{d\Delta p}{d\varepsilon} = \frac{\lambda}{k_w(\rho_c)} \\ \Delta p|_{\varepsilon=0} = 0 \end{cases} \quad \begin{cases} \frac{d(\bar{S}\varepsilon)}{d\varepsilon} = \sigma(\rho_c), \\ \bar{S}|_{\varepsilon=0} = s^* \end{cases} \quad (4.1-a,b,c)$$

or from (3.3):

$$\begin{cases} \frac{d\sigma}{d\varepsilon} = \frac{F(\sigma) - \lambda}{k_{oil}(\sigma)F(\sigma)p'_{cs}(\sigma)} \\ \sigma|_{\varepsilon=0} = s^* \end{cases} \quad \begin{cases} \frac{d\Delta p}{d\varepsilon} = \frac{\lambda}{k_w(\sigma)} \\ \Delta p|_{\varepsilon=0} = 0 \end{cases} \quad \begin{cases} \frac{d(\bar{S}\varepsilon)}{d\varepsilon} = \sigma \\ \bar{S}|_{\varepsilon=0} = s^* \end{cases} \quad (4.2-a,b,c)$$

The relations (4.1-b) for Δp are deduced by differentiation of (3.1-b), which defines the function $\Delta p(\rho_c(\varepsilon))$.

The equation (4.2-c) shows, that two functions $\bar{S}(\varepsilon)$ and $\sigma(\varepsilon)$ are expressed one through another; therefore it is sufficient to measure one of these functions. Another function does not give a new information.

Thus, to determine three functions $k_w(s), k_{oil}(s), p_c(s)$ it is necessary to measure three functions: $\rho_c = \rho_c(\varepsilon), \bar{S} = \bar{S}(\varepsilon)$, and $\Delta p = \Delta p(\varepsilon)$. Then, we obtain the following sequence of results:

$$\begin{cases} \Delta p(\varepsilon) \rightarrow k_w(\varepsilon), \text{ from (3.5-b)} \\ \rho_c(\varepsilon) \rightarrow k_{oil}(\varepsilon), \text{ from (3.5-a); } \Rightarrow k_w(\sigma), k_{oil}(\sigma), \rho_c(\sigma) \\ \bar{S}(\varepsilon) \rightarrow \sigma(\varepsilon), \text{ from (3.5-c)} \end{cases} \quad (4.3)$$

We have taken into account of the following dependence for the function in right-hand side of (4.1-a): $(F - \lambda)/(k_{oil}F) \equiv (k_w - \lambda(k_w + k_{oil}\bar{u})) / (k_w k_{oil})$.

Actually the ε -Method can be executed, if and only if the inlet face capillary pressure ρ_c and water pressure drop really may be measured. Two versions follow from these requirements: ε_0 -method, and multiple ε -method.

4.2. ε_0 -method

This version corresponds to the case when $\lambda=0$. It has been developed by Ramakrishnan and Capiello (1991). The curves in the figure 1 shows, that this case covers the biggest interval of saturation changes; thus it allows to reconstruct the whole of founded curves. The governing equations may be simplified to the following:

$$\begin{cases} \frac{d\rho_c}{d\varepsilon} = -\frac{1}{k_{oil}(\rho_c)}, & \Delta p=0, & \begin{cases} \frac{d(\bar{S}\varepsilon)}{d\varepsilon} = \sigma(\rho_c), \\ \bar{S}|_{\varepsilon=0} = s^* \end{cases} \end{cases} \quad (4.4)$$

One sees, that the water pressure is a constant, that may be measured at the outlet (p^*). Because we have only the one-phase flow, the pressure at the inlet is equal to the oil pressure. Then, by measuring the inlet pressure we obtain the capillary pressure as a difference between the inlet pressure and p^* .

The sequence (4.2) becomes more simple:

$$\begin{cases} \rho_c(\varepsilon) \rightarrow k_{oil}(\varepsilon), & \text{from (3.8-a)} \\ \bar{S}(\varepsilon) \rightarrow \sigma(\varepsilon), & \text{from (3.8-c)} \end{cases} ; \Rightarrow k_{oil}(\sigma), \rho_c(\sigma) \quad (4.5)$$

Thus, the oil permeability is easily reconstructed, however, the water permeability can not be determined here.

