

## Can We Really Measure the Relative Permeabilities Using the Micropore Membrane Method?

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### Abstract

We have performed experiments and simulations to study the accuracy of  $K_r$  determination by the membrane technique. We have developed a numerical simulator and an analytical model which both account for the membranes as a boundary condition. The analytical model is based on the assumption of constant  $K_r$  and linear  $P_c$  during a small pressure step. These assumptions lead to a saturation equation which can be solved by standard methods used for thermal problems. The conclusions of the experiments and the simulations are the following:

- for intermediate saturation values, the transient production is governed by membrane permeability, without any real influence of the  $K_r$ , when the membrane resistance (thickness/permeability) is larger than core resistance.
- for oil/water systems, the transient production is controlled by the very small mobility of the fluid at low saturation near the end-points. Its mobility can be determined with good accuracy, especially for very small  $K_r$  values. But, the mobility of the other fluid (at the highest saturation) cannot be determined.
- for oil/gas systems, the oil permeability can be determined, provided that the resistance ratio is smaller than 1 (membrane resistance/ core resistance).

Our conclusion is that transient production during  $P_c$  measurement with porous plate or membrane techniques cannot be used for  $K_r$  determination of liquid/liquid systems, except if the two following conditions are satisfied :

- 1) very low membrane resistance (smaller than core resistance)
- 2) independent measurement of the  $K_r(\max)$  at end-points by removing the membranes and using a standard one-phase flow.

### Introduction

In core analysis, there is a general trend towards simultaneous determination of relative permeabilities and capillary pressure curves during the same experiment. In this paper, we have studied the accuracy of relative permeability determination by using the transient production obtained during capillary pressure measurements with the membrane technique.

A few years ago, several studies (Richmond and Watson, 1988, Chardaire-Rivière, 1992) were devoted to the simultaneous determination of capillary pressure curves ( $P_c$ ) and relative permeabilities ( $K_r$ ) by automatic history matching during core flood experiments. It is now recognized that it is more accurate to measure separately the capillary pressure. However, the  $K_r$  functions can be derived using the transient of the production. Several papers have been published for the various methods. For centrifuge : O'Meara and Crump, 1985; Nordtvedt et al., 1993; Nimmo, 1990 with local saturation measurements using resistivity. For "porous plate" or "membrane" method: Jennings et al., 1985; Longeron et al., 1994; Christoffersen, 1995. In the membrane method, the transient is often used to estimate the equilibrium saturation (Guo and Hammervold, 1993). A new method combines the membrane method and a continuous injection (Kokkedee and Boutkan, 1993; Kokkedee, 1994).

We have used the transient method with membrane and have observed that it is possible to obtain a good history matching but with a very poor accuracy on the  $K_r$  values. This poor determination is due to both the hydraulic resistance of the membrane and the presence of the second fluid. In order

to determine the intervals of confidence for  $k_r$  determination, we have performed numerical simulations for various values of membrane resistances and fluid viscosities.

## Numerical Tools

We used two approaches, a numerical simulator with boundary conditions corresponding to the porous plate experiment and an analytical calculation suitable for small pressure increments and for sensibility studies.

For both calculations, the standard Darcy's equations are used together with the condition of no flow through the two membranes. The pressure drop  $\Delta P$  across the membrane is assumed to be proportional to the flow rate  $u$  (per unit surface area)  $\Delta P = R_m \mu u$  where  $\mu$  is the fluid viscosity and  $R_m$  a parameter that characterizes the hydraulic resistance of the membrane (the "resistance" of the membrane). For a porous sample, the resistance  $R_{pm}$  is equal to  $L/K$  (length of the sample over single-phase permeability). Generally the membrane resistance is derived from a single-phase flow through the porous sample with and without the membrane.

### numerical simulator

A 1-dimensional finite volume code has been developed. The main advantage of this method is that the flux conditions are naturally introduced. A Crank Nicholson scheme is adopted for time discretization. The non-linear terms are solved using a simple substitution technique. The final linear system is solved by a triangularisation method. Convergence tests based on the root of the square sum are taken at a precision of  $10^{-4}$ . The number of nodes was generally 80, uniformly distributed all along the porous medium.

### analytical model

Contrary to the centrifuge method or a displacement, the saturation is quite uniform along the sample during a small step of pressure increment. Consequently, we can assume that the relative permeabilities remain constant and that capillary pressure varies linearly with saturation. These approximations lead to a standard diffusion equation with constant coefficients for saturation. The problem is similar to heat exchange through a wall of finite thickness with mixed boundary conditions. The solution in the form of an infinite series is given in Carslaw and Jaeger (1959). The calculations are detailed in the Appendix. The model needs on the order of 100 terms in the series and the calculation of the roots and the series is developed in FORTRAN. This approach is very fast but the main advantage is to give a better understanding of the effect of the various parameters.

