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QUANTITATIVE EVALUATION AND CORRECTION OF GRAVITY EFFECTS ON CENTRIFUGE CAPILLARY PRESSURE CURVES

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Abstract

Interpreting centrifuge measurements, in terms of capillary pressure curves, requires a number of assumptions regarding core homogeneity and boundary conditions. In addition, accounting for the exact pressure field within the sample is an issue for reliable capillary pressure curve determinations. Evaluation and correction for the radial distribution of that field have been reported. In this paper the effect of gravity, which is superimposed on the centrifuge pressure field, is analyzed quantitatively. Both drainage and imbibition processes are considered. Gravity may affect both the formulation and the inversion of the centrifuge problem.

Of these two, changes in the inversion procedure are found to be more significant. However, for both imbibition or drainage cases, and for centrifuge geometry in use in common core analysis, the effect is less, to much less, than the effect of an error of 0.01 to 0.1 psi in the pressure determination. In the range of usual measurements, the gravity effect is kept lower than an error of 10 RPM (rotation per minute) on the rotation speed determination. That is below the accuracy of centrifuges now in use.

In practice, however, these errors may have some influence for low capillary pressures. A correction, related to the usual interpretation techniques, is presented. It allows the use of centrifuge measurements to be extended to samples with very low capillary level, if high accuracy rotation speed measurements can be achieved.

The correction is not crucial when accurate interpretation procedures are used. Nevertheless, its use is recommended as it compensates for a low, but systematic, bias due to gravity and may improve approximate interpretation procedures. Additionally it accounts globally for radial and gravity effects. It is as simple as the single radial correction, and can be applied as a pre-process to any procedure currently in use.

Introduction

The centrifuge has been extensively used to determine capillary pressure curves, S(Pc), for core samples since 1945 (Hassler and Brunner, 1945 ; Slobod et al., 1951). It requires a transformation, of fluid production measurements, based on assumptions regarding the physics of fluid displacement and the inversion of an integral equation linking the capillary pressure curve S(Pc) to experimental centrifuge data (Hassler and Brunner, 1945).

The validity of physical assumptions : outflow capillary boundary condition, no cavitation, equilibration time, end-piece effects, homogeneity of the core, etc., have been discussed and improved experimental procedures have been proposed (Hirasaki et al., 1988 ; Hirasaki and Rohan, 1993 ; O'Meara et al., 1988, 1992 ; SCA, 1993).

Improved inversion techniques have been proposed as well, providing more reliable interpretations (Hoffman, 1963 ; Luffel, 1964 ; van Domselaar, 1984 ; Rajan, 1986 ; Ayappa et al., 1989). Evaluation and correction for radial field distribution were presented recently (Christiansen, 1992 ; Forbes et al., 1994). Degradation due to gravity was discussed by Chen and Ruth, 1994.

In this paper we offer new insights to complete the process of inversion. The effect of gravity is evaluated quantitatively and a correction is given to compensate for potential gravity degradation, while still using usual interpretation techniques.

The Technique

The centrifuge method consists of measuring average fluid saturation in a core (Figure 1) at equilibrium during rotation at various angular velocities ω. The sample is initially filled with a fluid and spun within a second fluid. Due to the rotation, the inner fluid is forced out of the sample and the quantity expelled is measured to determine the average fluid saturation. When rotating, the core fluids are subjected to the centrifugal field, $1/2\rho\omega^2 r^2$, and to the gravity field, $-\rho gZ$. Where ρ is the fluid density, g the gravitational constant and (r, Z) refer to the cylindrical coordinates (Figure 1). At hydrostatic equilibrium, the pressure is therefore : $P=1/2\rho\omega^2r^2-\rho gZ+const.$ (Chen and Ruth, 1994).

The capillary pressure is then given by : $Pc(r, Z,\omega) = Const. - ½ \Delta\rho\ \omega^2\ r^2 + \Delta\rho\ g\ Z$

The value of the constant *Const.* is obtained from the boundary condition hypothesis, i.e. that the extremal Pc value is Pc=0, where the inner fluid is flowing out of the sample.

For cylindrical core sample in drainage experiment, Pc=0 is located on the border of the circular outflow face of the sample, the face the furthest from the axis of rotation, for $Z=-R$, if $g/\omega^2>R$ or, for $Z=-g/\omega^2$, if $g/\omega^2<R$, leading to : $Pc(r,Z,\omega)=1/2\Delta\rho\omega^2(r_3^2-r^2) + \Delta\rho gZ + 1/2\Delta\rho\omega^2(n+1) R^2$ with $n=2(g/\omega^2)/R-1$, if $g/\omega^2>R$ or $n=(g/\omega^2)^2/R^2$, if $g/\omega^2<R$. (r_3: radius to the center of the outlet face)

For cylindrical core sample in imbibition experiment, Pc=0 is on the top of the outflow face of the sample, the face the nearest from the axis of rotation, and $Pc(r,Z,\omega)=1/2\Delta\rho\omega^2(r_1^2-r^2)+\Delta\rho g(Z-R)$, that is again $Pc(r,Z,\omega)=1/2\Delta\rho\omega^2 (r_1^2-r^2) + \Delta\rho gZ + 1/2\Delta\rho\omega^2(n+1) R^2$ with $n= -2 (g/\omega^2)/R - 1$, (r_1 being the minimum radius from the rotation axis to the sample face)

By definition, the average saturation, <S>, in the core is:

$$<S> = \frac{1}{L\pi R^2}\int_{core} S_{r,Z,\omega}\,dv \qquad (1)$$

The local saturation is $S_{r,Z,\omega}$, it depends only on r, Z and ω, as the pressure does. L is the core length, R its radius, r the rotation radius, dv the elementary volume (rdrdθdZ in cylindrical coordinates).

Can gravity affect the formulation of the general saturation equation ?

Equation (1) is usually re-written linking $S_{r,Z,\omega}$ to the capillary pressure curves, S(Pc), by $S_{r,Z,\omega} = S(Pc_{(r, Z,\omega)})$. That may not be exact when gravity is considered in a drainage experiment.

For imbibition, $Pc(r,Z,\omega)=0$ is maintained on the top of the inner face of the sample, whatever the rotation speed. This ensures that during the experiment, Pc is continuously decreasing everywhere in the core when the rotation speed is increased. $S_{r,Z,w}$ is therefore actually varying according to $S_{r,Z,\omega} = S(Pc_{(r,Z,\omega)})$, where S(Pc) is the imbibition capillary pressure curve. The current formulation is perfectly valid.

For drainage experiment, uniqueness of the S(Pc) relationship, is no longer assured, due to gravity. The location of the minimum, Pc=0, moves when the rotational speed increases. It moves from the bottom of the outlet face (Z=-R), for low speed, along the border of that face to Z=0, for infinite speed (Figure 2). It leads to a peculiar Pc evolution during a "drainage centrifuge experiment". A location along the outlet border, where Pc=0 at a given time, displays positive Pc before and after that time. For instance, at the location the furthest from the centrifuge axis ($r^2=r_3^2+R^2$, Z=0), Pc is decreasing from ΔρgR (ω=0) to zero (ω infinite). In other words, Pc may be decreasing at certain locations within the core outlet region, when the rotational speed is increasing. That may lead to potential imbibition and hysteresis processes. If so, $S_{r,Z,\omega}$ may not be linked to the drainage capillary pressure curve $S(Pc_{(r,Z,\omega)})$, as assumed currently.

