IMPROVING THE ACCURACY OF NMR RELAXATION DISTRIBUTION ANALYSIS IN CLAY-RICH RESERVOIRS AND CORE SAMPLES

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Abstract

Deriving accurate T_2 relaxation spectra from nuclear magnetic resonance (NMR) CPMG measurements is critical to the determination of total and effective porosities, irreducible water saturations, and permeability from NMR logs and core measurements. Accurate spectra are also essential to estimate coefficients for the film model or Spectral Bulk Volume Irreducible (*SBVI*) model. This paper analyzes the effects of noise, inadequate sampling rate, and *CBW* on the estimation of T_2 distribution and effective porosity. Synthetic data were used so that it is easy to compare the results with underlying model inputs. A new method is described which utilizes the partial porosity distributions obtained from two echo trains acquired with different *TE*s. High signal-to-noise ratio echo data with sampling time *TE* of 0.6 ms are used to obtain the *CBW* T_2 distribution. These data are then used to reconstruct *CBW* contributions to the time domain early echoes of the conventional effective porosity echo data (*TE* = 1.2ms). This residual signal is then subtracted from the original echoes. The effective porosity distribution is estimated from the reconstructed echo train.

Introduction

Transverse relaxation time (T_2) measurements using CPMG sequence (Meiboom and Gill, 1958) is the most common NMR log acquisition and core measurement method. In this method, echo data are collected at a fixed time interval, the interecho time. Usually, a few hundred to a few thousand echoes are acquired to sample the relaxation decay.

The interecho time, *TE*, is one of the most important, controllable experiment parameters for CPMG measurements and can affect interpretation. In log acquisitions, *TE*s of 0.6 and 1.2 ms are used to manipulate the relaxation decay data to include or exclude clay bound water[†] (*CBW*) porosity. Both effective and total porosity, defined as the sum of *CBW* and effective porosity, can be obtained with these measurements (Prammer, 1996a). This method implicitly assumes effective porosity, calculated from a *TE* of 1.2 ms acquisition, effectively excludes the *CBW* porosity. This situation is generally valid for rocks that contain little clay, but with clay-rich rocks, a further correction may be required to improve the accuracy of the estimated effective porosity.

In this paper, we first analyze the effects of sampling rate, relaxation models and noise on the determination of T_2 spectrum using examples instead of mathematical details. Singular value decomposition (SVD) is the method (Prammer, 1996b) used in the inversion analysis, although most of the analysis used are equally applicable to other algorithms. In particular, we focus on the

†Not all clay bound water has a T_2 relaxation time less than 4 ms and not all clay bound water may be resolved with a *TE* of 0.6 ms. However, for the sake of simplicity we will refer to NMR T_2 relaxation data with $T_2 \leq 2.83$ ms resolved with a *TE* of 0.6 ms as *CBW*.

effects that impact *CBW* and porosity determinations. Then, we describe a time-domain method that corrects the effective porosity and T_2 spectrum utilizing the *CBW* porosity.

The analysis is also applicable to core analysis and is particularly useful for log and core data comparisons and integration. Smaller *TE* values are often used for laboratory core measurements and logs may also be acquired with non-standard *TE*s (e.g., 0.9 ms). As shown in this paper, caution must be exercised in comparing porosities estimated from relaxation decay data acquired with different *TE*s.

Effect of Inadequate Model Bins

The interpretation of the NMR core or log data is often started by inverting the time-domain CPMG echo decay into a T_2-parameter-domain distribution. In general, the T_2 of fluids in porous rocks depends on the pore-size distribution and the type and number of fluids saturating the pore system. Because of the heterogeneous nature of porous media, T_2 decays exhibit a multiexponential behavior. The basic equation describing the transverse relaxation of magnetization in fluid saturated porous media is

$$M(t) = \int_{T_{2\min}}^{T_{2\max}} p(T_2) \exp\left(-\frac{t}{T_2}\right) dT_2 \ , \qquad (1)$$

where effects of diffusion in the presence of a magnetic field gradient have not been taken into consideration. In CPMG measurements, the magnetization decay is recorded (sampled) at a fixed period, *TE*; thus, a finite number of echoes are obtained at equally spaced time intervals, $t = nTE$, where n is the index for n^{th} echo,

$$M^{\exp}(nTE) = \int_{T_{2\min}}^{T_{2\max}} p(T_2) \exp\left(-\frac{nTE}{T_2}\right) dT_2 + noise \ . \qquad (2)$$