We have checked that the numerical and analytical calculations give identical results for saturation profiles and effluent production. In addition, the results have been compared with simulations treating the membrane as a second porous medium. The results are the same provided a fine discretization of the membrane is used (even if its thickness is very small).

## Experiments

Our purpose was to measure the gas/oil  $k_r$  during intermediate drainage-imbibition cycles. Drainage and imbibition experiments were performed with paraffinic oil and gas in a high permeable sample (4 darcy, diameter 7.6 cm and thickness 2 cm, resistance  $R(\text{core})=5 \cdot 10^9 \text{ m}^{-1}$ ). The resistance of the cellulosic membrane (thickness/permeability) was 260 times larger than the resistance of the medium. Average saturations were measured with an absolute accuracy of 0.001.

We used both the analytical and numerical models for  $k_r$  determination and each step was studied independently. The main conclusions are the following:

- 1) each step can be fitted keeping a constant value for the resistance of the membrane.
- 2) for intermediate saturation, the kinetics of effluent production is mainly controlled by membrane resistance. The transient shown in Fig. 1a is used to determine the membrane resistance  $R(\text{membrane})=1.3 \cdot 10^{12} \text{ m}^{-1}$ . The value of oil relative permeability has a small influence (Fig. 1b). Changing the  $k_r(\text{max})$  from 1 to 0.01 has the same effect as changing the membrane resistance by a factor 2 (compare Fig. 1a and 1b).