Figure 2 gives the evolution of Pc on the border of the outlet face of the core, where that effect is maximum. The zone where Pc decreases ranges from 2ΔρgR to 0. One may note however, from the above equation, that the reverse variation of Pc (and potential departure from the drainage S(Pc) curve) may occur only if $g/\omega^2<R$ and if r is such as $r_3^2+R^2 > r^2 > r_3^2+R^2-(g/\omega^2)^2$, that is in a narrow zone near the outlet of the core. An evaluation of the relative volume of that zone shows that it represents less than 1% of the core volume and less than 0.1% if the rotation speed is higher than 300 RPM. In that zone, even if the saturation is very different from the drainage curve $S(Pc_{(r,Z,\omega)})$, it may have only a small effect on the evaluation of the integral of equation (1), when $S_{r,Z,\omega}$ is replaced by $S(Pc_{(r,Z,\omega)})$. For example, a discrepancy of 20 saturation units, on 1% or less of the integration volume, generates an error lower of less than 0.2 saturation units on integral (1), while the experimental error in <S> is usually about 1 saturation unit or more.

When the threshold pressure is higher than 2ΔρgR, (0.01 to 0.04 psi for 1"x1" sample), the saturation in that zone will stay at its maximum value, ensuring the uniqueness of the saturation curve involved.

This is always the case when spontaneous production, by gravity drainage is not observed before starting centrifuging.

For a drainage experiment, one can conclude that gravity may generate a difference between $S_{r,Z,\omega}$ and $S(Pc_{(r,Z,\omega)})$, for low rotation speed, in a narrow zone at the outlet part of the core only if spontaneous gravity production is observed before starting centrifuging.

Gravity can, therefore, potentially change the formulation of the saturation equation related to a drainage experiment. However the related change in the usual formulation (when replacing $S_{r,Z,\omega}$ by $S(Pc_{(r,Z,\omega)})$ in equation (1)), is not relevant, far below experimental error in <S>.

To summarize the general saturation equation is exactly, or very close to, the usual formulation :

$$<S> = \frac{1}{L\pi R^2}\int_{core} S_{(Pc(r,Z,\omega))}\, r\,dr\,d\theta\,dz \qquad (2)$$

$$Pc(r, Z, \omega) = ½\, \Delta\rho\, \omega^2 (r_3^2 - r^2) + \Delta\rho\, g\, Z + ½\, \Delta\rho\, \omega^2 R^2 (1+n)$$

for drainage : S(Pc) is the drainage capillary pressure curve and
$n=2 (g/\omega^2)/R - 1$, if $g/\omega^2 > R$ or $n=(g/\omega^2)^2 / R^2$, if $g/\omega^2 < R$.

for imbibition S(Pc) is the imbibition capillary pressure curve and
$n= -2 (g/\omega^2)/R - 1$

The problem is now to calculate S(Pc) from the measurement of <S>.

Reducing the saturation equation

Normalizing equation (2) requires a transformation, as presented by Forbes et al. (1994).

For drainage, that transformation is obtained by introducing
----normalized parameters : ----and normalized variables

$$B = 1 - \left(\frac{r_1}{r_3}\right)^2$$
$$N = \frac{R^2}{r_3^2 - r_1^2}\ ,\ M = \frac{g}{\omega^2 R} \qquad (3)$$
$$P_1 = \frac{1}{2}\Delta\rho\omega^2(r_3^2 - r_1^2)$$

$$x = \frac{1 - \left(\frac{r}{r_3}\right)^2 Cos(\theta)^2}{B}$$
$$y = 1 - \frac{\left(\frac{r}{r_3}\right)^2 Sin(\theta)^2}{NB}\ ,\ z = Z/R/\sqrt{y} \qquad (4)$$

B is known to represent the centrifugal aspect of the pressure field, related to the fact that the pressure varies versus r^2 and not linearly with r. N has been presented by Christiansen (1992), Christiansen and Cerise (1987) and Forbes et al. (1994), as appropriate to represent the magnitude of radial effects related to the curvature of the pressure field around the rotation axis. A new parameter, $M = (g/\omega^2)/R$, which is one fourth of the ratio between $2\Delta\rho g R$, the capillary pressure generated by gravity on top of the core, and $1/2\Delta\rho\omega^2 R^2$, the contribution of radial effect to the inlet capillary pressure, can be introduced. M is "a priori" an appropriate parameter to measure the extent of the effect of gravity.
Introducing x, y, z, B, N and M in (2), one obtains :

$$<S>_{B,N,M} = \frac{1+\sqrt{1-B}}{2}\int_{x=0}^{x=1}\frac{dx}{\sqrt{1-Bx}}\int_{y=0}^{y=1}\frac{2}{\pi}\sqrt{\frac{y}{1-y}}\,dy\int_{z=-1}^{z=1}\frac{dz}{2}\,S_{[P_1(x+Ny+2NMz\sqrt{y}+Nn)]} \qquad (5)$$

The integral is written **<S>$_{B,N,M}$** to stress that it depends on B (centrifugal effect), N (radial effect) and on M (gravity effect.

For imbibition, the same kind of transformation is performed, replacing r_1 by r_3 and r_3 by r_1, that is by introducing the normalized parameters : and the normalized variables

$$x=\frac{1-\left(\frac{r}{r_1}\right)^2 Cos(\theta)^2}{B} \qquad (6)$$

$$y=1-\frac{\left(\frac{r}{r_1}\right)^2 Sin(\theta)^2}{NB}, \; z=Z/R/\sqrt{y}$$

$$B=1-\left(\frac{r_3}{r_1}\right)^2$$

$$N=\frac{R^2}{r_1^{\,2}-r_3^{\,2}}, \; M=-\frac{g}{\omega^2 R} \qquad (7)$$

$$P_3=\frac{1}{2}\Delta\rho\omega^2(r_1^{\,2}-r_3^{\,2})$$

Introducing x, y, z, B, N and M in (2), one obtains :

$$<S>_{B,N,M}=\frac{1+\sqrt{1-B}}{2}\int_{x=0}^{x=1}\frac{dx}{\sqrt{1-Bx}}\int_{y=0}^{y=1}\frac{2}{\pi}\sqrt{\frac{y}{1-y}}dy\int_{z=-1}^{z=1}\frac{dz}{2}S_{[P_3(x+Ny+2NMz\sqrt{y}+Nn)]} \qquad (8)$$

Note that B, N, M and P_3 have negative value in this case.

Finally the **general "centrifuge problem"** can be summarized as follows :

1.The measurement process provides a data set {<S>, ω}, <S> being the measurement of the average saturation in the core and ω the corresponding rotation speed.

2.This is transformed into a data set {<S>,P}, P being P_1 for drainage experiments or P_3 in imbibition.

3.S(Pc) is obtained by inverting the integral equation : $<S>(P)=<S>_{B,N,M}(P)$ defined by equation (5) or (8).

We will now discuss how this "problem" is usually solved.