Equation (2) follows from Eq. (1) when noise and finite sampling are introduced. Because of the finite sampling of a continuous decay curve, information between the samples is not available. In order to estimate the unknown relaxation distribution function $p(T_2)$, a common approach is to use a set of pre-determined relaxation times and fit the amplitudes, T_{2i} (Prammer, 1996b). Using this approximation, the relaxation decay curve is modeled by

$$M(nTE) = pp_1 \exp\left(-\frac{nTE}{T_{21}}\right) + pp_2 \exp\left(-\frac{nTE}{T_{22}}\right) + \cdots + pp_k \exp\left(-\frac{nTE}{T_{2K}}\right). \qquad (3)$$

Mathematically, the multiexponential function is a valid approximation in that the function is linearly independent over distinct sampling points (Hamming, 1973). This property guarantees that a *unique* and *exact* solution can be found provided that there is no noise and that a sufficient number of the fitting bins, T_{2k}, are used to span all relaxation components in the underlying echo train. These strict conditions are not met in typical core and log NMR measurements; thus there are limitations on the quality of the approximate solution. The multiexponential inversion is known to be an extremely ill-conditioned problem, because the signal-to-noise ratio (*SNR*) is usually poor for NMR log data. Therefore, even small noise disturbances may substantially alter the solution.

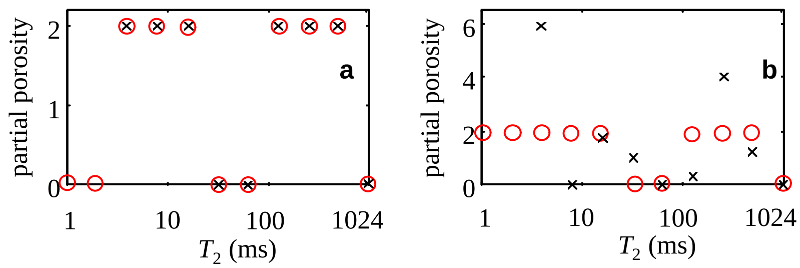

Figure 1. Comparison of 9 bin analysis of data containing (a) porosities corresponding to $T_2 \geq 4\text{ms}$ *and (b) additional porosities corresponding to 1ms and 2ms.* ("○": *model input and* "×": *fit*).

Figure 1 illustrates an analysis with a 9 T_2-bin (4, 8, 16, 32, 64, 128, 256, 512, 1024 ms) model of two synthetic noise-free echo train data sets of 300 echoes with a *TE* of 1.2ms. The model input in (b) contains two more short T_2 components (1 and 2 ms, respectively) than that in (a). When the underlying data include porosity outside of the T_2 bins in the fitting model (e.g., T_2 = 1ms and 2ms porosities), the inversion result (see plot (b)) may depart significantly from the underlying distribution as well as the effective porosity value.

Effect of Noise

The plots in Figure 1 suggests that an adequately large range of T_2 model bins which covers all T_2 components improves the accuracy of spectrum estimation of noise-free data. However, for noisy data, the situation is more complicated. This is shown in Figure 2. We used synthetic data with a moderate amount of Gaussian noise (1p.u.). The base model has partial porosities shown in Plots (a) and (d) with "•" symbols in Figure 2. The underlying distribution represents 12 p.u. *CBW*

