RELATIVE PERMEABILITY AND CAPILLARY PRESSURE: EFFECTS OF ROCK HETEROGENEITY

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Abstract

The importance of accurate modeling of heterogeneities for the determination of the two-phase flow functions is assessed. 1-D and 2-D synthetic and experimental cases are used to demonstrate the effects of rock heterogeneities on the flow functions. It is shown that inaccurate modeling of heterogenities can lead to large errors in the estimated two-phase flow functions even for moderate heterogeneities.

Introduction

Two-phase flow functions, capillary pressure and relative permeabilities, are typically determined through analyses of data acquired from measurements on some flooding process in the laboratory. As the flow functions are not directly measurable, they are inferred from the measured data utilizing some mathematical model of the physical process. This solution procedure is referred to as inverse modeling.

Frequently, the Johnson, Bossler, Naumann [1] (JBN) method is applied for solving the inverse problem. In this method, capillary pressure is neglected so that the relative permeability values can be calculated pointwise (or explicitly) for corresponding saturation values. It is also assumed that the porous media is homogeneous. However, absolute permeability and porosity often vary spatially within a core sample. Clearly, this will have an impact on the flow pattern within the core and also on the measured quantities. This should be accounted for in the interpreted flow functions.

Over the past ten years, an implicit methodology, in which basically a core flood simulator is used to history match the measured data, has been developed, tested, and reported in a series of papers (see, e.g., refs. 2-6). In this approach, the relative permeability and the capillary pressure functions are estimated simultaneously. Hence, the restrictive assumption regarding zero capillary pressure is avoided. Although the examples of this methodology so far has been directed towards cases for which the assumption of a homogeneous media may be adequate, in principle, no restrictions on the heterogeneity distribution are needed to apply this methodology.

Although few papers address determination of relative permeability and capillary pressure functions accounting for permeability and porosity heterogeneities, it is well known that the core plugs may indeed be heterogeneous (see, e.g., Hove et al. [7]). The core material subject to investigation might be composed of several individual core plugs (forming a so called “composite” core). The individual cores may differ both in porosity and permeability. It is not unrealistic to have a permeability contrast of 2:1 (i.e., the permeability of individual cores may differ by a factor of 2). However, also within a single core sample the permeability and porosity may vary. For example, small scale heterogeneities such as laminae and cross bedding may significantly impact the flow due to the large contrast in the magnitude of the permeability of the laminas. Such a permeability contrast may exceed 10:1 [8]. In addition, the permeability along and across the laminae at the same position in the core may differ significantly i.e., the permeability may be anisotropic.

In this paper, we demonstrate the importance of accounting for spatial permeability variation when estimating two-phase flow functions. We show how the relative permeability and capillary pressure functions may depart significantly from those of the rock sample whenever a simplified 1D heterogeneity or homogeneity assumptions are erroneously applied. The effects of anisotropy is not considered here, but are expected to be significant, as even a small anisotropy changes the local flow pattern. We plan to look into this in future work.

Methodology

In the Appendix we briefly outline the implicit method for determining the relative permeability and capillary pressure functions. We have in this work extended our capabilities to also handle situations for which the permeability and porosity can vary spatially. In practice this means that both the permeability and porosity can change from grid block to grid block in the simulator. Both 1D and 2D options have been implemented. As the capillary pressure function may vary with varying absolute permeability (especially for smaller permeabilities) [9], we have attempted to account for this effect: As an option, the Leverett j-function [9,13] has been implemented. For such cases, one Leverett j-function, representing the entire sample, will be estimated. The capillary pressure function will then vary in the grid blocks depending on its permeability and porosity.

Regardless of the method used for determination of the flow functions, there will generally be three sources for errors in the estimate [10]: (1) modeling errors; (2) bias errors; and (3) variance errors. The modeling errors are due to inadequate modeling of the flow process when estimating the flow functions. In the JBN method the capillary pressure is neglected altogether, and the media is assumed homogeneous. Although these assumptions may be adequate in some circumstances, they might lead to serious errors in others. The bias errors are due to utilizing a functional representation for the flow functions not capable of representing the true (although unknown) functions. The variance errors will always be present due to uncertainties in the measurements. When estimating the flow functions, one will generally aim at being at a level where the discrepancy between measured and simulated data can be described by random errors [11]. To achieve this, the modeling and bias errors must be much smaller than the variance errors. The expected value for the objective function (Sum of Square Residuals, *SSR*, see Eq. A-1) should then approach the degrees of freedom in the problem whenever the weighting matrix is chosen as the inverse of the covariance matrix of the measured data [11,12].