Solving the "centrifuge problem"

The oldest solution, (Hassler and Brunner, 1945) is the simplest, but the poorest, solution of the saturation equation. It consists of assuming B=0, N=0 and M=0. That is $<S>(P) = <S>_{0,0,0}(P)$ or :

$$<S>=\int_{x=0}^{x=1}dxS_{[Px]} \qquad (9)$$

Which is inverted as

$$S_{HB}(P)=<S>(P)+P\frac{d<S>}{dP}(P) \qquad (10)$$

This solution is denoted S_{HB}. It assumes that the pressure in the core varies linearly, being neither radial nor centrifugal, while gravity is neglected. This is actually the case for very short and narrow sample spun far from the rotation axis. As demonstrated by Forbes (1991, 1994), this solution may be significantly lower than the true S(Pc) solution.

Since 1945 numerous solutions have been provided, assuming $B\neq0$ but keeping N=0 and M=0. These solutions still neglect the curvature of the pressure field and the effect of gravity. These are solutions of the integral equation <S>(P) = <S>$_{B,\,0,\,0}$ (P) or :

$$\langle S\rangle = \frac{(1+\sqrt{1-B})}{2}\int_{x=0}^{x=1}\frac{S_{(xPc)}}{\sqrt{1-Bx}}dx \qquad (11)$$

Most of the corresponding solutions in use can be found in Ayappa et al. (1989), King et al. (1990), Bentsen and Anli (1977), Rajan (1986), Nordtvedt and Kolltveit (1991), Melrose (1988), Ruth and Wong (1988, 1990, 1991), Skuse et al. (1988), Glotin et al. (1990), Nordtvedt et al. (1990), Hermansen et al. (1991), Forbes (1991, 1993, 1994) and Golaz and Bentsen, (1980). Depending on the level of approximation, they can be good or poor solutions. A discussion of their respective drawbacks and qualities is available in Forbes, 1997. One may however keep in mind that they are all solutions of equation (11), which is an approximation of the complete equation (5).

More recently, radial effects, N≠0, M=0, have been considered. In that case, equations (5) and (8) reduce to <S>(P) = <S>$_{B,\,N,\,0}$ (P) or :

$$\langle S\rangle = \frac{1+\sqrt{1-B}}{2}\int_{x=0}^{x=1}\frac{dx}{\sqrt{1-Bx}}\int_{y=0}^{y=1}\frac{2}{\pi}\sqrt{\frac{y}{1-y}}\,dy\,S_{[P_1(x+Ny)]} \qquad (12)$$

Christiansen and Cerise (1987), Ayappa et al. (1989), Christiansen (1992) and Forbes et al. (1994) established the above equation or closely related forms. Forbes et al. (1994) provided a quantitative analysis of this equation and a way to account for it, while still using the different solutions previously developed for inverting equation (11).

It consists of replacing the data {P;<S>[P]} by {P/bo ; <S>[P] + a_o(<S>[a_oP] - <S>[P])} before processing usual solutions of (11), according to:

	P	a_o	b_o
Drainage	$1/2\Delta\rho\omega^2(r_3^2-r_1^2)$	$\frac{3/4N\,(1+(1-B)^{1/2})}{2(1+N)}$	$\frac{1+0.23N/(1+N)}{(1+N)}$
Imbibition	$1/2\Delta\rho\omega^2(r_1^2-r_3^2)$	$\frac{-1/4N(1+(1-B)^{1/2})}{2}$	$1-a_o\,(4-(1-B)^{1/2})/(2+(1-B)^{1/2})$

Such a correction significantly improves the interpretation in term of a capillary pressure curve, when N>0.03, that is for 1"x1" core sample in the usual Beckman centrifuge geometry, for instance (Forbes et al. 1994 ; Forbes and Fleury, 1995 ; Forbes 1997).

The final step to improve the inversion of the centrifuge problem is therefore to account for gravity. One must consider the complete saturation equation, that is equation (5), or (8), for B≠0, N≠0, M≠0, or <S>(P) = <S>$_{B,\,N,\,M}$ (P).

Accounting for gravity effects

The approach, presented in the Appendix, is similar to the one by Forbes et al. (1994) for accounting for radial effects. The equation (5) is "re-worked" to provide an evaluation of the integral <S>$_{B,0,0}$(P), for which inversion techniques are available (solutions of equation 11). This is a total correction accounting for both radial and gravity effects. The proposed total correction consists of changing the current determination {P; <S>[P]} by {P/b; <S>[P] +a_o(<S>[a_oP] - <S>[P])} before processing the usual solutions of equation (11), with

	P	B	N	M	C	1/b - 1/bo
Drainage	$1/2\Delta\rho\omega^2(r_3^2-r_1^2)$	$\frac{(r_3^2-r_1^2)}{r_3^2}$	$\frac{R^2}{(r_3^2-r_1^2)}$	$\frac{g}{\omega^2 R}$	$\frac{N(4+2(1-B)^{1/2})}{(5+(1-B)^{1/2})}$	M>1 : (4M-1.75) C 0<M<1 : $2.25M^{1.7}$ C
Imbibition	$1/2\Delta\rho\omega^2(r_1^2-r_3^2)$	$\frac{(r_1^2-r_3^2)}{r_1^2}$	$\frac{R^2}{(r_1^2-r_3^2)}$	$\frac{-g}{\omega^2 R}$	$\frac{N(4+2(1-B)^{1/2})}{(5+(1-B)^{1/2})}$	M<0 : 4M C

a_o and b_o are the parameters previously defined for the radial correction (see Appendix). The correction appears to be of the same kind as the radial correction, except that the b factor, for pressure correction, is pressure dependant (function of M), while it is a constant, b_o, when only radial correction is considered. Note that this is a pre-process correction, applied before processing the data through usual procedures. It can not be applied as a post-process correction, while the radial correction can (because b is pressure dependant while bo is not). However, the numerous procedures developed for inverting equation (11) can still be applied.

Validation

In order to check the validity of the above correction, the exact value of the integral $<S>_{B,0,0}(P)$ and the evaluation of that value given by the correction were compared. Many capillary pressure curves have been considered, in the form of polynomial relationships. For polynomial functions, the exact values of the integral $<S>_{B,0,0}(P)$, and of the full integral $<S>_{B,N,M}(P)$ can be calculated analytically. For every capillary pressure curve, every centrifuge geometry and any pressure, one can therefore produce the exact value of $<S>_{B,0,0}(P)$ and of the measurements $<S>=<S>_{B,N,M}(P)$. On these artificial measurements, one applies the above correction to retrieve the evaluation of the $<S>_{B,0,0}(P)$ curve.

Figures 3 and 4 provide examples of comparisons for drainage and imbibition cases respectively.The white dots represent a sampling of the $<S>_{B,N,M}(P)$ curve, that is the artificial data set of measurements, while the black dots are the correction of that sampling by the above procedure. These black dots can be compared with the line $<S>_{B,0,0}(P)$ which is the exact value.

In practice, the difference appears to be small, even for low levels of capillary pressure (see cases A and B on figures 3 and 4), that is capillary pressure lower than 0.1 psi, leading to large spontaneous production (>20 saturation units) under the effect of gravity, before centrifuging. These last cases are furthermore very unlikely to occur in core analysis. For initial spontaneous production (due to gravity before centrifuging) lower than 10 saturation units, the difference between the exact $<S>_{B,0,0}(P)$ and the correction has always been found to be less than 1 saturation unit (the usual uncertainty in centrifuge measurements).

