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USE OF PORE-SIZE DISTRIBUTIONS FROM MERCURY INJECTION TO DERIVE CORRELATIONS BETWEEN PORE-SIZE POPULATION STATISTICS AND ROCK/FLOW PARAMETERS.

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ABSTRACT

Capillary pressure data have long been used in the interpretation of pore structure. A full evaluation of the information which could be derived from such data is however limited to pore-throat size distribution. This work proposes the derivation of rock/flow parameters from the pore-throat population statistics.

INTRODUCTION

Capillary pressure data are used to assist in the interpretation of pore structure when the property of interest is the pore-throat size frequency distribution. Pore-throat sizes are calculated from Mercury-Injection capillary pressure data, under the assumption that pore-throats are equivalent to cylindrical capillaries. The Young-Laplace equation of capillarity is used to compute the pore-throat radius, r, from the equilibrium capillary pressure (Pc):

$$r = \frac{2\gamma \cos\theta}{P_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where γ is the Mercury/Air interfacial tension and θ is the contact angle measured in Mercury.

Although capillary pressure curves have long been used to interpret properties of porous media, a full evaluation of the information which could be derived from such curves is still lacking[1].

In a previous work[2], a model was proposed to fit pore-throat distributions. A stochastic approach to the problem was proposed. The goodness-of-fit was tested by Hypothesis Testing, and Statistical Inference was used to infer the pore-throat population parameters. The population parameters of the accepted distributions were then correlated to the lab-measured petrophysical properties of the rock samples.

In this work the model is extended to cover the case of bimodal pore-throat distributions and to generate an effective distribution from individual capillary pressure curves.

THE MODEL

Pore-size distribution data are computed from Mercury Injection capillary pressure measurements by Equation1, and are usually reported as discrete values of pore radius (in microns) versus percent pore volume, invaded by Mercury.

A key assumption in the model is that the reported percent is interpreted as the "*frequency*" of occurrence of a "*discrete random variable*" which is the reported pore-throat radius.

Calculation Algorithm

1. Convert, if necessary, the reported (lab) frequency distribution of the pore radius into a cumulative relative frequency distribution and express as percentages.
2. Check the statistical validity of the data by plotting pore-size x_i vs. Pr (x_i) on a log-probability paper. The plot should be a straight line.if the pore-size has a log-normal distribution.
3. Calculate the mean,

$$\mu = \sum_{i=1}^{n} \ln x_i \quad \Pr\ (x_i)$$

and variance, $\sigma^2 = E\ [(x - m)^2]$

$$= \sum_{i=1}^{n} (\ln x_i - m)^2 \ Pr\ (x_i)$$

where n is the number of data points.

4. For each x_i, evaluate $z_i = \dfrac{1nx_i - \mu}{\sigma}$
5. Calculate $F(z_i)$ from :

$$F\ (z_i) = \frac{1}{2}\left[1 + erf\ \left(\frac{z_i}{\sqrt{2}}\right)\right]$$

An example of the model fit to laboratory data of pore throat distribution is shown in Figure1.

Data Analysis

As a test of statistical hypothesis, the normality of the distributions and of their model fits can be judged by the Chi Square (χ^2) test. A graphical procedure can be employed to correlate porosity and permeability with the population parameters (m and s) of the accepted log-normally distributed samples.

In a previous work, the porosity of a certain Mid-Eastern carbonate reservoir was found to be correlatable with the coefficient of variation (σ/μ);

$$f = -10.673\ \ln\left(\frac{s}{m}\right) + 29.1 \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

whereas permeability was found correlatable with the population geometric mean radius (m);

$$K = 2.603\, m^{3.172} \quad \ldots\ldots(3)$$

In this work, consideration is given to those samples having deviations of the measured data in the steep part of the capillary pressure curves, i.e. in the region of small pore-throat sizes. Such samples exhibit multi-modal distributions of their pore-throats.

For the pore-throat data samples exhibiting a bi-modal distribution character, the two peaks are treated here as belonging to two discrete random variables X and Y having a joint distribution density function, *f(x,y)*.