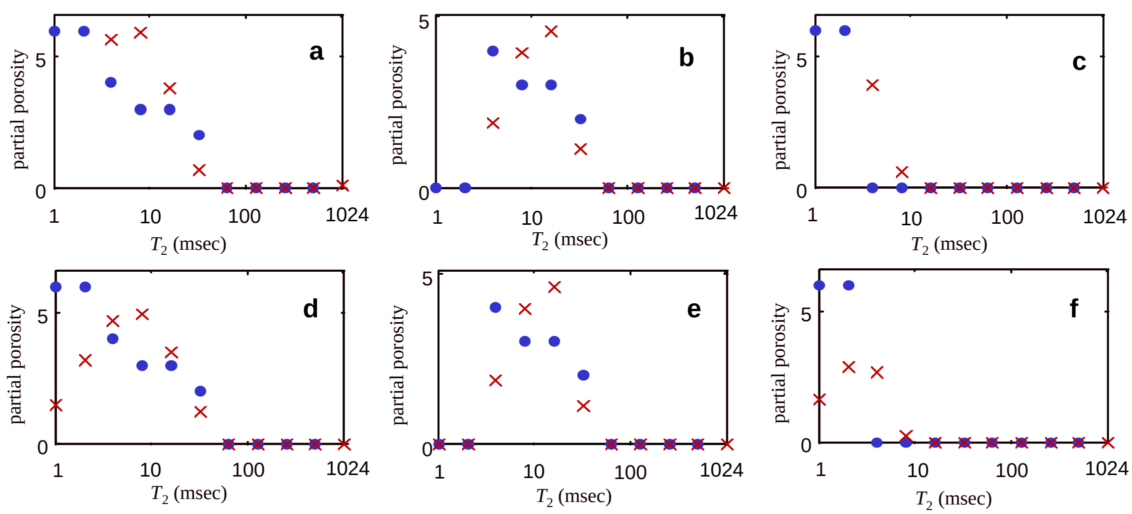

Figure 2. The effect of noise on inversion $T_{2k} = 2^{k+1}, k = 1,2,\ldots,9$ *(a-c) and* $T_{2k} = 2^k$, $k = 0,1,2,\ldots,10$ *(d-e).* ("●": *model and* "×": *fit*). *The underlying model data in Plots a and d contains 12 p.u. CBW and 12 p.u. BVI, the model in Plots b and e contains 12 p.u. BVI, and that in Plots c and f contains 12 p.u. CBW only. Comparing the top row and bottom row, it is clear that adequate* T_2 *bin selection is important but insufficient to recover the CBW porosities.*

porosity evenly distributed between the 1ms and 2ms T_2 bins and 12 p.u. of effective porosity, *MPHE*, with $T_2 \geq 4\text{ms}$. Then, the base model T_2 spectra are further divided into two cases: (1) has only 12 p.u. of effective porosity, shown in Plots (b) and (e), and the other case (2) represents only *CBW* (Plots (c) and (f)). The distributions are also tabulated in Table 1.

Two multiexponential fitting models with different fitting ranges are used. In Plots (a), (c) and (e), nine T_2 bins (4, 8, 16, 32, 64, 128, 256, 512, 1024 ms) were used, which is the typical range for effective porosity fitting. In the other three plots, eleven bins (1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024 ms) were used. In Fig. 2, the inversion results are shown with symbol "$\times$".

Table 1. Description of Models Shown in Fig. 2

Model T_2	1ms	2ms	4ms	8ms	16ms	32ms	≥64ms
(a) and (d)	6p.u.	6p.u.	4p.u.	3p.u.	3p.u.	2p.u.	0
(b) and (e)	0	0	4p.u.	3p.u.	3p.u.	2p.u.	0
(c) and (f)	6p.u.	6p.u.	0	0	0	0	0

The fitting results are summarized in Table 2. From Figure 2 and Table 2, we see that both 9- and 11-bin models can recover effective porosity sufficiently well for model data containing only effective porosity (case (b) and (e)). However, when present, the *CBW* porosity "leaks" into the effective porosity bins causing *PHIE* to be overestimated. Note that when noise is present, adding more fitting bins improves but does not completely eliminate the leakage problem.

Table 2. CBW and Effective Porosity with 9- and 11-Bin Analysis

	(a)	(b)	(c)	(d)	(e)	(f)
CBW (p.u., model || fit)	12 || 0	0 || 0	12 || 0	12 || 4.68	0 || 0	12 || 4.52
PHIE (p.u., model || fit)	12 || 16.27	12 || 11.55	0 || 4.57	12 || 14.42	12 || 11.55	0 || 2.95

Effect of TE

In general, the presence of noise deteriorates the accuracy of T_2 spectrum estimation for any T_2 distribution patterns, but the short relaxation time components are most sensitive. In this section, we analyze the effects of *TE* for cases with and without noise.

