COMBINED COMPLEX CONDUCTIVITY AND DIELECTRIC MEASUREMENTS ON CORE SAMPLES

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Abstract

The frequency dependence of electrical rock properties in the frequency range from 10^{-3} to 10^{9} Hertz has been analyzed with a Combined Conductivity and Permittivity Model (CCPM). The parallel connection of a low frequency and a high frequency constant phase angle element results in a simple transfer function for the whole frequency range. It could be shown that this function describes well the general dispersion behavior in the low as well as in the high frequency range. The combination of a conductivity measurement with a dielectric measurement provides four independent parameters that completely describes the complex electrical behavior over 12 orders of frequency. The low frequency parameters are essentially related to water content, water conductivity, surface area-to-porosity-ratio by simple, well known equations. The high frequency parameters are essentially related to water content and internal surface area. They are nearly independent on water conductivity. A measurement technique is proposed that allows a broad band complex measurement with an impedance and a network analyzer on the same sample without changing the sample shape.

Introduction

Electric and dielectric properties of rocks vary systematically with changes in water content. Any property selected for water content determination does not depend uniquely on water content. Thus a calibration procedure for each new rock type is necessary. The combination of different water content dependent properties reduces this disadvantage. The objective of the investigation is to improve the determination of water saturation on porous rock samples by applying simultaneously electric and dielectric measurements.

A water-wet silicate rock with an interconnected pore space is a lossy, dispersive dielectric medium whose electrical properties depend on water content, water composition and distribution of water phase. The quantities which describe the effective electrical or dielectric properties are the electrical conductivity and dielectric permittivity. Both quantities are complex and frequency dependent. The combination of an electric measurement in the low frequency range with a dielectric measurement in the high frequency range provides a set of four effective parameters that describes completely the broad band complex electrical transfer function. A conceptual model for the complex conductivity and dielectric permittivity of rocks is proposed that is applicable over a very broad frequency range. The rock structure and the thermodynamical state of the rock are characterized by measurable parameters used commonly in petrophysics and reservoir characterization. The so called Combined Conductivity and Permittivity Model (CCPM) is tested according to data obtained in an earlier investigation and to data found in literature.

Broad band transfer function

The electrical transfer function of a rock sample can be characterized by the dependence of the complex conductivity and the complex dielectric permittivity on frequency as well as on thermodynamic state parameters temperature, pressure and water content (DISSADO&HILL 1984). Based on Maxwell´s equations of electrodynamics, an effective current density J* is defined as the response of a time varying electrical field E. J* consists of two components: the conduction current $J_C=\sigma E$ and a the displacement current $J_D=\varepsilon \delta E/\delta t$:

$$J* = \sigma E + \varepsilon \P E / \P t \tag{1}$$

For a sinusoidal time dependence of the electrical field equation (1) reduces to

$$J*(\omega) = (\sigma + i\omega\varepsilon)E(\omega) \tag{2}$$

where σ is the specific conductivity and ε the dielectric permittivity. The term in brackets is the transfer function of the rock. A schematic representation is shown in fig.1. The transfer function may be formulated in terms of an effective conductivity σ* or effective dielectric permittivity ε* (OLHOEFT 1985):

$$\sigma + i\omega\varepsilon = \sigma* = i\omega\varepsilon* \tag{3}$$

The effective complex conductivity with a frequency dependent real and imaginary part is

$$\sigma*(\omega) = \sigma'(\omega) + i\sigma''(\omega) \tag{4}$$
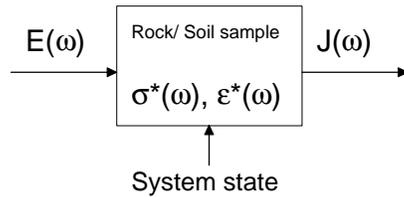

Fig. 1: Analysis of electrical transfer function.

In general, the frequency dependent response of a water containing rock between about some Millihertz and some Gigahertz may be characterized by the overlap of two ranges characterized by different electrical processes. The composite process forms a loss peak at an overlap frequency ω_C (JONSCHER, 1979, DISSADO & HILL, 1984).