In such case, the two geometric means are:

$$\mu_X = \sum_x \sum_y \ln x \; f(x,y) \quad \ldots\ldots(4)$$

$$\mu_Y = \sum_x \sum_y \ln y \; f(x,y) \quad \ldots\ldots(5)$$

and the variances:

$$\sigma^2_X = \sum_x \sum_y (\ln x - \mu_X)^2 \; f(x,y) \quad \ldots\ldots(6)$$

$$\sigma^2_Y = \sum_x \sum_y (\ln y - \mu_Y)^2 \; f(x,y) \quad \ldots\ldots(7)$$

Assuming the random variables X and Y are independent, then their covariance, σ_{XY}, is zero by theory; and hence the variables are *uncorrelated*.[3]

An effective geometric mean, μ_{eff}, is then defined as :

$$\mu_{eff} = \sqrt{m_X\, m_Y} \quad \ldots\ldots(8)$$

When the pore-throat samples with bimodal distributions are analyzed in this way, more samples pass the normality test and can be included in deriving the correlations for porosity and permeability. These correlations are shown in Figures 2 and 3.
When Equations 2 and 3 are solved simultaneously, a parametric correlation between K and ϕ results with σ as the parameter:

$\ln K = 0.297\phi - 3.172\ln\sigma - 7.692$……………………(9)

Figure 4 is a cross-plot of the resulting relationship.

A small number of the data samples also had measured end-point saturations (Swi and Sor). The corresponding volume fractions (Vwi and Vor) are found to be functions of the standard deviation as shown in Figures 5 and 6.

INTERPRETATION

Statistical analysis of the theoretical fit (by a log-normal distribution model) of pore-throat distributions in reservoir rock samples taken from various depths of carbonate reservoirs was carried out. The lab-measured porosity and permeability are found correlatable with the pore-size population parameters (μ and σ). Porosity is found to be inversely proportional to the coefficient of variation (σ/μ).

Since the coefficient of variation is a measure of the relative dispersion of the pore-throats about their mean μ, it is intuitive to suspect porosity to be a function of both σ and μ. Expressing porosity in the following manner,

Porosity ϕ = (pore volume/ Bulk volume)

$\phi \rightarrow \mu^3$ **.** (total number of pores/Bulk volume).

$\phi \rightarrow \mu^3$ **.** (Number density of pores).

shows that ϕ must depend on μ which in turn can depend on σ. Moreover, the number density of pores is a function of σ.

On the other hand, $\sigma^2 = E(x-m)^2 = \sum (x-m)^2 f(x)$

Hence, $\frac{s^2}{m^2} = \sum (\frac{x}{m} - 1)^2 \mathrm{f(x)}$ shows that σ/μ is a measure of the range of pore-size relative to μ. And since both ϕ and σ/μ are dimensionless, then so must be the constants in Equation 2.

Since permeability is a measure of the ease of flow through the pore-system, it should be sensitive to pore-throat size rather than to pore-throat-size range. This is intuitive, since permeability depends directly on continuity of pore channels but inversely on the wetted surface area[1], i.e. although large pores might constitute a small fraction of the number density of pores, there volume fraction, however, can be large enough to make them actually interconnect via large throats, or simply reduce the wetted surface area and, hence, reduce the pressure energy consumed by flowing a fluid through the larger pores. i.e. the impedance to flow caused by the preponderantly many small pores in limestones can be nullified by the existence of large interconnected pores. The analogy here to electric current conductance through resistances in parallel is clear.

From a different angle of view, when σ is held constant, variation in μ alone will vary ϕ only slightly because the number density varies inversely with μ, while K will change considerably due to its r^2 dependence on the pore throat and surface area. Equation 2 is therefore qualitatively justified.

CONCLUSION

1. Lab-reported pore-size distributions can be modelled by a log-normal distribution. The analysis of the results by the technique of statistical inference gives an expression for porosity as a function of the coefficient of variation of the pore size, and another for permeability as a function of the mean size.
2. Some lob-reported pore-throat distributions are subject to reasonable amount of sampling errors and/or lab measurement errors.
   In fact, several pore size distributions suffer from severe tail end effects; SCAL-reported distribution ends, especially the lower (i.e. small-size) end, deviate from the theoretically-expected linear plot of size vs. cumulative relative frequency on a log-probability paper. Over the lower end, the final small increment of volume is caused by mercury invading the small spaces associated with surface roughness of the pore wall. However, the largest throats are accessible for invasion at the exterior boundary of a rock sample without being screened by pore-throats. Other researchers ran into similar deviations of the measured data in the steep part of the capillary pressure curves, i.e. in the region of small sizes.[4,5] This problem can be circumvented by assuming a bi-modal distribution of pore-throats; one for the micro-pores and a second for the macro-pores.
3. When enough pore-throat size data are available, it should be possible to model relative permeability and to relate the different kinds of porosity and permeability correlations to sub-zones and facies of the reservoir. The judicious use of the resulting relationships should offer better estimates of the petrophysical properties and for their upscaling.