The proposed correction appears therefore to be reliable for any centrifuge measurement in core analysis, even if capillary pressure levels are low and if significant radial or gravity effects occur.

What is the extent of gravity effect ?

Having checked the proposed correction, one can use it for drawing some conclusions on the extent of the gravity effect. The above correction includes both radial and gravity effects. If one wants to separate the contribution of gravity, one has to compare with the radial correction.

The corrected pressure is P/b for total effect and P/b_o for radial effect, while the corrected saturation is the same in both cases. Gravity therefore generates only the pressure correction : $P_1/b-P_1/b_o$. That correction is drawn on figures 5 and 6, as a function of ω for 1" long core and liquid-liquid or gas-liquid systems.

The maximum gravity correction to pressure is in the range of $2\Delta\rho gR$, the extent of the gravity capillary pressure variation along the sample height. It can never exceed this level, that is 0.08 psi, gas-liquid cases or 0.016 psi for oil-brine cases, in current used geometries (R<1").

For imbibition, the pressure correction is constant and therefore is reduced in relative importance as the centrifuge capillary pressure increases. For drainage, it decreases rapidly with increasing rotational speed. It is only in the range of 0.01 psi for gas-liquid cases, or 0.002 psi for oil-brine cases, for rotation speeds higher than 500 RPM.

The obvious conclusion is that gravity may have an significant effect only for capillary pressure measurements below a few psi.

How the gravity correction compares with experimental error ?

If low levels of capillary pressure may occur, the gravity correction may be significant. How relevant is that correction, given the usual inaccuracy of 10 RPM in the rotational speed measurement?

Figures 7 and 8 show the equivalent error in rotational speed when the gravity correction is neglected, i.e., the variation in rotation speed producing an increase from P/bo to P/b.

For drainage (Figure 7), the pressure gravity correction is lower than an uncertainty of 10 RPM for rotational speeds higher than 200 RPM (for long radius geometry, r_3=21.6 cm.) to 300 RPM (for short radius geometry, r_3=8.6 cm.), for 1" long cores. Variation of these values is low when different core diameters or fluids are considered and low still for longer samples, to 200 RPM and 150 RPM respectively, for 3" long core for instance. The single rule to keep in mind, is that in drainage the gravity correction is certainly *below the uncertainty of experimental measurements* if the rotation speed is higher than 150/300 RPM, (i.e., if the capillary pressure is higher than 0.07 psi, liquid-liquid cases, or 0.35 psi, gas-liquid cases).

Considering imbibition (Figure 8), the gravity correction may be greater than the uncertainty of the measurement for rotational speeds as high as 1200 RPM for small radius geometry (1"x2" core, r_3=8.6 cm.). Such a geometry is however not likely to be used, because in imbibition experiments, space is needed, between the sample and the rotation axis to fit the production device in. For the usual geometry and a 1"x1" sample, the uncertainty exceeds the gravity correction below 400 RPM if the sample is settled at 10 cm from the rotational axis, 800 RPM for 1"x2" core, (Figure 8). The conclusion is that, for imbibition, *the gravity correction is lower than experimental error above 800 RPM in current centrifuges. It may be larger than experimental error for a large sample (2" in diameter) rotated close to the axis (r_3 about 12cm.), below 800 RPM* (i.e., for capillary pressures as high as 0.6 psi, liquid-liquid cases, or 3.5 psi, gas-liquid cases). Therefore gravity effects may be distinguishable from experimental error over a larger range of capillary pressure for imbibition than for drainage (Figures 7, 8). However its effect on pressure evaluation will never exceed 0.08/0.016 psi, as discussed previously (Figures 5, 6).

Interest of the total (including gravity) correction

The previous analysis shows that most centrifuge measurements are performed under conditions for which the gravity correction is either not needed, or needed but lower than measurement inaccuracy. Why then should the gravity correction be included when processing centrifuge data ?

Being able to correct for gravity when centrifuging, opens up the possibility of processing samples with low capillary pressure curve and therefore to extend the application of the centrifuge technique to unconsolidated and high permeability media. One should stress that the capability to process the data is necessary, but not sufficient. The main additional technical advance required is a centrifuge able to run at low speed (<300 RPM) with higher accuracy (error <5 RPM). However having developed the procedure to interpret the measurement is a significant first step.

There are in addition some reasons to use the total correction, accounting for gravity when interpreting current centrifuge measurements. Firstly, it may be necessary for imbibition, in a range of capillary pressure value (3 psi) which is likely to occur. Secondly, gravity introduces a systematic bias, low but always in the same direction. In most procedures currently in use for interpreting centrifuge measurements, the inversion scheme requires evaluation of the derivative $\langle S\rangle'_{B,0,0}(P)$, or an implicit fitting procedure. Subtle change in the pressure correction may actually be amplified by the interpretation procedure. Most inversion techniques do not check their consistency with centrifuge data. As a result, corrections in pressure data, even low ones, may improve that consistency. A good example can be found in Chen and Ruth, 1994, where the effect of gravity may appear much higher than the actual effect (that is one order of magnitude above the maximum range of the maximum potential gravity effect on pressure shift, $2\Delta\rho gR$, see their Figure 4). The reason is that the "parameter estimation method" in use is too sensitive to the simplistic parametric function pre-assumed for the capillary pressure curve. The use of a more accurate interpretation method would have shown much less influence of gravity.

A practical reason to use the total correction, is it is as easy to use as the radial correction, which is actually needed (Forbes et al., 1994 ; Fleury and Forbes, 1995). Since it consists of a pre-process correction, applied before any of the usual schemes for interpretation, there is a potential improvement and no additional cost to use it.

Thus, use of the proposed correction is recommended. It will mainly account for radial effect and will correct for gravity degradation when needed. It will reduce error generated by the interpretation procedure.

Conclusions

Gravity damage to centrifuge capillary pressure determinations has been analyzed quantitatively.

Gravity may change the formulation of the centrifuge saturation equation, but the change is far below uncertainties of saturation measurements.

Gravity may have a more significant influence on the inversion of the saturation equation. It has been related to an increase of the pressure, by a value necessarily lower than the extent of capillary pressure due to gravity along the core height, $2\Delta\rho gR$, that is less than 0.01 to 0.1 psi. Gravity may therefore have an effect only for very low levels of capillary pressure. It is usually far below the current uncertainties of measurements. In practice, only pressure measurements, lower than 0.35 psi (drainage) or 3 psi (imbibition), obtained for rotational speeds lower than 150-300 RPM (drainage) or 400-800 RPM (imbibition) may be changed, depending on the centrifuge and sample geometry.

A total correction for radial and gravity effects, has been proposed. It allows accounting for gravity and thus measurements at low capillary pressure levels. Therefore the use of the centrifuge technique may be extended to unconsolidated and high permeability samples. It is stressed however that high accuracy centrifuges will be needed to do so.

Even if the gravity effect is usually low in current centrifuge measurements, it is finally recommended to process the correction, accounting for gravity, because it is no more costly than applying the radial correction, it compensates for low but systematic bias, and improves approximate interpretation procedures. It is the most complete way to interpret centrifuge capillary pressure measurement.