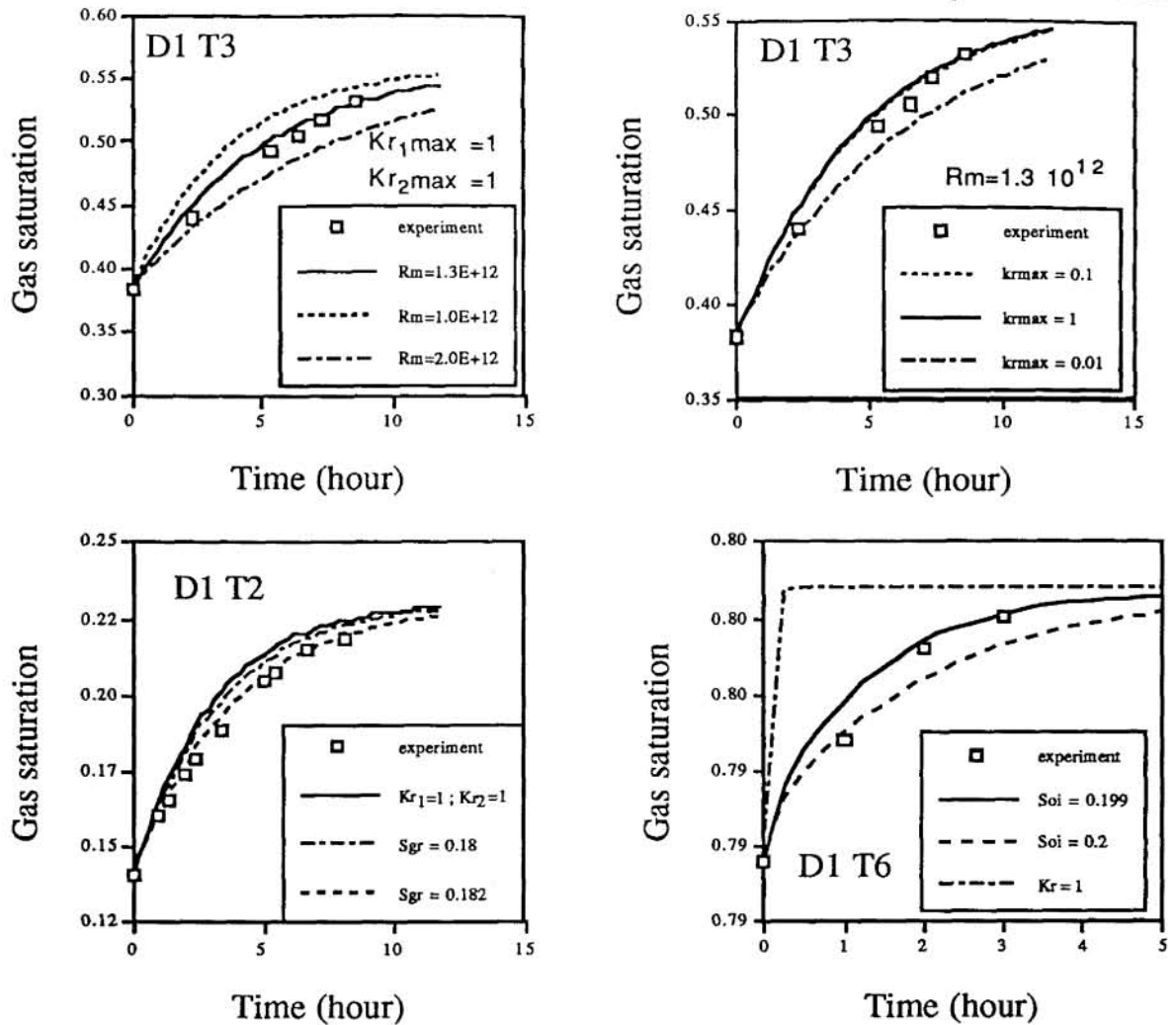


Fig. 1 - Experimental results. transient productions during gas/oil drainages.

3) near the end-points, the transient production is controlled by the fluid that is at the lowest saturation. At low gas saturation, the fit is controlled by the end-point saturations from 0.18 to 0.182 in a Corey representation (Fig. 1c). At high gas concentration, the fit is highly sensible to a change from 0.199 to 0.200 in oil saturation (Fig. 1d).

4) there is no determination of the  $K_r$  values of the fluid at higher saturation ( $K_r > 10^{-3}$ ). In the experiment, the membrane resistance is very high compared to the medium, around 260 times (Table 1). In order to choose a more appropriate membrane for this gas/oil system and also for water/oil experiments, we performed a systematic simulation to determine the accuracy of the  $K_r$  measurement.

## Accuracy Tests

In the transient method, there are two effects that lead to a poor determination of the relative permeability of fluid 1 :

- 1) the pressure drop in the membrane
- 2) the pressure drop in fluid 2 which is important when this fluid is at low saturation.

The two effects are studied independently.

## Principle of the simulation

The principle of the calculation is the following:

- 1) The "theoretical" values for  $P_c$ ,  $K_r$  and membrane resistance  $R_m$ , are known.

- 2) To study the effect of membrane resistance, we assume that  $R$  is known with an uncertainty of  $\pm 10\%$  (leading to  $R^+$  and  $R^-$ ). The  $10\%$  value seems reasonable to account for the variation of membrane properties due to pore clogging, mechanical constrain, temperature effect, etc.
- 3) A small pressure increment around saturation  $S$  is simulated, replacing  $R$  by  $R^+$  or  $R^-$  and keeping the  $Kr_1$  at its true value.  $Kr_2$  is then determined by fitting the simulated effluent production. The two extreme values  $R^+$  or  $R^-$  lead to two limits for  $Kr_2$ . Repeating the procedure for different saturation determine the interval of confidence for  $Kr_2$ . The confidence interval for  $Kr_1$  is determined by keeping the true value for  $Kr_2$ .
- 4) To study the effect of the second fluid, we use a membrane with no resistance ( $R=0$  and consequently  $r=0$ ).  $Kr_1$  is varied by  $\pm 10\%$  and the two extreme values for  $Kr_2$  are determined by fitting the effluent production. The roles of the fluids are then exchanged to determine the confidence interval for  $Kr_1$ .

**Fluid and core data**

**Table 1 Capillary pressure**

$S_w$	$P_c$ (Pa)
.175	70000
.2	38500
.225	31500
.3	21000
.4	14000
.45	12300
.65	10500
.825	10000
.9	7000
1	0