Acknowledgement

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Nomenclature

r	pore-throat radius, micron
g	Mercury/Air interfacial tension, mN/m
θ	contact angle, degrees
Pc	capillary pressure, psi
E	refers to the expectation (probability) of an event
ln	natural logarithm
Pr	probability
σ^2	variance
σ	standard deviation, micron
μ	mean, micron
F(z)	standard log normal distribution function

erf	error function
i	a subscript
n	an integer
ϕ	porosity, %
K	permeability, mD

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TABLE I

ROCK SAMPLES AND THEIR PORE-SIZE POPULATION PARAMETERS

ROCK SAMPLE DESCRIPTION						POPULATION PARAMETERS DETERMINED BY MODEL			
Well No.	Sample No.	Facies	Rock Properties			χ2	μ(x) micron	σ (x) micron	σ (x)/μ(x)
			φ %	K, mD	Depth, ft				
W-1	71	M3AU2	27.42	3.2	8448.1	15.517	1.376	1.522	1.10373
	71A	M3AU2	27.92	6.9	8449.1	12.590	1.523	1.616	1.06308
	131	M3AL	27.8	4.7	8458.2	18.941	1.305	1.323	1.01347
	201	LM1	17.9	0.76	8473.8	19.229	0.726	1.448	1.99601
	201A	LM1	22.29	0.02	8475.2	14.940	1.058	1.507	1.42670
	261	LM2	26.81	3.9	8483.1	15.407	1.194	1.336	1.11792
	261A	LM2	26.56	5.05	8484.1	22.993	1.09	1.234	1.13107
W-2	21	-	26.4	4.2	8099.7	18.950	1.366	1.284	0.93891
	21A	-	30.26	5.8	8100.5	18.216	1.363	1.284	0.94276
	111	G1	22.6	630	8430.71	46.545	9.05	3.975	0.43921
	111A	G1	19.96	14.5	8431.4	18.631	1.787	2.915	1.63457
	151	WP1	15.2	0.44	8480	32.085	0.705	1.507	2.12860
	151A	WP1	12.52	0.44	8480.8	6.109	0.571	2.691	4.73017
W-3	31	M1	30.3	26	7974.1	9.625	1.647	1.522	0.92333
	31A	M1	29.9	20	7974.1	6.669	2.056	1.751	0.84815
	81	M3AL	30.81	6.9	8032.5	18.864	1.3	1.284	0.98408
	81A	M3AL	31.85	7	8033.3	16.368	1.318	1.297	0.98107
	101		27.9	530	9109.8	40.879	10.02	3.857	0.38624
	101A		32.9	3250	9110.6	94.474	24.51	1.859	0.07560
W-4	51	M2	28.3	1400	8034.71	23.756	8.654	3.819	0.44061
	71	R2	33.57	54	8048.1	18.399	3.17	1.552	0.48971
	101	M3AL	28.85	8.35	8080.09	16.557	1.735	1.419	0.81895
W-5	4C	G1	26.75	5	8233.9	19.705	1.018	1.419	1.38685
	6C	P	33.3	18	8239.01	13.763	1.989	1.336	0.67495
	15C	R/W	27.73	20	8272.31	18.513	1.616	1.433	0.89041
	17C	-	25	10	9202.1	25.673	1.926	1.733	0.89942
	20C	-	28.05	39.5	9223.9	11.266	2.357	2.46	1.04305
	23C	-	29.1	16	9237.01	18.396	2.114	1.896	0.89628
W-6	103C	VI	29.85	6.35	8099.1	22.414	0.916	1.363	1.48820
	112C	M1	19	42	8170.7	10.850	1.812	4.4	2.42440
	114C	M1	25.3	58	8176.51	17.526	3.821	2.534	0.66563
	116C	R	15.8	4	8185.7	19.386	2.436	3.034	1.24780
	119C	R	33.65	70	8189.8	16.266	3.175	2.054	0.64747
	123C	M1	27.2	33	8197.7	17.208	2.166	1.507	0.69270
	125C	R	30.1	23	8201.5	15.960	2.315	1.492	0.64418
	133C	LM	29.2	3	8306.7	25.388	1.133	1.209	1.06492
	142C	P	22.68	6.4	8452.69	7.784	1.189	2.014	1.69052
	146C	P/W	22.5	2.1	8477.6	33.971	1.017	1.336	1.31051
W-7	204C	IV	22.2	1.1	8338.7	22.252	0.611	1.259	2.06134
	210C	M2	24.6	10	8418.31	8.479	1.522	2.387	1.56605
	212C	R	26.9	16	8429.1	16.166	2.209	1.804	0.81715
	216C	LM	26.55	5.85	8500.69	49.185	1.077	1.197	1.11660
W-8	301C	IV	30.25	3.4	7949.41	32.780	0.87	1.22	1.40829
	309C	M2	5.4	0.02	8033.4	19.899	0.201	2.363	11.76777
	316C	M3AU2	26.3	5.6	8072	10.180	1.328	1.632	1.23035
	319C	LM2	32.4	6	8139.9	17.945	1.524	1.22	0.79978
	326C	P	25.6	10	8290.91	9.184	1.324	1.878	1.42032