In the low as well as in the high frequency range the frequency dependence is determined by electrical interfacial phenomena. Both processes, that of the low frequency side of ω_C and that of the high frequency side of ω_C, show an identical behavior with respect to amplitude and phase angle, but with different magnitudes. This main feature is a constant phase angle and an identical power law dependence of the real and imaginary component of conductivity (see figure 2):

$$\sigma''(\omega) / \sigma'(\omega) = \tan[(1-p)\pi / 2], \quad \omega < \omega_C \tag{5a}$$

$$\sigma''(\omega) / \sigma'(\omega) = \tan[(n)\pi / 2], \quad \omega > \omega_C \tag{5b}$$

In the low frequency range the electrical properties are controlled by ohmic conduction processes. The complex conductivity exhibits a constant phase angle behavior. The real and the imaginary component have the same power law frequency dependence, determined only by the amplitude factor σ_O and the exponent $1-p$. A dc-component is ignored:

$$\sigma^*(\omega) = \sigma_o\left(i\omega / \omega_o\right)^{1-p}, 10^{-3} < \omega < 10^{3}\,Hz \tag{6}$$

In the high frequency range dielectric displacement processes control the frequency response. The Cole-Cole-model is often used to describe the transition from low frequency range to the high frequency constant phase angle behavior. The limiting behavior in the high frequency range is a power law frequency dependence, characterized by another exponent n. The Cole-Cole-model in terms of conductivity (KULENKAMPFF & SCHOPPER, 1988) is

$$\sigma^*(\omega) = \sigma_{DC}\left[1 + \left(i\omega / \omega_o\right)^{n}\right], 10^{2} < \omega < 10^{9}\,Hz \tag{7}$$

The conductivity amplitude in the high frequency range depends on the characteristic frequency ω_O and on σ_{DC}. A lot of papers show that both, the equation (6) and (7) are well applicable to experimental data in the lab scale (VINEGAR&WAXMAN, 1984, BÖRNER, 1992, KULENKAMPFF & SCHOPPER, 1988, TAHERIAN et al. 1990). But most of the experimental analysis are focused on one limited frequency band. Therefore general characteristics of the frequency dependence are not recognizable and the parameters used to validate the models are limited to two or three orders of frequency.
In order to analyze a broad frequency range that includes more than 12 orders of frequency, σ_{DC} in equation (7) was substituted by the frequency dependent expression (6) to obtain a simple broad band transfer function (KULENKAMPFF et al. 1993):

$$\sigma^*(\omega) = \sigma_o\left[\left(i\omega / \omega_o\right)^{1-p} + \left(i\omega / \omega_o\right)^{n}\right] \tag{8}$$

The function has a similar shape like the model of anomalous low frequency dispersion, developed by DISSADO & HILL (1984). Parameters of the model are σ_O and $1-p$ from equation (6), n and ω_O from equation (7). ω_O is a material specific equivalent frequency. But its significance for rock structure is unclear. If no dispersion is observed ($1-p=0$, $n=1$) it can be interpreted as an inverse time constant. In equation (8) the material parameter ω_O may be substituted by the material-independent normalizing frequency ω_N. Furthermore two separate amplitude factors are introduced: For the low frequency expression (equation 6) $\sigma_{N,lf} = \sigma_o\left(\omega_N / \omega_o\right)^{1-p}$, and for the high frequency expression (equation 7) $\sigma_{N,hf} = \sigma_o\left(\omega_N / \omega_o\right)^{n}$. The modified transfer function for the effective complex conductivity is then:

$$\sigma^*(\omega) = \sigma_{N,lf}\left(i\omega / \omega_N\right)^{1-p} + \sigma_{N,hf}\left(i\omega / \omega_N\right)^{n} \tag{9}$$

According to equation (3) the effective complex conductivity σ^* has the same meaning like the effective complex permittivity ε^*.
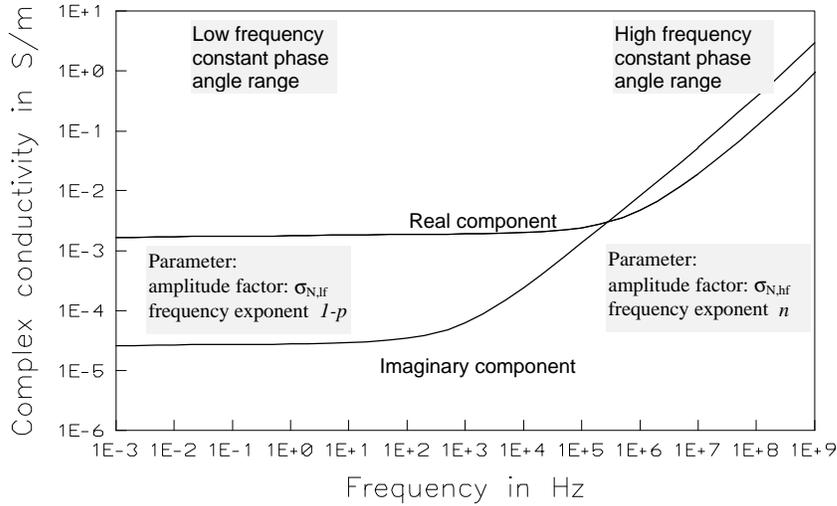
Fig. 2: General frequency dependence of complex conductivity of water containing silicate rocks.