For the simulation we have used three fluid systems

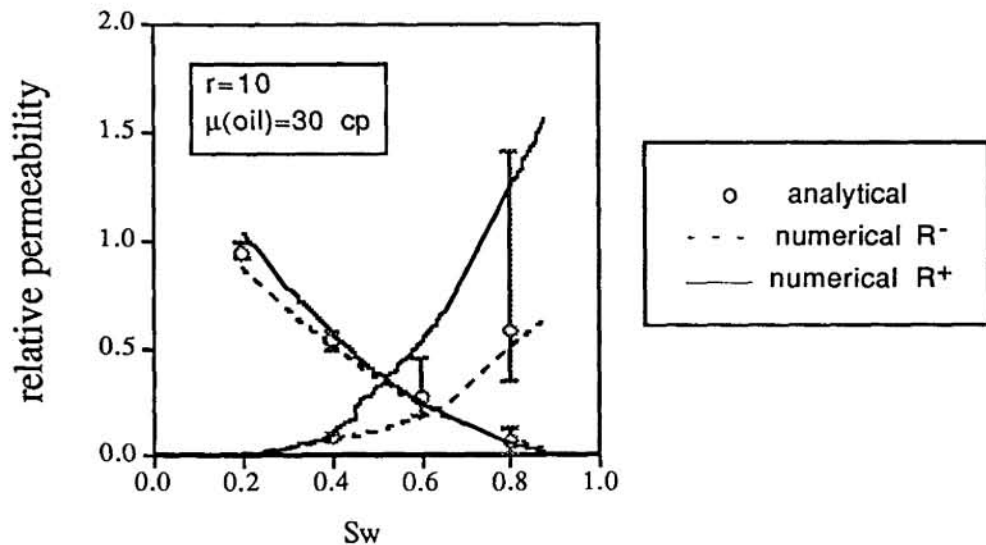
- water (1 cp) and 1cp oil,
- water (1 cp) and 30cp oil,
- oil (1 cp) and gas (0.01 cp).

The rock sample has a permeability of 0.162 Darcy, porosity 0.237, length 2 cm and diameter 4 cm (resistance  $R(\text{core})=1.23 \cdot 10^{-13} \text{ m}^{-1}$ ).

There is only one membrane (water-wet) and the ratio  $r = R(\text{membrane})/R(\text{porous medium})$  is varied from 1 to 100 (resistance from  $1.23 \cdot 10^{-13} \text{ m}^{-1}$  to  $1.23 \cdot 10^{-11} \text{ m}^{-1}$ ).

For all the fluid systems, we have simulated a first drainage with initial wetting fluid saturation equal to 1 and final saturation equal to 0.175. The relative permeabilities are calculated using a Corey function with exponent equal to 2 and maximum values equal to 1. The  $P_c$  curve is given by Table 1.

We have verified that the numerical and the analytical simulators gave similar results (Fig. 2). We will use mainly the analytical model.