Without noise, the accuracy of recovering T_2 components less than *TE* is dependent on round-off errors and the amount of regularization. Thus, it is algorithm dependent. Figure 3 shows a set of inversion results ($\times$) using the same model data ($\bullet$) but sampled at different *TE*s. The total sampling time is kept constant: $NE_i \cdot TE_i = 100\,\text{ms}$. The results are obtained using singular value decomposition (SVD) with minimum regularization. In this case, the amplitudes associated with shortest T_2 components ($T_{2i} \geq TE/7$) can be accurately estimated. This accuracy requires that all echoes be used including the first echo. The only source of error is the numerical round-off during the matrix inversion in the analysis. This indicates that without noise and using a *TE* of 0.6ms (e.g., the sampling parameter used for MRIL *CBW* logging), it is possible to recover T_2 components as fast as 0.1ms. Therefore, improving *SNR* is equally important to reducing *TE* to recover short T_2 components. High *SNR* data is achieved with C/TP logging acquisition (Prammer, *et al.*, 1996a) by stacking 50 or more echo packets.

When noise is present, an inadequate sampling rate can affect the calculated relaxation distribution, particularly for the fast relaxing components. Because there are only a few non-vanishing echoes representing the decay of a short T_2 component, the T_2 relaxation spectrum resolvability suffers more for short T_2 components as *SNR* decreases. The spectrum resolving power was studied experimentally by Whittall *et al.* (1991) and found to be proportional to $SNR \cdot \sqrt{NE}$. This suggests that data acquired with an unfavorably long *TE*, and, thus a smaller *NE*, can be compensated for by improving the *SNR*. This is usually achieved by repetitive measurements and data stacking. Because *TE* may be limited by hardware, it is desirable to improve *SNR* to achieve the same T_2 spectrum resolution associated with a shorter *TE* data acquisition.

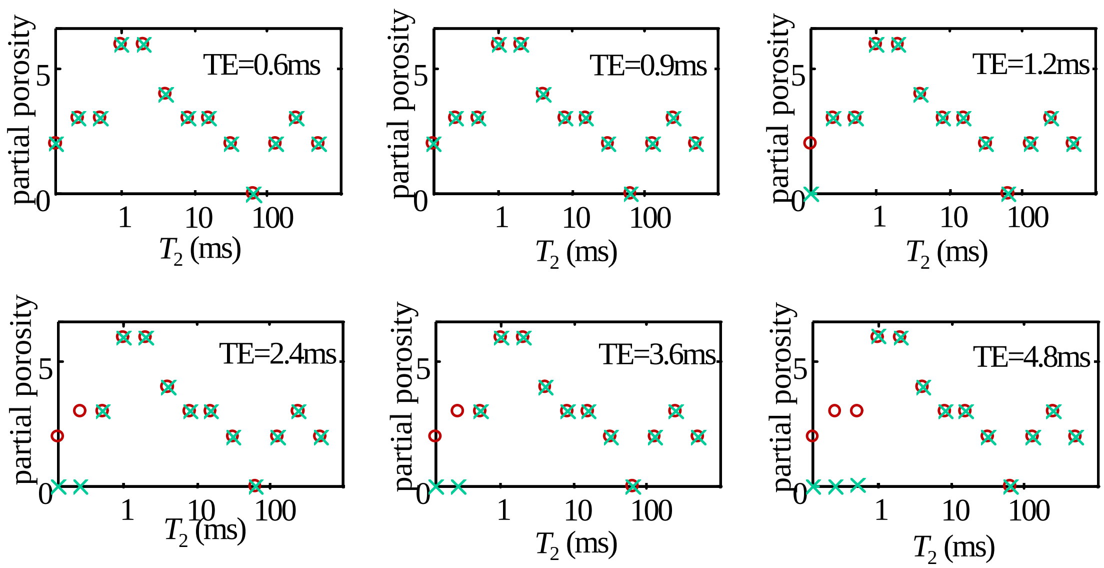

Figure 3. Effect of inadequate sampling of relaxation decay without noise contamination (○: *input model*, ×: *fitting results*)