NOMENCLATURE

Latin

r : Radial distance from the centrifuged axis to a point in the centrifuged core
r_1 : r at the inner core face
r_3 : r at the outer core face
Pc, P : Capillary pressure
P_1 : Pc evaluated at r_1
P_3 : Pc evaluated at r_3
S : Wetting phase saturation
$<S>$: Average wetting phase saturation
S_{HB} : Hassler and Brunner saturation
B, N, M : Dimensionless factors (see text)
r, Z, θ : cylindrical coordinates linked to the centrifuge axis (see Figure 1)
x,y,z :Integration variables (see text)
a_o, b_o, b :Parameters (see text)
g : Gravitational constant
R : Radius of core cylinder plug
dv : elementary volume

Greek

ρ : Phase mass density
Δρ : Difference of the phase densities
ω : Centrifuge angular velocity
ε : Free parameter (see text)

Metric units

Pc : pascal, Pa.
r : meter, m.
ρ : kilogram per cubic meter, kg/m^3.
ω : radian per second, rad/s.

Conversion factors

Pc :

from/to	mbar	bar	Pa	MPa	psi
mbar	1	10^{-3}	10^{2}	10^{-4}	$1.45\ 10^{-2}$
bar	10^{3}	1	10^{5}	10^{-1}	14.5037
Pa	10^{-2}	10^{-5}	1	10^{-6}	$1.45\ 10^{-4}$
MPa	10^{4}	10	10^{6}	1	$1.45\ 10^{2}$
psi	68.94	$6.894\ 10^{-2}$	$6.894\ 10^{3}$	$6.894\ 10^{-3}$	1

ω :

from/to	rad/s	RPM
rad/s	1	9.549
RPM	1.04710^{-1}	1

r :

from/to	m	cm	inch
m	1	10^{2}	39.37
cm	10^{2}	1	$3.937\ 10^{-1}$
inch	$2.54\ 10^{-2}$	2.54	1

ρ :

from/to	kg/m^3=g/l	pound per cubic inch
kg/m^3=g/l	1	$3.6127\ 10^{-5}$
pound per cubic inch	$2.7680\ 10^{4}$	1

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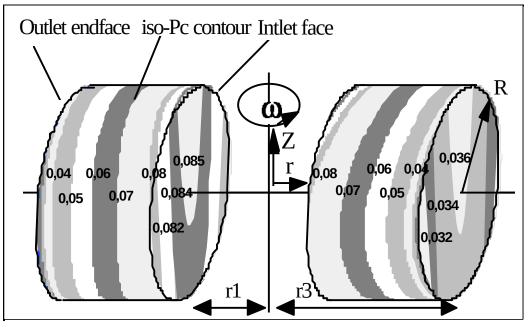

Figure 1 : The Centrifuge geometry. Core samples are rotated around a vertical axis. The actual capillary pre field distribution is drawn for a 2"x2" core with ar air-brine system rotating at 100 RPM at r3=8.6cm. Iso-Pc are indicated in psi.

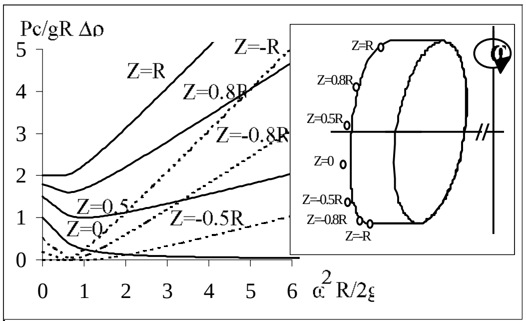

Figure 2 : Evolution of Pc on the border of the outlet face during a centrifuge drainage. By effect of gravity, Pc ma be decreasing when the rotation speed is increased. The gray area displays the core zone where such reverse variation of Pc may occur. That zone is usually much less than 1% of the core volume.

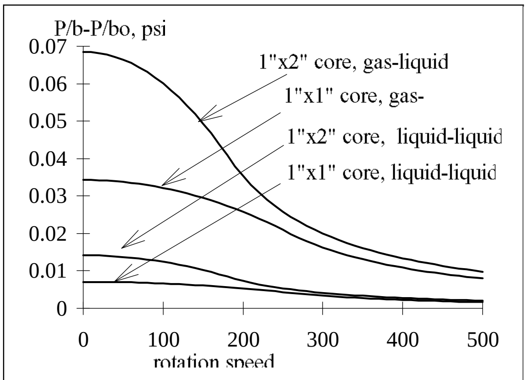

Figure 5 : Extension of gravity correction to pressure, drainage measurements. The maximun value is in the range of the gravity capillary pressure along the heigh of the core : 2 Δρ gR

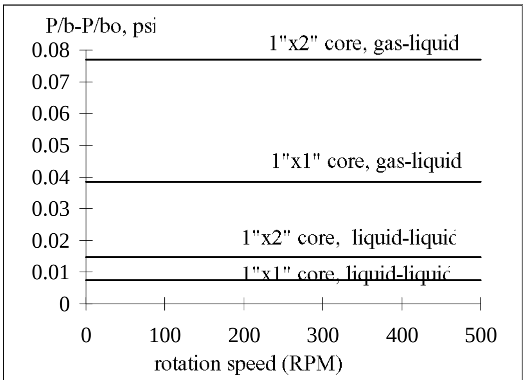

Figure 6 : Extension of gravity correction to pressure, for imbibition measurements. The maximun value is in the range of the gravity capillary pressure along the height of the core : 2 Δρ gR

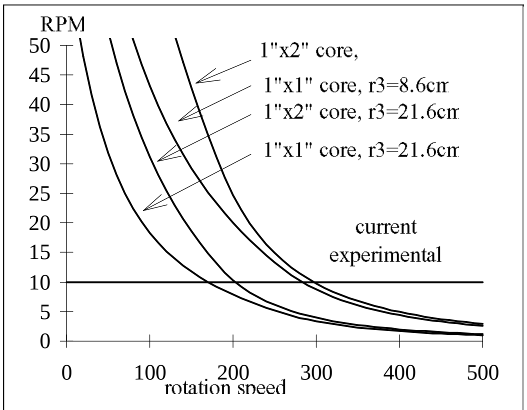

Figure 7 : Equivalent error in rotation speed, when neglecting gravity correction in drainage centrifuge measurements.

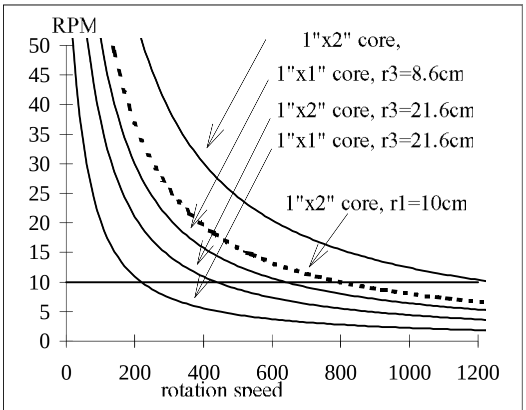

Figure 8 : Equivalent error in rotation speed, when neglecting gravity correction in imbibition centrifuge measurements.