**Fig. 2 - Comparison of sensibility tests performed with the numerical and analytical simulators.**

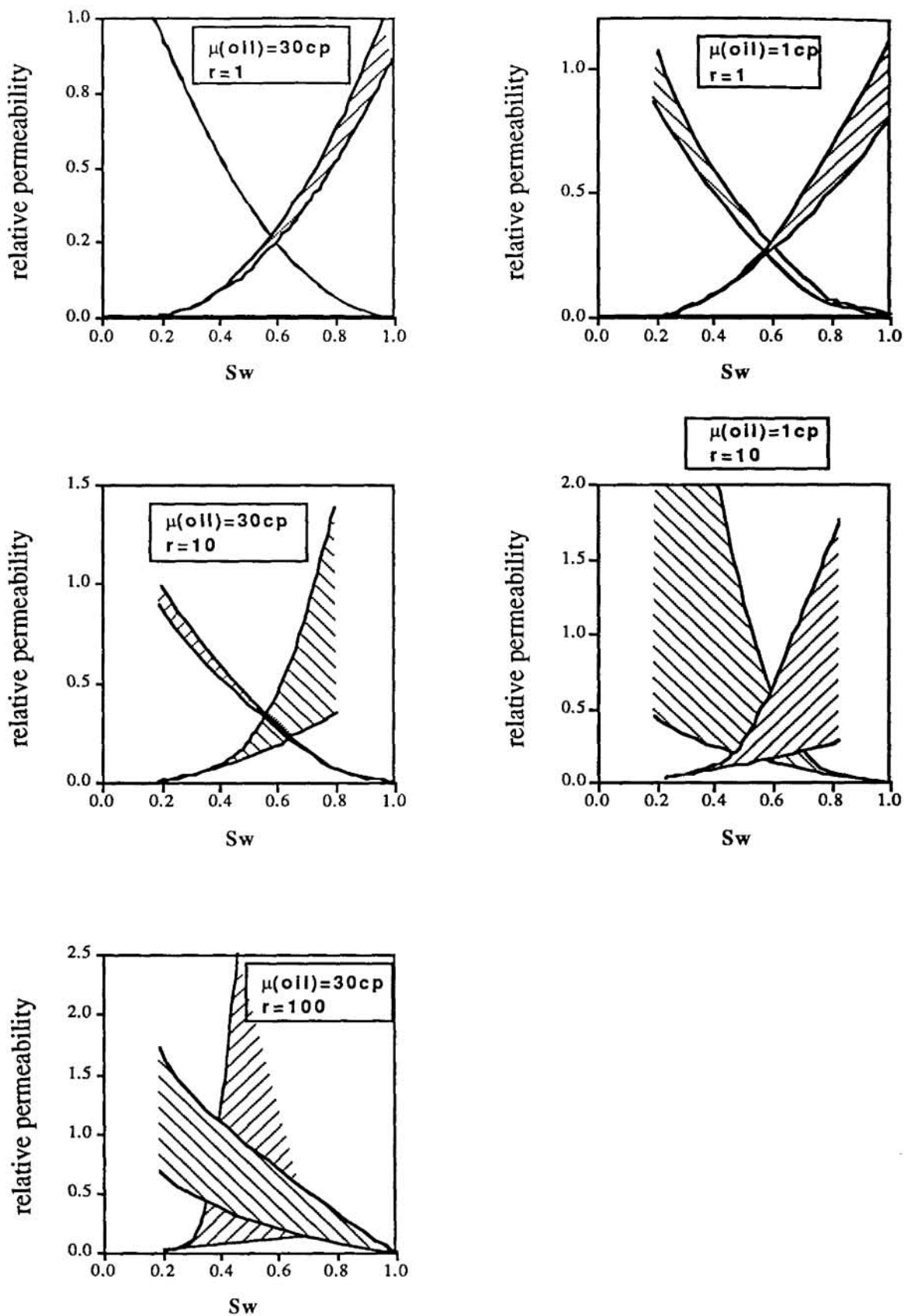


Fig. 3 - Influence of membrane resistance in water/oil systems with various oil viscosities.

## Results

The results are summarized in Fig. 3 and 4. The shaded area represent the zone of uncertainty.

### Effect of membrane resistance only (Fig. 3)

- the uncertainty increases with the membrane resistance.
- increasing the viscosity of one fluid increases also the accuracy on the corresponding  $K_r$ . However, the time scale of the transient increases in the same ratio as the viscosity (compare Fig. 3b for 30 cp oil and 3 e for 1cp oil).
- For an oil viscosity of 1 cp, the accuracy is acceptable only for resistance ratios of the order or smaller than 1 (Fig. d). For a ratio 10, which is a common value in membrane experiments, there is absolutely no accuracy (Fig. 3 e).

### Effect of the other fluid (Fig. 4)

Simulations were performed with negligible membrane resistance ( $r=0$ ) and a variation of  $\pm 10\%$  for one of the relative permeability. For any fluid system, the pressure drop in the fluid at low saturation controls the kinetics of effluent production near the end-points. There is no possibility for determining the  $K_r$  of the fluid at higher saturation and the shape of the shaded area agrees with this limit. However, the importance of uncertainty in  $K_r$  determination depends on fluid viscosity :

- for water/oil systems (oil and water viscosities equal 1cp), there is no accuracy on the determination of the  $K_r$  value (Fig. 4a).
- for oil/gas systems, the determination of oil relative permeability is good till a saturation of 0.8.