Fourier analysis of exponential decays provides insight on why estimation of short T_2 components are more vulnerable to noise. According to the sampling theory, the maximum frequency that can be recovered is

$$f_{\max} = \frac{1}{2TE} . \qquad (4)$$

While the Fourier transform (FT) of a single T_2 decay component is

$$\tilde{M}(f) = FT[M(t)] = FT\left[\exp\left(-\frac{t}{T_2}\right)\right] \propto \frac{T_2(1 - i2pfT_2)}{1 + (2pfT_2)^2}, \qquad (5)$$

which peaks at zero frequency while the tail depends on the T_2 decay component. Figure 4 is a plot of power spectra ($|\tilde{M}|^2$) from zero to f_{max} of exponentials with different T_2 constants. It shows that long T_2 components have more energy in the low frequency region and, thus, are easy to distinguish from white noise, which contributes to all frequencies. On the other hand, short T_2 components have relatively flat power spectra and lack a distinctive frequency characteristics, and, thus are hard to distinguish from white noise and other fast decaying T_2 components.

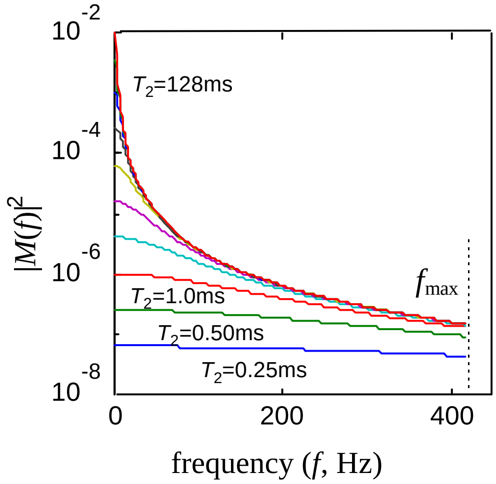

Figure 4. Power spectrum of exponentials with different T_2s. All curves assume TE = 1.2ms.

We demonstrate in Figure 5 the effects of noise and inadequate *TE* on T_2 spectrum estimation