Figure 1

Example of Log-normal Distribution Model Fit to Pore-throat Size Distribution Data

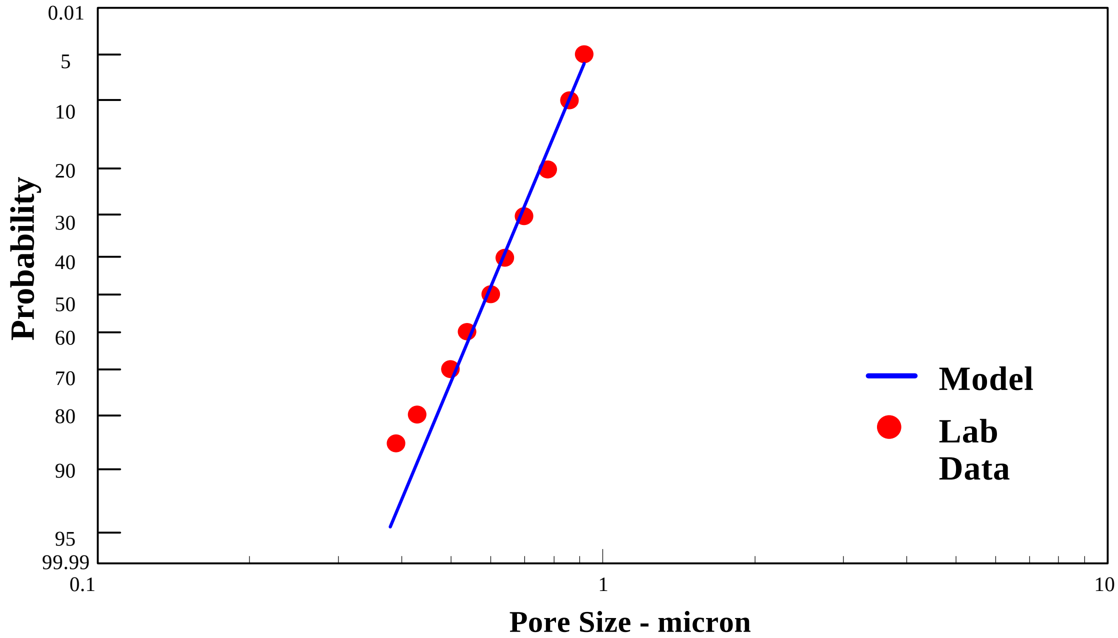

Figure 2

Porosity vs Coefficient of Variation. ; Accepted Samples

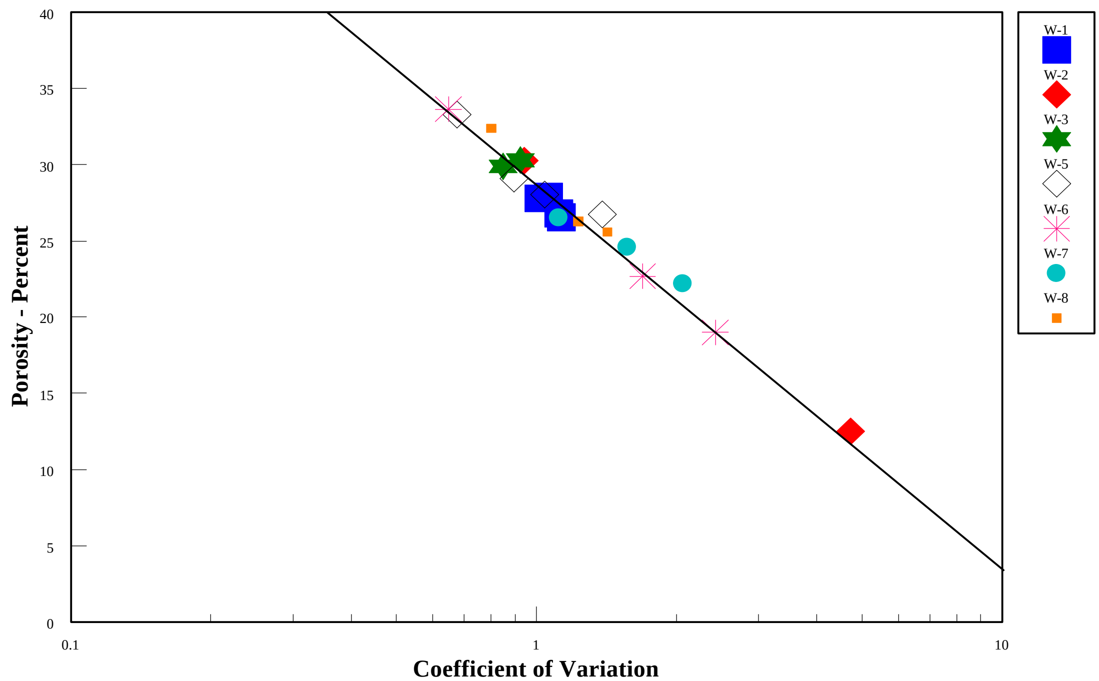