4.3. Multiple ε -method

The second version allows to determine the water phase permeability too. It requires to make two sequential series of experiments. The first series corresponds to the ε_0 -method described above. After this, one executes a new series corresponding to the ε -method, but when $\lambda \neq 0$. It is necessary to measure the functions $\rho_c^I(\varepsilon)$ and $\bar{S}^I(\varepsilon)$ in the first series, and $\bar{S}^{II}(\varepsilon)$ in the second one.

We have the following algorithm of data interpretation:

$$\begin{cases} \rho_c^I(\varepsilon) \rightarrow k_{oil}(\varepsilon), & \text{from (3.8-a)} \\ \bar{S}^I(\varepsilon) \rightarrow \sigma^I(\varepsilon), & \text{from (3.8-c)} \end{cases} ; \Rightarrow k_{oil}(\sigma^I), \rho_c^I(\sigma^I) \\ \bar{S}^{II}(\varepsilon) \rightarrow \sigma^{II}(\varepsilon) \text{ from (3.5-c); } \rightarrow k_w(\varepsilon) \text{ from (3.5-a); } \Rightarrow k_w(\sigma^{II})$$

5. OTHER FORMS OF INTEGRAL METHODS

Theoretically it is possible to propose two other groups of methods. We differ λ -method and ε - λ -method as in the case described above. The first corresponds to the case, when only parameter λ is varied and ε is fixed. The second can be realized when both parameters ε and λ are varying simultaneously.

In this case we have to use the integral equations directly, or their finite-difference approximation. For example, it is easy to deduce the following expression in form of recurrent relations in the case of λ -method:

$$\frac{k_{oil}(\rho_i^{(n)})F(\rho_i^{(n)})}{F(\rho_i^{(n)}) - \lambda^{(n)}} \approx \begin{cases} \frac{k_{oil}(\rho_i^{(n-1)})F(\rho_i^{(n-1)})}{F(\rho_i^{(n-1)}) - \lambda^{(n-1)}}, & n \geq 2 \\ -\varepsilon\rho_i^{(1)}, & n = 1 \end{cases}$$

where n is the number of the experiment. The first number corresponds to the experiment at the value of λ near to 1. This equation is an approximate equivalent to relation (3.5-a). Analogous approximations may be deduced for other equations (3.5-b,c). We omit these deductions.

The practical importance of these methods is very low. They may be only applied after the development of precise methods for phase pressure measurement.

6. DISTRIBUTED METHODS

Let us suppose that only one experiment has been made in the sample, and the saturation distribution $s(x)$ has been measured along the sample length.

6.1. Duality principle

The governing equations are (2.3), or after some obvious transformations:

$$\begin{cases} \frac{dp_c}{dx} = \frac{F(p_c) - \lambda}{k_{oil}(p_c)F(p_c)}, & \frac{d(p-p^*)}{dx} = \frac{\lambda}{k_w(p_c)} \\ p_c|_{x=1} = 0 & p-p^*|_{x=1} = 0 \end{cases} \quad (6.1-a,b)$$

or instead of (6.1-a):

$$\begin{cases} \frac{ds}{dx} = \frac{\varepsilon[F(s) - \lambda]}{k_{oil}(s)F(s)p'_{cs}(s)} \\ s|_{x=1} = s^* \end{cases} \quad (6.2)$$

It is easy to see, that equations (6.1) coincide to (4.1), if one formally changes the variables: $x \rightarrow 1-x$, $p_c \rightarrow \rho_c$, $p-p^* \rightarrow \Delta p$, $s \rightarrow \sigma$. The duality principle follows from this:

- 1) *The saturation distribution $s(1-x)$ within the medium at $\varepsilon=1$ coincides to the function $\sigma(\varepsilon)$, resulting from the integral ε -method, at the same values of the inlet fractional flow λ .* (6.3)
- 2) *The same analogy takes place for the pairs of functions:*
 $p_c(1-x)$ and $\rho_c(\varepsilon)$; $(p-p^*)(1-x)$ and $\Delta p(\varepsilon)$.

The comparison of pictures on Fig.1 proves this principle.

Thus we can transfer general ideas of the techniques developed for integral methods to the distributed ones.