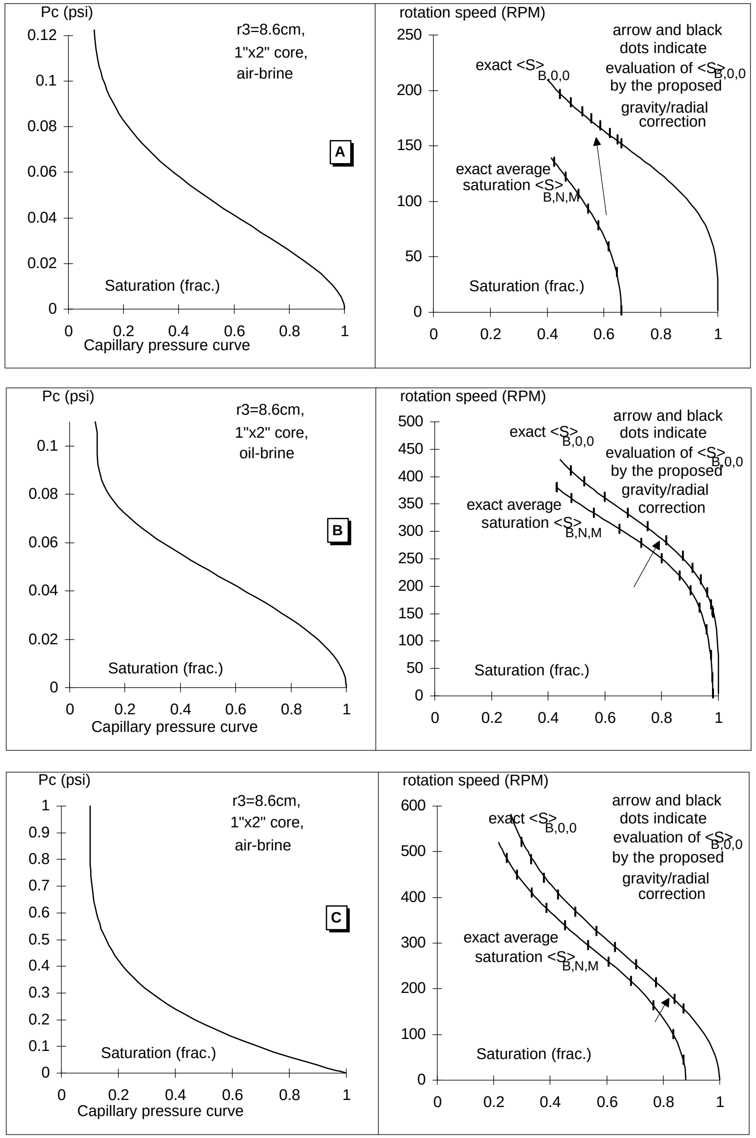

Figure 3 : Evaluation of the proposed gravity/radial correction in drainage.

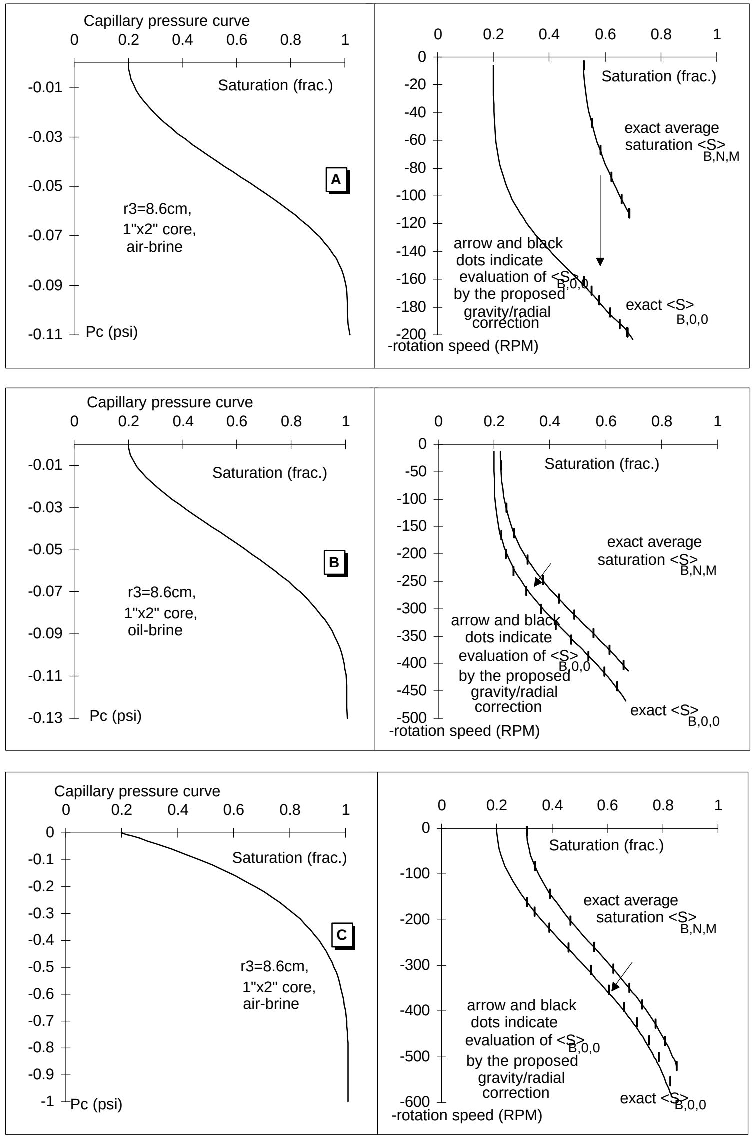

Figure 4 : Evaluation of the proposed gravity/radial correction in imbibition.

The black dots show the corrected values from values sampled on the exact observed average saturation (white dots) for different rotation speeds.

The black dots have to be compared with the line <S>B,0,0 which is the exact value to be reached by the correction.

Even for very unfavorable cases (A, B) which are very unlikely to happen in core analysis, the correction appears to be appropriate.

Appendix

The following shows an appropriate procedure for correcting for radial and gravity effect on centrifuge capillary pressure measurements.

One considers the complete saturation equation, that is including all effects (centrifugal, radial, gravity) and valid for drainage and imbibition experiments (see text). One then applies the procedure described by Forbes (1994) for quantifying and correcting for radial effects :

$$<S>_{B,N,M} = \frac{1+\sqrt{1-B}}{2}\int_{x=0}^{x=1}\frac{dx}{\sqrt{1-Bx}}\int_{y=0}^{y=1}\frac{2}{\Pi}\sqrt{\frac{y}{1-y}}\,dy\int_{z=-1}^{z=1}\frac{dz}{2}\,S_{[P_1(x+Ny+2NMz\sqrt{y}+Nn)]} \qquad (1)$$

The pressure, P_1 (x + N y + 2NM z $y^{1/2}$ + Nn),

is written P_e (1+e) (x + N y + 2NM z $y^{1/2}$ + Nn) /(1+mN)

where $P_e = P_1$ (1+mN)/(1+e), e is a free parameter

1+mN is the maximum absolute value of (x + N y + 2NM z $y^{1/2}$ + Nn),

that is for drainage, m= 1 + 2 M + n

for imbibition, m= 1 - 2 M + n, if M >1, or m= 1 - 2 M^2 + n, if M <1.

Now we consider the part (1+e)(x + N y + 2NM z $y^{1/2}$ + Nn)/(1+mN)

that is, (1+e)**(**x + 1/(1+mN) (-mNx+ N y + 2NM z $y^{1/2}$ + Nn)**)**

or, at the first order in N/(1+mN) : x + ex +N/(1+mN) **(**-mx+ y + 2 M z $y^{1/2}$ + n **)** + ...