In the so-called Combined Conductivity and Permittivity Model (CCPM) the low frequency part is characterized by the amplitude $\sigma_{N,lf}$ and the frequency exponent *1-p*. The high frequency part is characterized by an other amplitude factor $\sigma_{N,hf}$ and exponent *n* (see figure 2). Table 1 gives a summary of parameters and derived expressions of equation (9).

Table 1: Summary of parameters and derived terms of equation (9).

	Low frequency range 10^{-1}-10^{+1} Hz	high frequency range 10^{8}-10^{9} Hz
amplitude factor	$\sigma_{N,lf}$ in S/m	$\sigma_{N,hf}$ in S/m
frequency exponent	*1-p*	*n*
constant phase angle	$\varphi = \tan[(1-p)\pi/2]$ in rad	$\varphi = \tan[n\pi/2]$ in rad
real component	$\sigma'(\omega) = \sigma'(\omega=\omega_N)\omega^{(1-p)}$; in S/m $\sigma'(\omega=\omega_N) = \sigma_{N,lf}\cos[(1-p)\pi/2]$	$\sigma'(\omega) = \sigma'(\omega=\omega_N)\omega^{n}$; in S/m $\sigma'(\omega=\omega_N) = \sigma_{N,hf}\cos[n\pi/2]$
imaginary component	$\sigma''(\omega) = \sigma''(\omega=\omega_N)\omega^{(1-p)}$; in S/m $\sigma''(\omega=\omega_N) = \sigma_{N,lf}\sin[(1-p)\pi/2]$	$\sigma''(\omega) = \sigma''(\omega=\omega_N)\omega^{n}$; in S/m $\sigma''(\omega=\omega_N) = \sigma_{N,hf}\sin[n\pi/2]$

The existence of a true dc-conduction component in the low frequency range is ignored because it is impossible to separate it from the measured frequency spectrum. A high frequency limiting value of permittivity ε_∞ seems to exist in the Gigahertz range. ε_∞ is often obtained from the highest measured frequency or by calculation from the permittivities of rock-components using mixing laws. The CCPM excludes these limiting cases (onset of dc-conduction and existents of a frequency independent permittivity). Therefore the applicability of the CCPM is focused on water containing rocks with significant water-solid interaction and The CCPM is restricted on the frequency range of 10^{-3} to about 10^{9} Hz.