6.2. Particularity of the saturation distribution at the end of the sample

The figure 1 shows the singularly behavior of the function $s(x)$ at the end of the medium. The such a behavior is caused by property of the function $k_{oil}(s)$, which tends to zero when

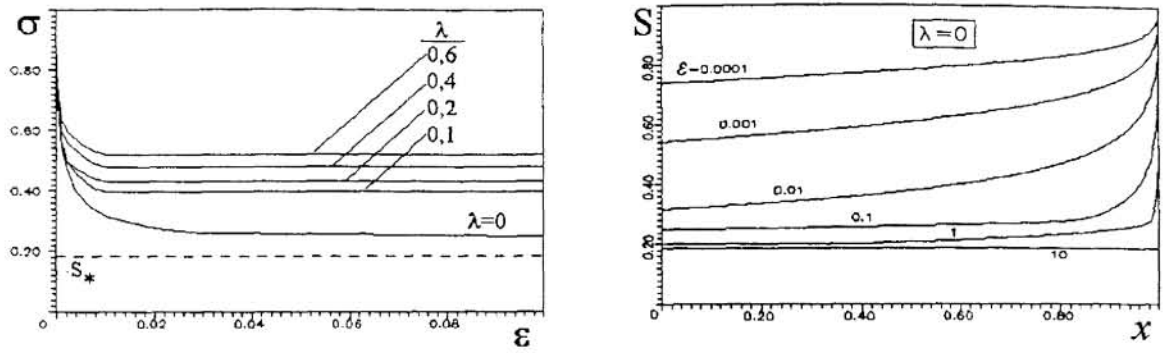


Fig. 1. Dependence $\sigma(\epsilon)$ of the inlet saturation on the flow rate (on the left), and the saturation distribution $s(x)$ along the sample (to the right)

$s \rightarrow s^*$. Accordingly the percolation theory this function shows the following behavior near the point $x=l$

$$k_{oil}(s) \sim b(s^* - s)^\beta, \quad s \rightarrow s^* \quad (\beta > 0) \tag{6.4}$$

where b, α are the constant values.

It easy to obtain by means of the series expansion method the following expansion for the solution of (6.2), which describes the saturation distribution near the end of the medium:

$$s(x) \sim s^* - a(1-x)^\alpha, \quad x \rightarrow 1; \quad \alpha = \frac{1}{1+\beta}, \quad a = \left(\frac{\epsilon(1-\lambda)}{p'_{cs}(s^*)\alpha b} \right)^\alpha \tag{6.5}$$

Thus, the point $x=l$ is a branching point.

This relation (6.5) will be used in the one of method versions.

6.3. Direct distributed method

All the distributed methods have a considerable limitation, which may be formulated by the following theorem. The initial non-stationary equation (1.1) shows that the saturation distribution only depends on two undefined functions, $F(s)$ and $\psi(s) \equiv k_{oil}(s)p'_{cs}(s)$. Thus, knowing only the saturation distribution in space and time it is impossible to determine functions $k_{oil}(s)$ and $p_c(s)$ separately, as well as the function $k_w(s)$. The direct method allows the determination only of the functions $F(s)$ and $\psi(s)$.

Let us assume the two experiments have been executed at different values of the parameter pair (λ, ϵ) , and two corresponding saturation distributions have been obtained: $s_1(x)$ for (λ_1, ϵ_1) , and $s_2(x)$ for (λ_2, ϵ_2) . Naturally, $\epsilon_1 \neq 0$ and $\epsilon_2 \neq 0$.

Let us assume that functions $s_i(x)$ are monotonous, then the inverse functions exist: $x_1(s)$ and $x_2(s)$. We obtain from (6.2) the following solution with respect to the functions $F(s)$ and $\psi(s)$, which is nontrivial, if $\lambda_1, \lambda_2 \neq 0$ simultaneously:

$$F(s) = \frac{\epsilon_1 \lambda_1 (dx_1/ds) - \epsilon_2 \lambda_2 (dx_2/ds)}{\epsilon_1 (dx_1/ds) - \epsilon_2 (dx_2/ds)}, \quad \psi(s) = - \frac{\epsilon_1 [F(s) - \lambda_1] (dx_1/ds)}{F(s)} \tag{6.6}$$

One of λ_i may be equal to zero; both ϵ_i may be identical. The corresponding simplifications are obvious.