Figure 3

Permeability vs Mean Radius; Accepted Samples

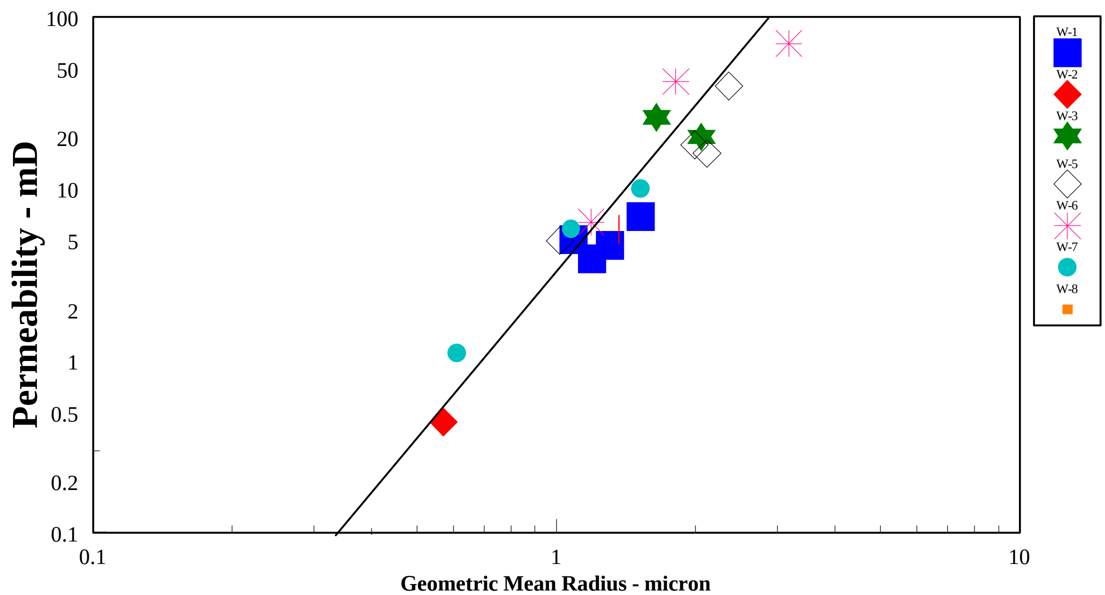

Figure 4

Parametric Cross Plot of Permeability vs Porosity; Example 1

(Standard Deviation as a Parameter)