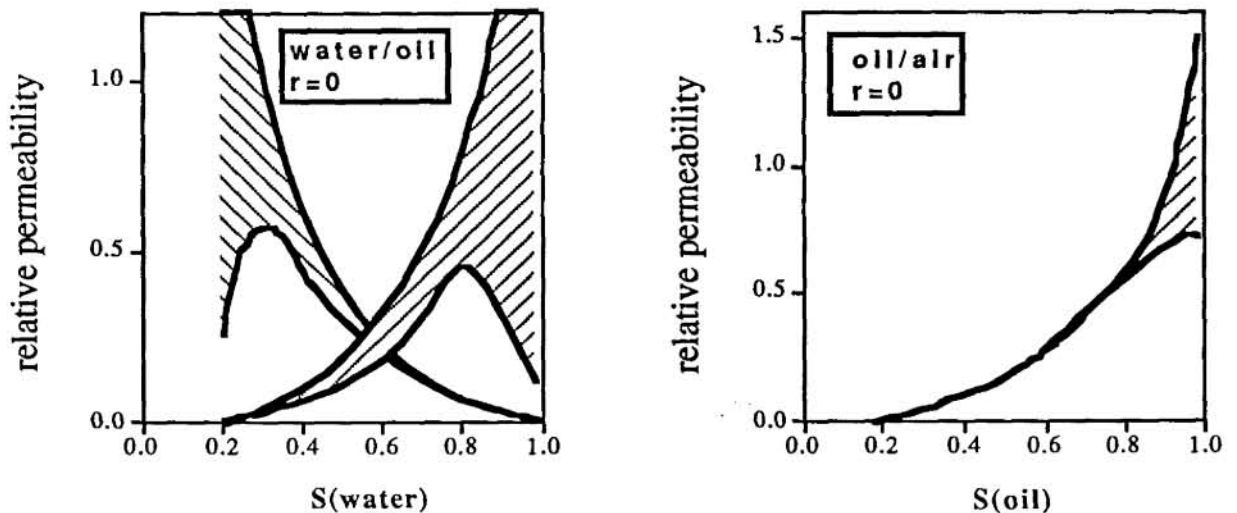


Fig. 4 - Influence of viscosity contrast when membrane resistance is negligible in water/oil system and gas/oil systems

## Conclusion

At the end of this study we can answer the question raised in the title. Is it possible to really determine the  $K_r$  curves from a porous plate experiment? The answer is no for water/oil systems, and yes for  $K_r$ (oil) in a oil/gas system provided that the resistance ratio is smaller than 1.

However, the parts of curves corresponding to average and low fluid saturations can be derived provided that membrane resistance is smaller than the core resistance. In this condition, the determination can be improved if we can measure independently the end-point permeability after removal of the membrane.

## Acknowledgements

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## APPENDIX. An Analytical Solution for the Porous Plate Experiment

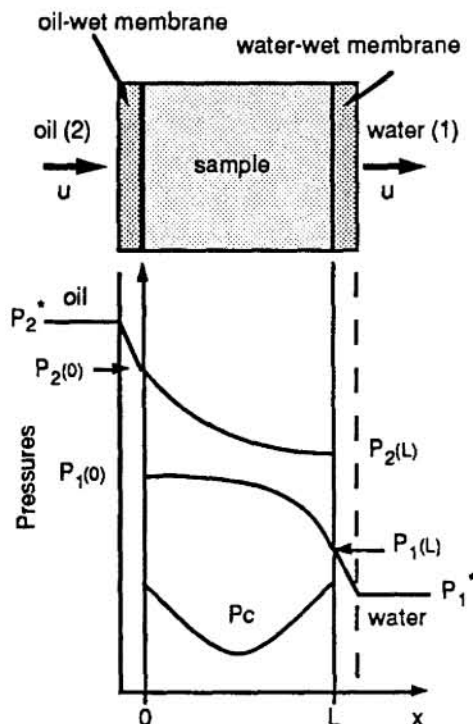


Fig. A1 - Schematic of setup and corresponding pressure variation in oil and water across the sample.

We will consider the example of oil displacing water (drainage) in a standard experiment with two porous plates or two micropore membranes (water-wet and oil-wet). At  $t=0$  the saturation is uniform at  $S_{wi}$  and the capillary pressure is  $P_{c_i}$  (Fig. A2). The pressure is suddenly changed to  $P_{c_j}$ . The saturation varies to reach the final value  $S_{wf}$  (Fig. A3). The purpose of the calculation is to find the saturation profile  $S(x)$  and the effluent production  $u$  as functions of time  $t$ .

If the increments of pressure and saturation are small, we can use the following assumptions :

- 1) the relative permeabilities remain constant during the step
- 2) the capillary pressure varies linearly with saturation during the step.

Even with these assumptions the calculation of the saturation profile and production is not straightforward. However, the problem is similar to heat transfer in a wall of finite thickness and the solutions of such a problem have been studied in detail. The basis of our calculation can be found in Carslaw and Jaeger (1959) for a symmetrical case. We will describe the more general case where the boundaries

condition are different for the two sides of the wall.

Darcy's law for fluids 1 (water) and 2 (oil) is

$$u_1 = -\frac{k_1}{\mu_1} \frac{\partial P_1}{\partial x} ; u_2 = -\frac{k_2}{\mu_2} \frac{\partial P_2}{\partial x} ; u = u_1 + u_2 \quad (1)$$

where  $u_i$  is Darcy's velocity (flow rate by unit sectional area). The total velocity  $u$  does not depend on  $x$  (incompressible fluid). The capillary pressure is

$$P_c = P_2 - P_1 \quad (2)$$

Fluid 2 is flowing through the membrane at  $x=0$  (noted 2) and fluid 1 through the membrane at  $x=L$  (noted 1).