Test and experimental data

Complex conductivity or permittivity data measured over a broad frequency range were published by POLEY et al. (1978), OLHOEFT (1985), LOCKNER & BYERLEE (1984), DISSADO & HILL (1984), TAHERIAN et al. (1990), KULENKAMPFF et al. (1993),
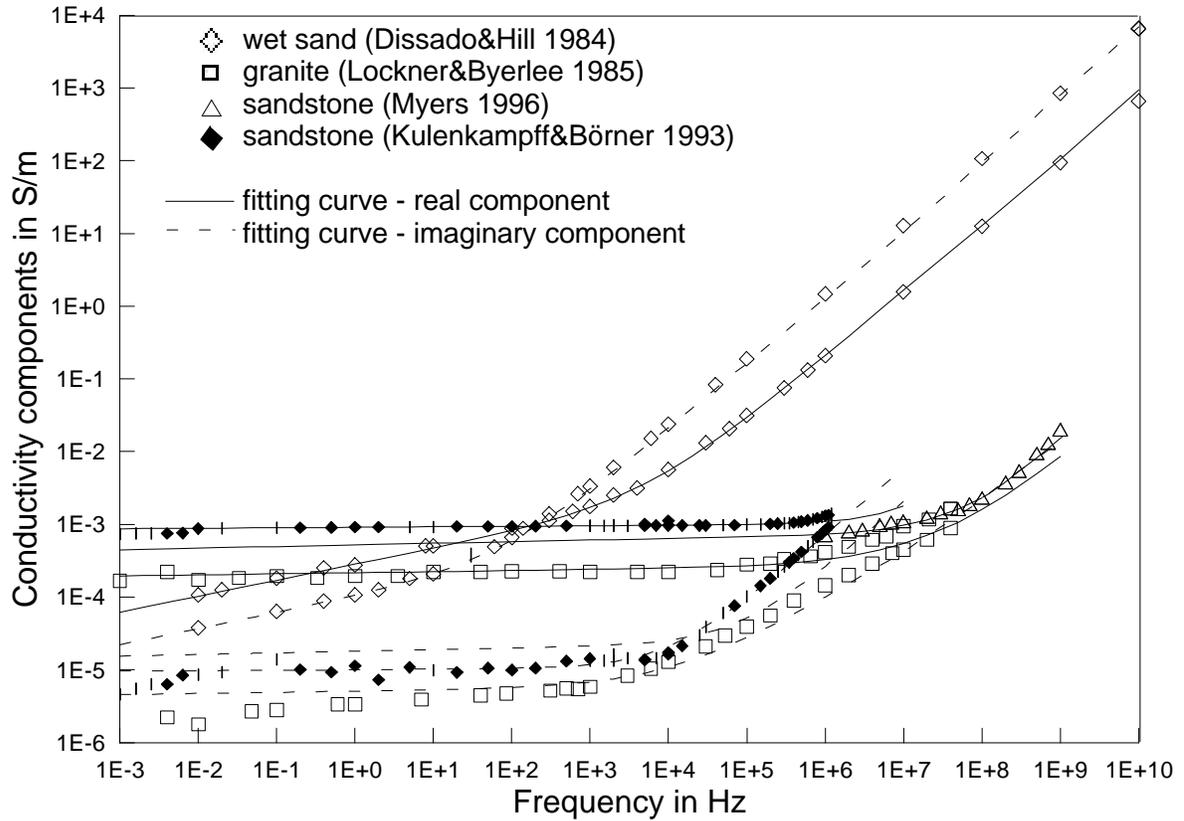
Fig. 3: Complex conductivity spectra of rocks with different structure and state.

BÖRNER (1995), MYERS (1991, 1996). A set of selected broad band complex measurements is given in fig. 3. The plot includes our own measurements on sandstones published previously in KULENKAMPFF et al. (1993) and data from other authors. The frequency range is different for each spectrum but it covers in most cases more than five orders of frequency. It is obvious that all samples show a general frequency dependence of conductivity according to (9). Furthermore, it can be seen that only a measurement over a broad frequency range gives the opportunity to observe the constant phase angle behavior in low as well as in the high frequency range. Fitting curves according to equation (9) are plotted together with the measured data. It can be seen that data could be well fitted within a considerable error range. The data confirmed the existence of a constant phase angle range in the low and in the high frequency section. On this basis the general complex transfer function according to (9) may be derived from two small band complex measurements: one in the low frequency constant phase angle range and another in the high frequency constant phase angle range. Measurements in the overlay range are not required.

Conceptual model of broad band complex conductivity

The four parameters of the transfer function (9) $\sigma_{N,lf}$, $\sigma_{N,hf}$, *1-p* and *n* are related to pore structure and thermodynamic state of the rock. The corresponding relationships are often modeled with an approach that uses the parallel conduction of volume and interface related