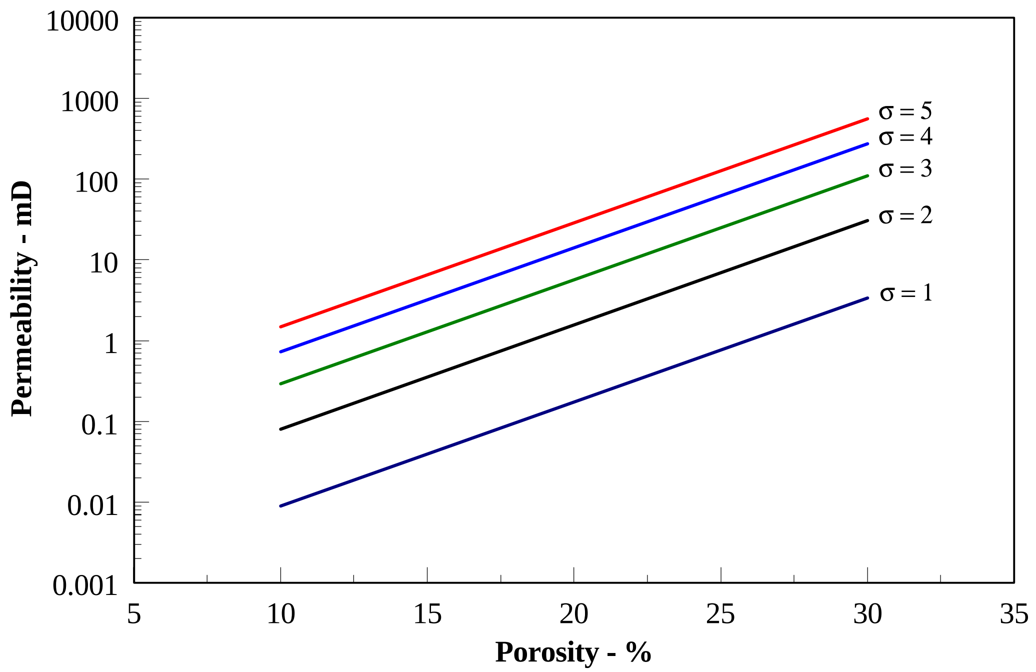

Figure 5

Initial Water Volume Fraction vs Standard Deviation; Accepted Samples

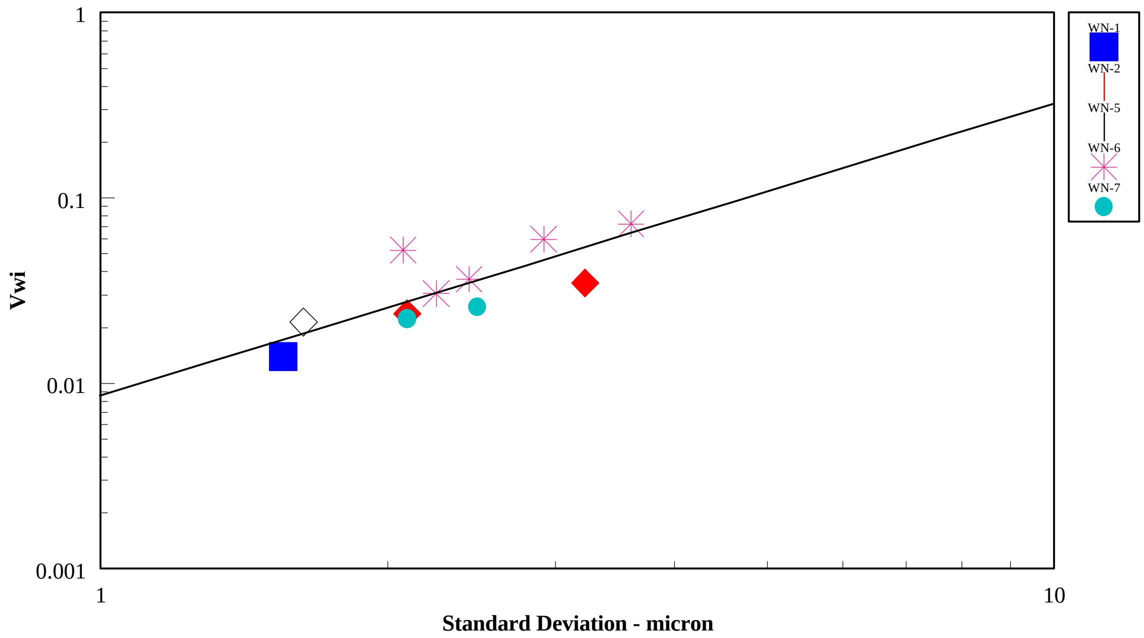

Figure 6

Ρεσιδυαλ Οιλ ςολυμε Φραχτιον ϖσ Σταν δαρδ Δεϖιατιον; Αχχεπτεδ Σαμ πλεσ