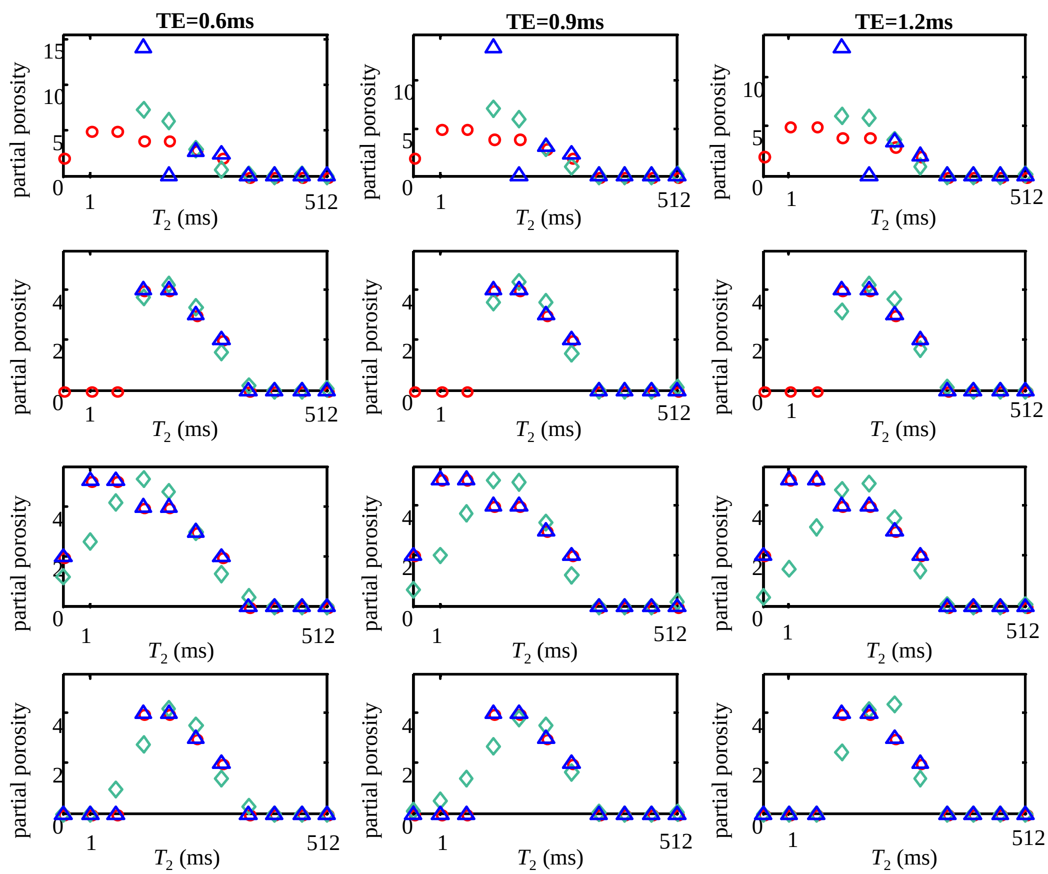

Figure 5. Effect of inadequate TE on T_2 spectrum and porosity estimation for noisy data (○: model input, Δ: fitting results for noise-free data, ◊: fitting results for noisy data). The results demonstrated that noise and the T_2 bin range selection are the dominant factors for the T_2 spectral resolvability. Without noise, it is possible to resolve T_2 components which are fractions of TE if fitting range is properly chosen.

using synthetic echo data with 1p.u. of Gaussian noise added. The relaxation record length (120ms) is the same for all these echo trains. Thus, the number of echoes is inversely proportional to *TE*, which are 200, 150, and 100 for *TE*s of 0.6, 0.9, and 1.2 ms, respectively. The input model bins in the three plots on the first and third rows are identical (i.e., 13p.u. *BVI* porosity and 12p.u. of *CBW* porosity). In the figure's first row, the fitting model contains logarithmically spaced T_2 bins from 4 to 512ms, as one usually uses in the T_2-log data processing. The plots in the third row show the results when the fitting model contains bins from 0.5 to 512ms. We see that the spectrum corresponding to the short T_2 bin data is *TE* dependent for noisy data, but is insensitive to *TE* for noise-free data with adequately chosen fitting model (third row). If fitting ranges are smaller than the data distribution range (first row), then the spectra are distorted for both noisy and noise-free data. As a comparison, we show on the second and fourth rows the fitting results for input models that do not contain *CBW*.

Discussion on Definition of Effective Porosity

NMR effective porosity is often interpreted to be the summation of the capillary bound fluid (*BVI*) and movable fluid (*BVM*), but clay bound porosity (*CBW*) is not included. Conventionally, it is calculated as the summation of the partial porosity, i.e.:

$$MPHE = \sum_{j=1}^{K} pp_{T_{2j}} \ , \qquad (6)$$

where the summation in the equation includes all partial porosities with $T_2 \geq 4\text{ms}$. In order to examine the effect of *CBW* porosity on the effective porosity determination, the examples shown in Figure 5 were used to calculate *MPHE*. The results are shown in Table 3.

Table 3. Example of aEffects of Fitting Model, TE, and Noise on MPHE Determination

	Fitting with 4ms - 512ms model and effective porosity is calculated by *MPHE*=Σ_i pp_i for all T_{2i}			Fitting with 0.5ms - 512ms model, effective porosity calculated with (1) *MPHE*=Σ_i pp_i for T_{2i}≥4ms and (2) *MPHE*=Σ_i pp_i for all T_{2i}, respectively					
TE	0.6ms	0.9ms	1.2ms	0.6ms		0.9ms		1.2ms	
Input signal	12 p.u. CBW and 13 p.u. BVI								
MPHE definition	Sum over 4-512ms bins			(1)	(2)	(1)	(2)	(1)	(2)
0.5-512ms, noisy	17.41	17.19	16.38	14.33	22.23	14.52	20.88	14.41	19.37
0.5-512ms,noise-free	19.27	18.84	18.45	13.00	25.00	13.00	25.00	13.00	25.00
4-512ms,noisy	12.88	12.83	12.56	13.14	12.12	13.89	11.80	12.25	12.25
4-512ms,noise-free	13.00	13.00	13.00	13.00	13.00	13.00	13.00	13.00	13.00