6.4. Combined distributed-integral method

In order to determine the phase permeability functions the following sequence of experiments may be executed. The part I consists of one series of experiments at $\lambda=0$ and different values of ε , corresponding to the integral ε_0 -method. Part II consists in the one experiment at $\lambda \neq 0$ and any ε (the value of ε about of 1 is more convenient), corresponding to the distributed method. After first part we obtain the function of capillary pressure $p_c(s)$ and relative oil permeability $k_{oil}(s)$. The second part gives the distribution $s(x)$ or the inverse function $x(s)$. Then we determine the function $F(s)$ from (6.2), and after this the water relative permeability:

$$F(s) = \frac{\lambda \varepsilon (dx/ds)}{k_{oil}(s) p'_{cs}(s) + \varepsilon}, \quad k_w(s) = \frac{k_{oil}(s) \bar{\mu} F(s)}{1 - F(s)}, \quad (6.7)$$

where ε and λ are values of the parameters corresponding to the part II of the method.

6.5. Distributed half-parametric method

This version partially uses the theoretical laws of some functions' behavior.

The method is based on the following idea. The relation (6.5) shows that the end of the distribution $s(x)$ is very sensitive to the function $k_{oil}(s)$, because the parameter β of this function, introduced accordingly to (6.4), determines the singularly behavior of saturation distribution near the point $x=1$. Therefore, by treatment of the saturation distribution end we can seek the parameter β , hence the behavior of the oil permeability near s^* .

Moreover, if we assume that the function $k_{oil}(s)$ has a following form:

$$k_{oil}(s) = \left(\frac{s^* - s}{s^* - s_*} \right)^\beta, \quad s \in (s_*, s^*) \quad (6.8)$$

which coincides with the relation (6.4), when $b=b \equiv (s^* - s_*)^{-\beta}$, then defining the parameter β we obtain a satisfactory approximation for the whole curve in interval (s_*, s^*) .

The techniques of the method is as follows. Let us assume that we know the saturation distribution $s(x)$. We have from (6.5), where two parameters β and a are unknown:

$$Y = \frac{1}{1+\beta} X + \ln a; \quad Y \equiv \ln(s^* - s), \quad X \equiv \ln(1-x) \quad (6.9)$$

That is a linear equation with respect to parameters $1/(1+\beta)$ and $\ln a$, which can be solved by regression methods. For example, the mean square method leads to the following results:

$$\beta = \frac{\sum X_i^2 / \sum X_i - \sum X_i / N}{\sum Y_i X_i / \sum X_i - \sum Y_i / N} - 1, \quad a = \exp \left\{ \frac{1}{N} \left(\sum Y_i - \frac{1}{1+\beta} \sum X_i \right) \right\} \quad (6.10)$$

where the summing is made from 1 to N , by N experimental points.

Thus, the function $k_{oil}(s)$ is determined.

Knowing functions $k_{oil}(s)$ and $\psi(s)$ we obtain the capillary pressure function as

$$f(s) \equiv p'_{cs}(s) = \begin{cases} \frac{\psi(s)}{k_{oil}(s)}, & s \neq s^* \\ -\frac{\varepsilon(1-\lambda)(\beta+1)(s^*-s_*)^\beta}{a_1^{1+\beta}}, & s = s^* \end{cases} \Rightarrow p_c(s) = \int_0^s f(\bar{s})d\bar{s} \quad (6.11)$$

The relation for $f(s)$ at $s=s^*$ results from the definition (6.5) of the parameter a .

In practice, the integration will be replaced by the summing over discrete system of points, that leads to recurrent equations:

$$p_c(s_i) = \begin{cases} 0, & i=1 \\ \frac{f_i + f_{i-1}}{2}(s_i - s_{i-1}) + p_c(s_{i-1}), & i \geq 2 \end{cases} \quad (6.12)$$

The sequence of actions is as follows:

- i) one makes one experience at $\lambda=0$ and at free values of ε , one measures the function $s(x)$ and one builds the inverse function $x(s)$;
- ii) one determines the function $\psi(s)$ from (6.6), which is reduced to $\psi(s) = -\varepsilon(dx/ds)$;
- iii) one builds the graphics $Y(X)$ and one fixes its linear part corresponding to (6.9);
- iv) one computes the parameters (6.10) by regression method, applied to the linear part of the function $Y(X)$;
- v) one reconstructs the oil permeability (6.8);
- vi) one determines the function $f(x)$ from (6.11);
- vii) one computes the capillary pressure (6.12).

To obtain the water permeability, it is necessary to make the second experiment at $\lambda \neq 0$ and free value of ε , and apply relations (6.6), (6.7) after this.