electrical components because the electrical conductive phases are continuously distributed in the rock. The conceptual model of electrical processes in the pore space is shown in fig. 4. In the first stage the figurative model reflects the electrical processes at the level immediately below the scale of investigation. The second stage demonstrates the averaged multiphase system (LUCKNER & SCHESTAKOW 1991). In stage three the figurative model of stage two is abstracted to the extent that electrical processes can be directly described with mathematical models. The parallel conduction model is one of the simplest models but the only one with a high degree of clarity to allow effective application.
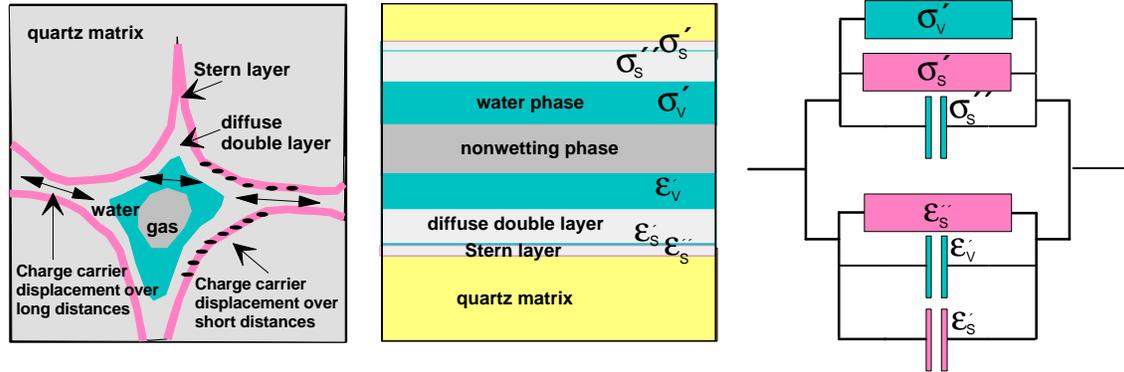

Fig. 4: Conceptual model of electrical processes in the pore space and the corresponding equivalent circuit.

The rock is divided into compartments: silicate solid phase, wetting water phase and non-wetting gas or hydrocarbon phase. The interaction between the water phase and the solid phase results in two further phases that can be identified with the Stern-layer and the diffuse part of the electrical double layer (fig. 4). The dominating electrical processes are located in the water phase and in the interface region. The evidence of existence of volume and interface related processes is given by the measured transfer function. The distribution of the electrical processes in the pore space may be obtained from the experimental investigation of functional relations between measurable electrical parameters of the transfer function and thermodynamic state parameters.

The equivalent circuit shown in fig. 4 is a combination of known models for the low frequency complex conductivity (or induced polarization, VINEGAR & WAXMAN, 1984) and models of the high frequency complex dielectric permittivity (e.g. MYERS, 1996).

The low frequency constant phase angle behavior ($10^{-3} < \omega < 10^{3}$) can be explained using two equations, that relate the real and the imaginary part of $\sigma_{N,lf}$ to water saturation S_W, water conductivity σ_W, Formation factor F, surface area-to-porosity ratio S_{PV} and a function $f(\sigma_W)$, that considers the variation of charge density in the interface:

$$\boldsymbol{s}'\!\left(\boldsymbol{w}_N\right) = \boldsymbol{s}_V + \boldsymbol{s}_S = \frac{S_W^n}{F}\left[\boldsymbol{s}_W + f\!\left(\boldsymbol{s}_W\right)S_{PV}\,S_W^{-1}\right] \tag{10a}$$

$$\boldsymbol{s}''\!\left(\boldsymbol{w}_N\right) = \boldsymbol{s}_P = \frac{l}{F}\left[f\!\left(\boldsymbol{s}_W\right)S_{PV}\,S_W^{n^*}\right] \tag{10b}$$

The previously presented model (BÖRNER 1992) includes basic elements of the models of VINEGAR & WAXMAN (1984) and RINK & SCHOPPER (1974). The real part consists of the electrolytic volume conductivity σ_V and the interface conductivity σ_S. The imaginary part considers the capacitive effect of the interface (polarization) σ_P. The electrolytic volume conductivity is (1) proportional to water saturation and (2) increases linearly with increasing water conductivity σ_W according to the Archie equation. σ_W is a function of the state parameters salinity c and temperature T. Both the interface conductivity σ_S and the polarization σ_P show (1) a linear dependence on the surface area-to-porosity ratio S_{PV}, (2) a proportionality to the water content, (3) a nonlinear weak increase with increasing water conductivity according to $f(\sigma_W)$. l is the nearly salinity independent ratio of σ_S and σ_P. The frequency dependence according to equation (5a) is then:

$$1 - p = \frac{2}{p} \arctan \frac{S_P}{S_V + S_S} \tag{11}$$

1-p was found to be between 0 and 0.03 for silicate sedimentary and crystalline rocks. Values up to 0.3 were found in the literature. The dispersion in the low frequency range is related to charge carrier transport effects over relatively long distances near the pore wall surface. DISSADO & HILL (1984) identified this process as an inter cluster transport of charge carriers near the surface. In this sense, clusters are larger structures in the pore network like constrictions in the pore channels (VINEGAR&WAXMAN, 1984). The process strongly depends on state parameters (S_w, c, T), which controls the distance of water-solid interaction.