Two fitting models with 4-512ms and 0.5-512ms are used to obtain the T_2 spectra. For the latter, *MPHE* values were calculated with (1) T_2≥4ms bins and (2) all T_2 bins, respectively. Obviously, case (2) is equvalent to the MRIL total porosity definition. The various defintions of *MPHE* are used here to assess the CBW influence. We see that to a different degree, all three estimation approaches shown in Table 3 do not correctly exclude the *CBW* effect on the computed effective porosity. The trend observed in Figure 5 is statistically valid even though each individual fitting result may be subject to random noise and inversion algorithm specific characteristics. That is, for all the *TE* values shown, part of the *CBW* porosity would be interpreted as *MPHE*.

The *TE* dependence of NMR logs may also affect log and core *MPHE* interpretations. The standard MRIL® T_2 log uses *TE* of 1.2ms. Laboratory NMR measurements usually are acquired with short echo spacings ($TE \leq 0.5\text{ms}$). As we see from the above, *MPHE* from these measurements may not be identical if fast relaxing T_2 porosity exists. While the T_2-cutoff value between *CBW* and *BVI* is debatable, the use of short *TE*s is favorable when the resolution power, $SNR \cdot \sqrt{NE}$, is considered; however, log and core analysts need to be aware of possible discrepancies in *MPHE* estimates obtained from data acquired using different *TE* values.

Effective Porosity Corrections

In the last section, we demonstrated that the effective porosity can be overestimated and is *TE* dependent if there is a *CBW* porosity. The degree of misinterpretation depends on several factors such as the *CBW* mean T_2 and *SNR*. Since the systematic error is difficult to correct by simply subtracting in the T_2 domain, a time-domain correction procedure can be applied before inversion. The essence of the correction is to filter out the *CBW* contribution in the 1.2 *TE* echo train. Here we use T_{CBW} = 2.83 ms as the T_2 cutoff between *CBW* and *BVI*, which is the mid-point in log scale between the last *CBW* T_2 bin (2ms) and the first *BVI* bin (4ms).

CBW is acquired with the new MRIL C/TP tool and with a *TE* of 0.6 ms (labeled Data set B in Fig. 6). Typically, 50 repeat acquisitions of 10 echoes are obtained at each depth with this mode and a single echo train of several hundred echoes is acquired with a *TE* of 1.2 ms for the effective porosity. Thus, the *SNR* is approximately 7 times better with *CBW* echo data than the echo data for an effective porosity log. The wait time (e.g., 20 ms) in *CBW* logs is considerable shorter than the wait time used for fully polarized T_2 log acquisitions. Consequently, T_2 components corresponding to *CBW* are fully polarized; longer components are only partially polarized. *CBW* porosity distribution is obtained by inverting these echoes using a multiexponential model with six bins (T_{2i} of 0.5, 1, 2, 4, 8, 256 ms), and the sum of the first three bins is the *CBW* porosity. The last three bins are used to account for partially recovered porosities and are discarded (See Steps 1 and 2 in Figure 6). Because *SNR* is high, the *CBW* can be estimated relatively accurately.

In order to remove the contribution of clay bound water porosity from the regular 1.2 ms *TE* echo train (labeled Data set A in Figure 6), the forward calculation of the *CBW* echo decay contribution is performed using the three *CBW* bin partial porosities (as described in the step 3 in Figure 6). *CBW* has little effect on the echo signals beyond 5 times its highest T_2 component ($\text{i.e.,} 5 \times 2.83\text{msec} = 14.15\text{msec}$). Thus, the forward modeling of the *CBW* echo decay signal is only necessary for echo 1 to echo 12. The constructed *CBW* echo decay amplitdues are subtracted from the corresponding echo amplitudes in the Data set A echo train (See Step 4). The corrected echo train is reconstructed with the 12 new echo amplitudes plus the remainder of the original echo amplitudes in set A. The corrected echo train is, therefore, not contaminated by *CBW* porosities. Finally, the effective T_2 distribution and effective porosity are calculated by inverting the reconstructed echo train.