$$P_2(0) - P_2^* = R_2 \mu_2 u ; P_1(L) - P_1^* = R_1 \mu_1 u \quad (3)$$

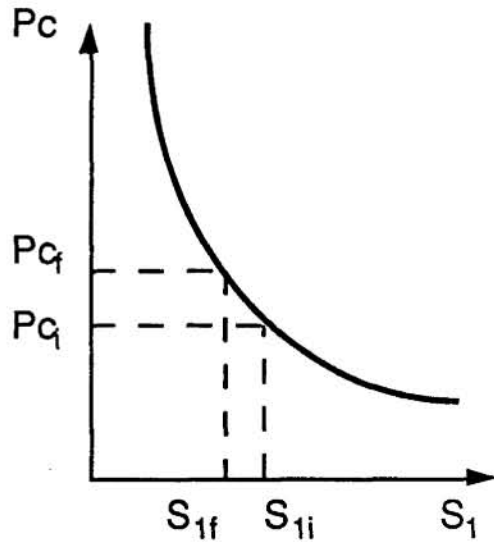


Fig. A2 - Variation of capillary pressure and saturation during a step of drainage

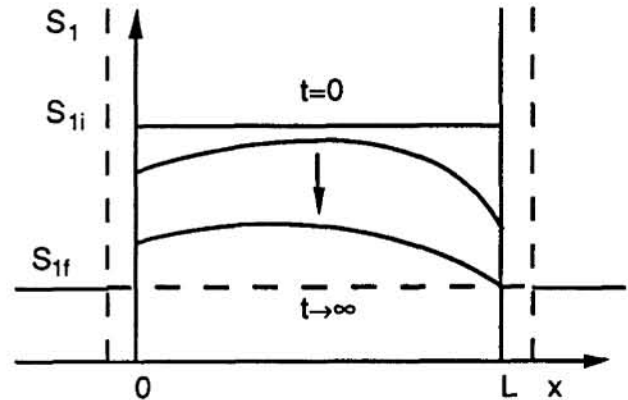


Fig. A3 - Schematic of the variation of saturation profile with time

$P^*$  are the pressures at the outside of the membranes (kept constant during a step) and  $P(0)$  and  $P(L)$  are the pressure at the contact between the membranes and the porous medium (Fig. A2).

Mass balance for fluid 1 is (where  $\epsilon$  is porosity)

$$\epsilon \frac{\partial S_1}{\partial t} + \frac{\partial u_1}{\partial x} = 0 \quad (4)$$

Boundary conditions: The membrane permeable to water prevent any oil flow. Consequently  $u_1(0)=0$  and  $u_2(L)=0$ . Darcy's law implies the following boundary conditions

$$u = \frac{k_2}{\mu_2} \frac{\partial P_c}{\partial x} \Big|_0 ; u = -\frac{k_1}{\mu_1} \frac{\partial P_c}{\partial x} \Big|_L \quad (5)$$

For deriving the pressure equation, we use the fact that the total velocity does not depend on  $x$ .

$$\int_0^L (u_1 + u_2) dx = uL = - \left[ \frac{k_1}{\mu_1} (P_1(L) - P_1(0)) + \frac{k_2}{\mu_2} (P_2(L) - P_2(0)) \right] \quad (6)$$

Introducing the capillary pressure and the external fixed pressures  $P^*$ .



$$u(L + \Lambda(R_1\mu_1 + R_2\mu_2)) = \frac{k_1}{\mu_1} (P_{c_f} - P_c(0)) + \frac{k_2}{\mu_2} (P_{c_f} - P_c(L)) \quad (7)$$

with the total mobility

$$\Lambda = \frac{k_1}{\mu_1} + \frac{k_2}{\mu_2} \quad (8)$$

Initial condition: At  $t=0$ , saturation is uniform  $S_1=S_{1i}$

Combined the Darcy's equations (Eq. 1) the assumptions of constant Kr and linear Pc lead to a simplified form of the Buckley-Leverett equation

$$\frac{\partial S_1}{\partial t} = \frac{a}{\varepsilon} \frac{k_1 k_2}{\mu_1 k_2 + \mu_2 k_1} \frac{\partial^2 S_1}{\partial x^2} \quad (9)$$

We will use as saturation variable the difference  $S = S_1 - S_{1f}$  and note the additional pressure drop due to the membranes in the form of an additional length  $L_R$  of porous medium such as

$$L_R = \Lambda(R_1\mu_1 + R_2\mu_2) \quad (10)$$

using the linear equation for capillary pressure, the pressure equation is then

$$u(L + L_R) = a \frac{k_1}{\mu_1} S(0) + a \frac{k_2}{\mu_2} S(L) \quad (11)$$

For the calculation of the solution we will use a simplified notation for the basic flow equations to have a notation similar to a thermal problem.