Note that we develop here in N/(1+mN) and not in N, to ensure that the third term above is low enough, that is lower than N. This is the case by definition of m and because N is significantly lower than 1. One will also choose e later on, within the same order of magnitude than N.

Then one can expand the saturation term of the integrand above through the first order of the regular Taylor series, we obtain :

S[P_1 (x+Ny+2NMz$y^{1/2}$+Nn)] ≈ S[P_e x] + P_e S'[P_e x] **(** ex + N/(1+mN)(-mx+ y + 2 M z $y^{1/2}$ + n) **)**....

We note S'[P_e x] to denote the derivative dS/dP for P=P_e x. Integrating, one obtains :

$$\begin{aligned} <S>_{B,N,M[P_1]} \approx\ & \frac{1+\sqrt{1-B}}{2}\int_{x=0}^{x=1}\frac{dx}{\sqrt{1-Bx}}\int_{y=0}^{y=1}\frac{2}{\Pi}\sqrt{\frac{y}{1-y}}\,dy\int_{z=-1}^{z=1}\frac{dz}{2}\,S_{[P_\varepsilon x]} \;+ \\ & \frac{1+\sqrt{1-B}}{2}\int_{x=0}^{x=1}\frac{dx}{\sqrt{1-Bx}}\int_{y=0}^{y=1}\frac{2}{\Pi}\sqrt{\frac{y}{1-y}}\,dy\int_{z=-1}^{z=1}\frac{dz}{2}\left(\varepsilon x+\frac{N(-mx+y+2Mz\sqrt{y}+n)}{1+mN}\right)P_\varepsilon S'_{[P_\varepsilon x]} \end{aligned} \qquad (2)$$

Now, one can simplify that expression, reminding that :

$$<S>_{B,0,0[P_\varepsilon]} = \frac{1+\sqrt{1-B}}{2}\int_{x=0}^{x=1}\frac{dx}{\sqrt{1-Bx}}\,S_{[P_\varepsilon x]} \;;\; <S>'_{B,0,0[P_\varepsilon]} = \frac{1+\sqrt{1-B}}{2}\int_{x=0}^{x=1}\frac{x\,dx}{\sqrt{1-Bx}}\,S'_{[P_\varepsilon x]} \qquad (3)$$

and (see Forbes et al. 1994)

$$\begin{aligned} & \frac{1+\sqrt{1-B}}{2}\int_{x=0}^{x=1}\frac{dx}{\sqrt{1-Bx}}\,P_\varepsilon S'_{[P_\varepsilon x]} \\ & \approx \frac{1+\sqrt{1-B}}{2}\left(<S>_{B,0,0[P_\varepsilon]} - S_{[0]}\right) + \frac{1+\sqrt{1-B}}{2}\,\frac{4-\sqrt{1-B}}{2+\sqrt{1-B}}\,P_\varepsilon <S>'_{B,0,0[P_\varepsilon]} \end{aligned} \qquad (4)$$

and

$$\int_{y=0}^{y=1} \frac{2}{\Pi} \sqrt{\frac{y}{1-y}} dy = 1 \;\; ; \int_{y=0}^{y=1} \frac{2}{\Pi} y \sqrt{\frac{y}{1-y}} dy = 3/4 \qquad (5)$$

and

$$\int_{z=-1}^{z=1} \frac{dz}{2} = 1 \;\; ; \int_{z=-1}^{z=1} \frac{zdz}{2} = 0 \qquad (6)$$

Simplifying from these expressions, one obtains :

$<S>_{B,N,M}[P_1] \approx <S>_{B,0,0}[P_e] + a_M(<S>_{B,0,0}[P_e] - S[0])$
$\quad + P_e <S>'_{B,0,0}[P_e] (e - b_M (1+mN) + 1)$ (7)

where :

$$a_M = \frac{1+\sqrt{1-B}}{2} \frac{3/4N+nN}{1+mN} \;\; ; b_M = \frac{1+\frac{mN}{1+mN} - a_M \frac{4-\sqrt{1-B}}{2+\sqrt{1-B}}}{1+mN} \qquad (8)$$

We denote a_o and b_o the value of a_M and b_M for M=0, that is when gravity is neglected. a_o and b_o are the two parameters presented by Forbes et al. (1994) to allow radial correction on the form :

$<S>_{B,0,0}[P_1/b_o] \approx <S>_{B,N,M}[P_1] + a_o (<S>_{B,N,M}[a_o P_1] - <S>_{B,N,M}[P_1])$

In order to obtain a comparable expression from equation (7), we introduce a_o as follows :

$<S>_{B,N,M}[P_1] \approx <S>_{B,0,0}[P_e] + a_o(<S>_{B,0,0}[P_e] - <S>_{B,N,M}[0]) + P_e<S>'_{B,0,0}[P_e] (e-b_M(1+mN)+1)$
$\quad + a_M (<S>_{B,0,0}[P_e] - S[0])$
$\quad + a_o (<S>_{B,N,M}[0] - <S>_{B,0,0}[P_e])$ (9)

$<S>_{B,N,M}[0]$ being the value of $<S>_{B,N,M}[P_1]$ for w=0, that is the average saturation after spontaneous production by gravity (drainage or imbibition), before starting the centrifuging.

The objective is now to select e in order to make the sum of the 3 last terms negligible.

evaluation of $<S>_{B,0,0}[P_e] - S[0]$:

$<S>_{B,0,0}[P_e] - S[0]$ is evaluated at first order as $P_e <S>'_{B,0,0}[P_e]$ (note that $<S>_{B,0,0}[0] = S[0]$)

evaluation of $<S>_{B,N,M}[0] - <S>_{B,0,0}[P_e]$:

$<S>_{B,N,M}[0]$ can be evaluated from equation (7). As we have let e be a free parameter, we can choose $e=eo=b_M(1+mN) - 1$. Therefore , $P_{eo} = P_1 (1+mN)/(1+eo)=P_1/b_M$ and

$<S>_{B,N,M}[P_1] \approx <S>_{B,0,0}[P_1/b_M] + a_M (<S>_{B,0,0}[P_1/b_M] - S[0])$

Making w=0 leads to

$P_{1[w=0]} = 0$, $a_{M[w=0]} = (1+(1-B)^{1/2})/4$, and $(P_1/b_M)_{[w=0]} = 2DrgR / [2-(1+(1-B)^{1/2})(4-(1-B)^{1/2})/4/(2+(1-B)^{1/2})]$,

(change g by -g for imbibition)

providing : $<S>_{B,N,M}[0] \approx <S>_{B,0,0}[(P_1/b_M)_{[w=0]}] + a_{M[w=0]} (<S>_{B,0,0}[(P_1/b_M)_{[w=0]}] - S[0])$

Now we re-consider P_e as defined before for any value of e. Subtracting $\langle S\rangle_{B,0,0}[P_e]$ on both sides leads to

$$\langle S\rangle_{B,N,M}[0] - \langle S\rangle_{B,0,0}[P_e] \approx (1+a_{M\,[w=0]})\;(\langle S\rangle_{B,0,0}[(P_1/b_M)_{[w=0]}] - \langle S\rangle_{B,0,0}[P_e]) + a_{M\,[w=0]}\;(\langle S\rangle_{B,0,0}[P_e] - S[0])$$