The relationship between high frequency electrical parameters, pore space parameters and fluid properties was analyzed e.g. by POLEY et al. (1978), LANGE (1983), TAHERIAN et al. (1990), MYERS (1991, 1996) and others. Other studies are focused on the medium frequency range with the overlay of low and high frequency behavior (ENDRES & KNIGHT, 1992, KULENKAMPFF et. al. 1993, RUFFET et al. 1991, LOCKNER & BYERLEE 1985).

JONSCHER (1979) and DISSADO & HILL (1984) found that the high frequency constant phase angle behavior above the overlay frequency ω_C is a general feature of many wet materials. In equation (9) the high frequency amplitude factor $\sigma_{N,hf}$ may be explained using empirical equations for the real part and for the imaginary part. The frequency dependence of dielectric permittivity is found up at least to 1.1 GHz (SHERMAN, 1987). At frequencies above 1 GHz the real part of conductivity seems to be neglectable (WANG & SCHMUGGE, 1978). The imaginary part of $\sigma_{N,hf}$ includes the volume permittivity term ε_V and the interface contribution ε_S. ε_V represents the sum of the specific permittivities ε_i of the rock compartments (matrix, water, gas/hydrocarbon) weighted by their volumetric content V_i. ε_V increases strongly with increasing volumetric water content or water saturation respectively. In the case of water saturated rock, ε_V increases with increasing porosity. ε_V is relatively unaffected by the salinity (SCHÖN, 1996). The permittivity contribution ε_S is related to charge carrier displacement in the electrical double layer. It represents the high frequency capacitiv effect of the double layer. Therefore ε_S increases with increasing internal

surface area S_V. The real component of conductivity in the high frequency range is determined by the interface related loss component ε_L. ε_S and ε_L are explained with the deformation of the electrical double layer by charge carrier motion over short distances (intra cluster transport) at the matrix surface (DISSADO & HILL, 1984). In the constant phase angle range the real and imaginary component of high frequency complex conductivity are then:

$$s''(w_N)/iw = e_V + e_S = \Sigma e_i V_i + e_S(S_V) \tag{12a}$$

$$s'(w_N)/iw = e_S(S_V) \tag{12b}$$

Using equation (5b) the frequency dependence in the high frequency range is:

$$n = \frac{2}{p} \arctan \frac{e_V + e_S}{e_S} \tag{13}$$

The frequency exponent n ranges between 1 (no dispersion) and 0.5 (the case of a single diffusion limited process according to the Warburg impedance). Equations that are similar to (12) are also used in the time domain reflectometry technique (TDR) in soil physics.

Tab. 2 gives some examples of measured parameters that verify the electrical broad band transfer function for different rock types. The data were obtained by fitting equation (9) to the experimental data published by different authors.

Tab. 2: Parameters of the electrical transfer function

Sample type	state	$\sigma_{N,lf}$ in S/m	$\sigma_{N,hf}$ in S/m	1-p	n
wet sand, DISSADO&HILL, 1984	unsaturated	2.0E-04	1.0E-06	0.22	0.9
granite, LOCKNER&BYERLEE, 1985	saturated	2.1E-04	5.0E-09	0.015	0.64
sandstone (MYERS, 1996)	saturated	5.0E-04	1.2E-10	0.022	0.93
sandstone B6r	saturated, fresh water	9.0E-04	5.0E-10	0.007	0.92
sandstone B6a	saturated, fresh water	1.0E-03	5.0E-10	0.01	0.94
sandstone B12	saturated, fresh water	1.2E-03	5.0E-10	0.015	0.93