$$\frac{\partial S_1}{\partial t} = D \frac{\partial^2 S_1}{\partial x^2} \quad (12)$$

$$u = \beta S(0) + \beta' S(L) \quad (13)$$

and, at  $x=0$ ,  $u=u_2$  (because  $u_1=0$ ), and the opposite at  $x=L$

$$u = h \left. \frac{\partial S}{\partial x} \right|_0 ; u = -h' \left. \frac{\partial S}{\partial x} \right|_L \quad (14)$$

with the parameters

$$D = \frac{a}{\varepsilon} \frac{k_1 k_2}{\mu_1 k_2 + \mu_2 k_1} \quad (15)$$

$$h = \frac{a k_2}{\mu_2} ; h' = \frac{a k_1}{\mu_1}$$

$$\beta = \frac{h'}{L + L_R} ; \beta' = \frac{h}{L + L_R}$$

## Solution

The principle of the calculation is described in Carslaw and Jaeger, page 114 (1959) but for a symmetrical case. We will generalize the calculation for the unsymmetrical case corresponding to our problem. It can be shown, by using Laplace transforms for instance, that a general solution of the mass-balance diffusion equation is of the form

$$S(x, t) = \exp(-D\alpha^2 t)(A \cos \alpha x + B \sin \alpha x) \quad (16)$$

The solution must satisfy the boundary and initial conditions. Eliminating  $u$  between the boundary conditions (Eq. ) leads to the following relationships between  $A$  and  $B$

$$B' = \frac{B}{A} = \frac{\alpha h' \sin \alpha L}{\alpha h + \alpha h' \cos \alpha L} = \frac{\beta + \beta' \cos \alpha L}{\alpha h - \beta' \sin \alpha L} \quad (17)$$

To verify this equation,  $\alpha$  must be solution of the following equation

$$(\beta h' + \beta' h) \cos \alpha L - \alpha h h' \sin \alpha L + (\beta h + \beta' h') = 0 \quad (18)$$

We have seen that any function

$$S(x, t) = A_n \exp(-D\alpha_n^2 t)(\cos \alpha_n x + B' \sin \alpha_n x) \quad (19)$$

satisfies the boundary conditions if  $\alpha_n$  is one of the solution of Eq. 16, and  $B'=B/A$  (Eq. 17). We will find a solution which also fit the initial condition  $S(x,0)=f(x)$  as an infinite series

$$S(x, 0) = \sum_{n=1}^{\infty} A_n (\cos \alpha_n x + B' \sin \alpha_n x) = \sum_{n=1}^{\infty} A_n X_n \quad (20)$$

The method to determine the  $A_n$  coefficients is similar to the developments in Fourier's series by an integration term by term

$$\int_0^L X_n \sum_{n=1}^{\infty} A_n X_n dx = \int_0^L X_n f(x) dx \quad (21)$$

We follow the demonstration used by Carslaw and Jaeger to demonstrate that all the cross terms are zero because we have the relationship  $\beta' h' = \beta h$ . The quadratic term is then integrated

$$I_n = \int_0^L X_n^2 dx = \frac{1}{2\alpha_n} [(1 - B'^2) \sin \alpha_n L \cos \alpha_n L + 2B' \sin^2 \alpha_n L + (1 + B'^2) \alpha_n L] \quad (22)$$

The  $A_n$  are now calculated for a uniform initial saturation. Finally, the solution is

$$S(x, t) = S_i \sum_{n=1}^{\infty} \exp(-\alpha_n^2 D t) \frac{1}{I_n \alpha_n} (\sin \alpha_n L - B' \cos \alpha_n L + B') (\cos \alpha_n x + B' \sin \alpha_n x) \quad (23)$$

The flow rate can be calculated from Eq. (13)

$$u(t) = h S_i \sum_{n=1}^{\infty} \exp(-\alpha_n^2 D t) \frac{B'}{I_n} (\sin \alpha_n L - B' \cos \alpha_n L + B') \quad (24)$$

Effluent production  $V(t)$ , for unit sectional area, is the integral of flow rate:

$$V(t) = h S_i \sum_{n=1}^{\infty} \frac{1 - \exp(-\alpha_n^2 D t)}{\alpha_n^2 D} \frac{B'}{I_n} (\sin \alpha_n L - B' \cos \alpha_n L + B') \quad (25)$$