Developing each term of the left part at first order provides:

$$\langle S\rangle_{B,N,M}[0] - \langle S\rangle_{B,0,0}[P_e] \approx (1+a_{M\,[w=0]})\;((P_1/b_M)_{[w=0]} - P_e)\,\langle S\rangle'_{B,0,0}[P_e] + a_{M\,[w=0]}\;P_e\;\langle S\rangle'_{B,0,0}[P_e]$$

That is $\langle S\rangle_{B,N,M}[0] - \langle S\rangle_{B,0,0}[P_e] \approx [\,(1+a_{M\,[w=0]})\,P_1/b_{M\,[w=0]}\,/\,P_e\;\; -1\,]\;P_e\;\;\langle S\rangle'_{B,0,0}[P_e]$

Reporting these last 2 evaluations in equation **(9)** provides :

$$\langle S\rangle_{B,N,M}[P_1] \approx \langle S\rangle_{B,0,0}[P_e] + a_o\,(\langle S\rangle_{B,0,0}[P_e] - \langle S\rangle_{B,N,M}[0]) + P_e\,\langle S\rangle'_{B,0,0}[P_e]\,(\,e - b_M\,(1+mN) + 1 + (a_M - a_o) + a_o\,((1+a_M)\,P_1/b_M)_{[w=0]}\;/\,P_e\,)$$

As, $\quad P_e = P_1\,(1+mN)/(1+e)$

$$\langle S\rangle_{B,N,M}[P_1] \approx$$
$$\langle S\rangle_{B,0,0}[P_e] + a_o\,(\langle S\rangle_{B,0,0}[P_e] - \langle S\rangle_{B,N,M}[0])$$
$$+ P_e\,\langle S\rangle'_{B,0,0}[P_e]\,(\,(e+1)(1+a_o((1+a_M)\,P_1/b_M)_{[w=0]}/P_1/(1+mN)) - (b_M\,(1+mN) - a_M + a_o)\,)$$

As e is still a free parameter, we choose

$$e+1 = [\,b_M\,(1+mN) - (a_M - a_o)\,]\,/\,[\,1 + a_o\,((1+a_M)\,P_1/b_M)_{[w=0]}\;/P_1/(1+mN)\,]$$

Therefore , $\quad \langle S\rangle_{B,N,M}[P_1] \approx \langle S\rangle_{B,0,0}[P_e] + a_o\,(\langle S\rangle_{B,0,0}[P_e] - \langle S\rangle_{B,N,M}[0])$

for $\quad P_e = P_1/b$ and $b = [\,b_M - (a_M - a_o)/(1+mN)\,]\,/\,[\,1 + a_o\,((1+a_M)\,P_1/b_M)_{[w=0]}\;/P_1/(1+mN)\,]$

b depends on B, N, and M, or w, according to b_M, a_M, a_o, m, $((1+a_M)P_1/b_M)_{[w=0]}$ and P_1 as defined above. Full analytical expression of b, as a function of M, or w, is not simple to be written. However, plotting 1/b versus M provides curves close to straight lines, for any B or N values, fitted as

$$\begin{aligned}
&\textit{for drainage} \quad M>1 && \frac{1}{b}=\frac{1}{bo}+\frac{4+2\sqrt{1-B}}{5+\sqrt{1-B}}N(4M-1.75)\\
&0<M<1 && \frac{1}{b}=\frac{1}{bo}+\frac{4+2\sqrt{1-B}}{5+\sqrt{1-B}}2.25NM^{1.7}\\
&\textit{for imbibition} \quad M<0 && \frac{1}{b}=\frac{1}{bo}+\frac{4+2\sqrt{1-B}}{5+\sqrt{1-B}}4NM
\end{aligned} \tag{10}$$

One may also remind that, for drainage, bo can be simplified as : (1+0.23N/(1+N))/(1+N), Forbes et al., 1994.

These simpler expressions are more convenient to be manipulated, and will be kept in the following.

We consider now $\langle S\rangle_{B,N,M}[P_1] \approx \langle S\rangle_{B,0,0}[P_1/b] + a_o(\langle S\rangle_{B,0,0}[P_1/b] - \langle S\rangle_{B,N,M}[0])$, written as $\langle S\rangle_{B,0,0}[P_1/b] \approx \langle S\rangle_{B,N,M}[P_1] + a_o/(1+a_o)\;\;(\langle S\rangle_{B,N,M}[0] - \langle S\rangle_{B,N,M}[P_1])$

Following the same way, used by Forbes et al. (1994) for evaluating radial effect, $(\langle S\rangle_{B,N,M}[0] - \langle S\rangle_{B,N,M}[P_1])/(1+a_o)$ is replaced by $(\langle S\rangle_{B,N,M}[a_o\,P_1] - \langle S\rangle_{B,N,M}[P_1])$, assuming that it is a first order approximation for low value of a_o.
One therefore obtains : $\langle S\rangle_{B,0,0}[P_1/b] \approx \langle S\rangle_{B,N,M}[P_1] + a_o\,(\langle S\rangle_{B,N,M}[a_o\,P_1] - \langle S\rangle_{B,N,M}[P_1])$

This expression allows to calculate $\langle S\rangle_{B,0,0}$ from the measurement $\langle S\rangle_{B,N,M}$. It includes both the radial and gravity correction. Note that if gravity is neglected, that is if $M\approx 0$, one obtains $b=b_o$. The correction reduces consistently to the radial correction :

$$\langle S\rangle_{B,0,0}[P_1/b_o] \approx \langle S\rangle_{B,N,M}[P_1] + a_o\,(\,\langle S\rangle_{B,N,M}[\,a_o\,P_1\,] - \langle S\rangle_{B,N,M}[P_1]\,)$$

Also note that, while a_o and b_o are constants (depending only on the centrifuge and sample geometry), b, varying with M, is pressure dependant, functions of w or P_1.

To sum up, the proposed total (radial/gravity) consists in changing the current determination,
{P ; <S>[P]} by {P/b ; <S>[P] + a_o(<S>[a_oP] - <S>[P])} with :

	P	B	N	M	ao	bo	C	1/b - 1/bo
Drainage	$1/2Drw^2(r_3^2-r_1^2)$	$\frac{(r_3^2-r_1^2)}{r_3^2}$	$\frac{R^2}{(r_3^2-r_1^2)}$	$\frac{g}{w^2R}$	$\frac{3/4N\ (1+(1-B)^{1/2})}{2(1+N)}$	$\frac{1+0.23N/(1+N)}{(1+N)}$	$N\frac{(4+2(1-B)^{1/2})}{(5+(1-B)^{1/2})}$	M>1 : (4M-1.75)C 0<M<1 : $2.25M^{1.7}$ C
Imbibition	$1/2Drw^2(r_1^2-r_3^2)$	$\frac{(r_1^2-r_3^2)}{r_1^2}$	$\frac{R^2}{(r_1^2-r_3^2)}$	$\frac{-g}{w^2R}$	$\frac{-1/4N(1+(1-B)^{1/2})}{2}$	$1-a_o\frac{(4-(1-B)^{1/2})}{(2+(1-B)^{1/2})}$	$N\frac{(4+2(1-B)^{1/2})}{(5+(1-B)^{1/2})}$	M<0 : 4M C