6.6. Examples

We have computed the direct steady-state problem (6.2), which gives to us the saturation distribution $s(x)$. This distribution with introduced small oscillations has been associated to the "experimental data". The oil relative permeability function has been taken in the form (6.8), when $\beta=3$, $s_*=0.2$, $s^*=0.8$. After this, the half-parametric distributed method has been applied.

The Fig. 2 shows the dependence $Y(X)$, where the initial linear interval corresponds to the

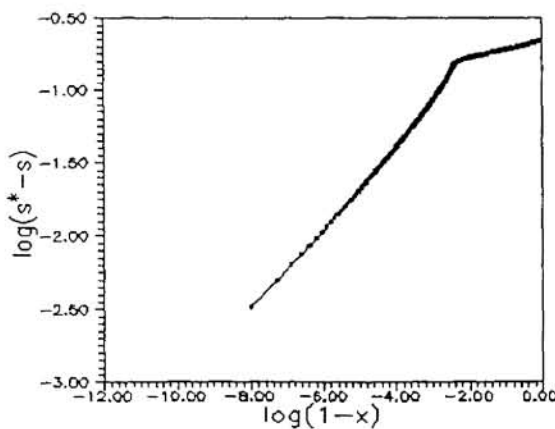


Fig. 2. Interpretation of the saturation distribution end accordingly the relation (6.11)

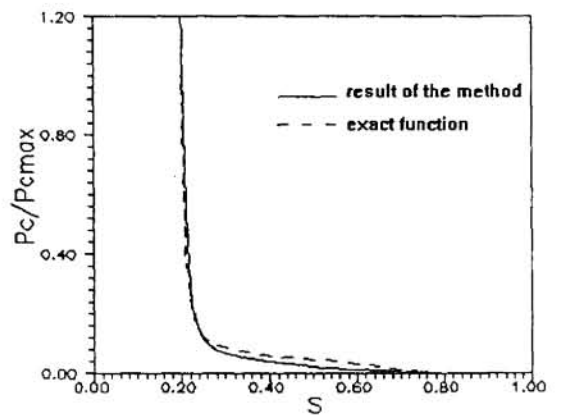


Fig. 3. Capillary pressure function; results of the distributed method (solid line) and the exact curve (dashed)

relation (6.9). This part of the curve has been processed to determine the parameters (6.10).

The final result for capillary pressure function determination is shown in the Fig.3.

7. RELATIVE PERMEABILITIES DEPENDING ON FLOW VELOCITY

The two-phase flow when relative permeabilities depend on flow velocity is a special case appropriated to the gas-condensate mixture in particular. Then the problem consists in determination of two functions $k_w(s,q)$, $k_{oil}(s,q)$, $P_c(s,q)$, where q is the dimensionless total flow rate.

It is easy to show that the half-parametric method allows to determine all the mentioned functions, if it is repeated step by step for different magnitudes of the flow rate.

The algorithm of method is invariant for each step.

CONCLUSIONS

One of the main results of this paper is to recommend the combined methods for the determination of all the three function of capillary pressure and relative permeabilities. The ε_0 -method (that is named also as a "semi-dynamic" method) should be combined with other integral method, when both fluid phases should be injected in the sample. The distributed method must be supplemented with an integral ε_0 -method, or with the theoretical correlations. As example we show the possibilities of three new combined methods: integral multiple ε -method, distributed-integral method and half-parametric distributed method. All the versions require to make the usual measurements, such as the inlet-outlet pressure and/or the saturation distribution along the sample.

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REFERENCES

- Brown H.W. *Capillary pressure investigations*, Petroleum Transactions, AIME, v. 192, 1951.
- Ramakrishnan T.S., Cappiello A., *A new technique to measure static and dynamic properties of a partially saturated porous medium*. Chem. Eng. Sci., 1991, v. 46, No 4, pp.1157-1163.
- Lenormand R., Eisenzimmer A., Zarcone C. *A novel method for the determination of water/oil capillary pressures of mixed wettability samples*. Proc. 1993 Soc. Core Analysts Conf., Houston, Texas, August, 9-11, 1993.
- Virnovsky G.A., Guo Y., Skaeveland S.M. *Relative Permeability and Capillary Pressure Determined from Steady-State Flow Experiments*, Proc. 8th European IOR, Symposium in Vienna, Austria, May, 15-17, 1995, pp.60-69.
- Marle Ch.M., *Multiphase Flow in Porous Media*. Gulf. Publishing, Editions Technip, Paris, 1981.