These corrections are only necessary for zones with *CBW*. When the *CBW* approaches zero the correction approaches zero and the subtraction does not alter the original data. The accuracy of the correction is obviously related to the *SNR* of the 0.6 ms *TE* echo train data. We have conducted a simulation with 500 sets of synthetic data with *SNR*=60. The estimated *CBW* values using the method described in step 2 of Figure 6 has an 80% confidence interval within ±10% accuracy of the model *CBW* value. Increasing the number of echoes (*TE* = 0.6 ms) does not appear to improve *CBW* estimate accuracy which is consistent with the fact that not much *CBW* information is carried beyond the 10^{th} echo.

Discussion and Summary

1. We have demonstrated with synthetic data that in clay-rich rocks the presence of *CBW* porosity may affect the accuracy of the effective porosity estimate. By subtracting the *CBW* contribution from the effective porosity echo train data, the error in the effective porosity data can be minimized or eliminated.

2. An inadequate sampling rate (1/*TE*) has little effect on noise-free data other than that arising from roundoff error, if the fitting model bin range is sufficiently broad. The effect becomes significant when data are noisy.

3. NMR-based effective porosity data should be qualified with the *TE* value used in the measurements. Comparison of effective porosity from core and log data must account for the *TE* values used to collect the data.

Nomenclature

A&B	-	Symbol for two acquired echo train amplitudes
A_{new}	-	Symbol for corrected echo amplitudes
BVI	-	Bulk volume irreducible
BVM	-	Bulk volume movable
CBW	-	Clay bound water porosity
M	-	Echo amplitude
$\tilde{M}$	-	Fourier transform of *M*
MPHE	-	Effective porosity
NE	-	Total number of echoes
$p(T_2)$	-	Relaxation distribution function
$pp_{T_{2i}}$	-	Partial porosity corresponding to T_{2i}
SBVI	-	BVI calculated from the film model
T_1	-	Longitudinal relaxation time
T_2	-	Transverse relaxation time
T_{2i}	-	i^{th} T_2 component
T_{CBW}	-	T_2 cutoff for *CBW*
TE	-	Interecho time, also is the echo train sampling period
TW	-	Wait time
f_{total}	-	Total porosity

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Step 1:
Input data set **A** & **B**
A. NMR T_2 log data (echo train with TE=1.2ms): **A(1.2*j)**
B. NMR Clay bound water data (echo train with TE=0.6ms, high S/N): **B(0.6*k)**

→

Step 2:
Invert data set **B** with multiexponential model and T_2 bins=0.5,1, 2,4,8,256. This step yields partial porosity results pp=[$pp_{.5}$ pp_1 pp_2 pp_4 pp_8 pp_{256}].

↓

Step 3:
Calculate the effect of CBW on T_2 log (Set **A**)

$$cor(n) = pp_{0.5}\exp\left(-\frac{n\cdot 1.2}{0.5}\right) + pp_1\exp\left(-\frac{n\cdot 1.2}{1}\right) + pp_2\exp\left(-\frac{n\cdot 1.2}{2}\right),\ n = 1,\cdots,12$$

↓

Step 4:
Subtract the CBW contribution from T_2 log data (Set A) for the first 12 echoes

$$A'(n) = A(n) - cor(n),\ \text{n} = 1,2,\ \ldots,\ 12 \qquad \text{(depth-by-depth)}$$

Reconstruct new echo train which has removed the CBW effects:

$$A_{new}(n) = [A'(1), A'(2), \ldots, A'(12), A(13), \ldots, A(NE)]$$

↓

Step 5:
Invert A_{new} (n) to obtain T_2 distribution and effective porosity.

↓

Step 6:
Combine the CBW distribution in Step 2 and the effective porosity distribution in Step 5 to obtain total porosity

Figure 6. Illustration of Effective Porosity Correction Procedure