Conclusions

The electrical transfer function of rocks may be described using the combination of a low frequency and a high frequency constant phase angle element. It seems to be sufficient to measure complex electrical properties on rock sample in a small frequency band in the Hz-range and near the GHz-range (see fig. 5). This allows a simple parameter estimation because the fundamental electrical properties are determined by two electrical processes alone, the low frequency dispersion process and the high frequency dispersion process. Thus the general frequency dependence may be reconstructed if the amplitude and the frequency exponent of both processes are known. However it is necessary to obtain more experimental data to check the CCPM model. These investigations must include the whole frequency range that is relevant in petrophysics. Of the same importance is the realization of defined and throughout time-independent state conditions in the sample. In fig. 6 a measurement

technique is proposed that without changing the sample shape allows a broad band complex measurement with an impedance and a network analyzer on the same sample. An impedance analyzer such as proposed by OLHOEFT (1985) measures the impedance by digitalization of the input and output signal at discrete frequencies. The sample is stimulated with an AC-signal source and the actual voltage across the sample is monitored between two separate terminals. The main components of the vector analyzer (HP 8752C) are the signal source and the receiver. The source drives a signal at a single frequency to the rock sample. The receiver detects the reflected and the transmitted signals from the sample. The problem is the combination of different terminals that are required by the two measuring techniques. The terminals or electrodes must not be allowed to influence each other.
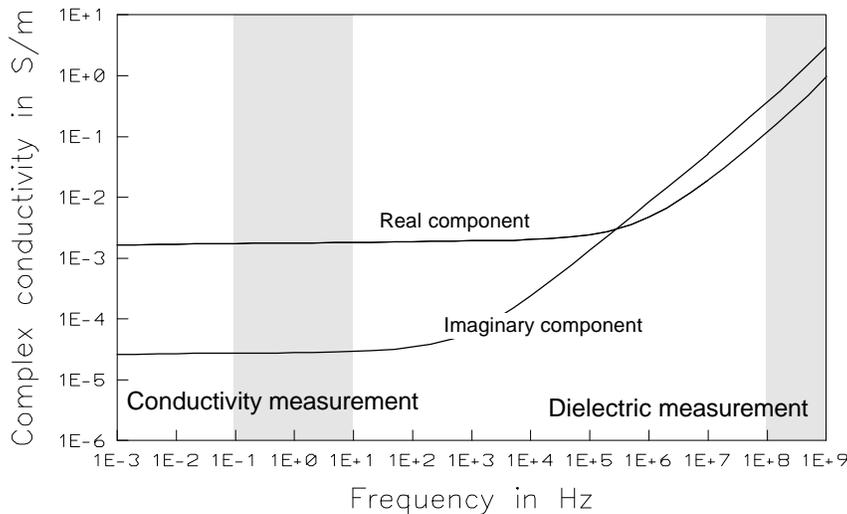

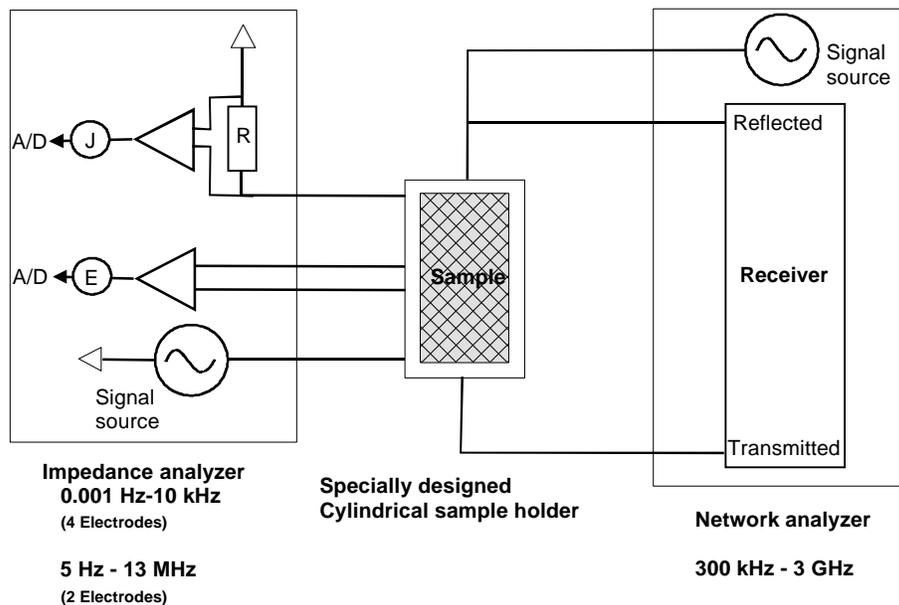Fig. 5: Frequency ranges suitable for a combined measurement of conductivity and permittivity.

Fig. 6: System for a combined conductivity and dielectric permittivity measurement.

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Acknowledgements