In this work, the investigation is directed towards those modeling errors done when estimating the relative permeabilities and capillary pressure functions on 2D heterogeneous samples, but assuming the samples to be simplified 1D heterogeneous or homogeneous. In our studies, we utilize a set of simulated data generated from a simulator with both a 2D and a 1D heterogeneous absolute permeability distribution. We then attempt estimating the relative permeability and capillary pressure functions used as input to the simulation (or, the “true” flow functions), from the simulated data, and assuming a 1D heterogeneous absolute permeability distribution or a single absolute permeability for the entire sample. In this process, no measurement errors are added to the simulated data prior to estimating the flow functions; here we limit the investigation to study the impact of modeling errors on the estimated properties. However, as we know with which accuracy the different data groups may be measured, we would know which value the *SSR* should approach in the case of no modeling errors and the presence of variance errors (provided that the problem was properly weighted), namely the degrees of freedom in the problem [12]. Consequently, as a rough measure for whether or not the impact from the modeling errors would be distinguishable from the variance errors, we compare the *SSR* obtained in these cases (in which modeling errors are present, but variance errors are not), with the corresponding expected values in cases with variance errors.

Outline of Cases

The synthetic cases investigated in this work are shown in Figure 1. The core sample is divided into two different pieces (denoted C1 and C2) with different permeability. The line dividing them can be altered through an angle a in the center of the core. If a is 90° we will have a composite core with two individual cores with equal length but with different permeabilities (Figure 1b). Decreasing this angle towards 0°, passing the angle $\overline{q}$ (the angle where the dividing line goes from the lower-left corner to the upper-right corner), we will eventually end up with a horizontally layered core (Figure 1f). With a in the range $\left[0^\circ - 90^\circ\right)$ we will have 2D cases, with $a = 90^\circ$ we will have 1D cases. The permeability contrasts considered between the two different core pieces for the synthetic cases are 100:50mD (contrast 2:1, for one 1D case only), 50:100mD (contrast 1:2), and 50:500mD (contrast 1:10). In the caption of Figure 1, e is the angle corresponding to one grid block at one of the corners i.e., $\overline{q} - e$ being the first case where a continues fluid flow in C1 may be possible.

To demonstrate the importance of accurate modeling of heterogeneities for the determination of two-phase flow functions, a set of flow functions (see for example solid lines in Figure 2) are first chosen. For the capillary pressure we use a Leverett j-function for the entire sample, and scale each individual grid block with its permeability to find the capillary pressure function for that specific grid block. The flow scenario studied is an oil flood from a core saturated 100% with water (a “drainage” case). After 1000 min of oil injection, the rate is increased (a so called ”rate-bump”) and the injection of oil proceeds. The total simulation time is 2000 min. The core and fluid properties are listed in Table 1. Using the above described experimental design, production and pressure drop data were generated for a series of 1D and 2D heterogeneity cases. The simulated data were used as input to the estimation procedure. The relative permeability and capillary pressure (Leverett j-function) were estimated using an average value of the absolute permeability.

In this work we consider both 1D and 2D heterogeneities. The 1D examples allow us to compare flow functions based on accurate modeling of 1D heterogeneities (e.g. composite core) with flow functions based on the assumption that the core sample is homogeneous (with appropriate average value for the permeability). Flow functions accounting for 2D heterogeneity patterns are compared to flow functions based on simplified heterogeneity patterns such as a 1D heterogeneity distribution and a homogeneous core with appropriate averages for the permeability.

Results and Discussion

We present the results focusing on two measures: (i) we study the relative permeability and capillary pressure (Leverett j-function) compensation for the modeling errors, and (ii). we investigate and compare the initial SSR of the different cases. The initial *SSR* is the *SSR*-value we obtain for the objective function when the averaged absolute permeability (i.e., the approximated or homogeneous permeability) and the true flow functions are inputted to the simulator.

Synthetic 1D Cases

First we consider the permeability contrasts 2:1 and 1:2 in a case with a equal 90° (see Figure 1b)). In the 2:1 case, the permeabilities of C1 and C2 are be taken to be 100mD and 50mD, respectively. In the 1:2 case, the two cores have switched places. Generating the synthetic data with permeability contrast 2:1 and trying to find estimate of the flow functions with a homogeneous core sample, the oil relative permeability curve is lowered while the water relative permeability curve is shifted dramatically upwards, see Figure 2. For the permeability contrast 1:2, we see the opposite effect: The oil relative permeability curve is slightly increased while the water relative permeability curve is dramatically shifted downwards. The Leverett j- functions are also changed, especially for the case with permeability contrast 2:1.

Next, we consider the case when the permeability contrast was increased from 1:2 up to 1:10. In Figure 3 we see that the relative permeability compensation has not particularly increased, while the Leverett j-function compensation has increased significantly. As will be discussed later, the small differences in compensation in the relative permeability curves for weak and strong contrast for $a = 90^{\circ}$ is not seen for other angles.

We investigated cases for which we kept the permeability of C1 equal to 50mD. The permeability of C2 was varied from 50.1mD up to 500mD. For a series of different permeabilities for core C2, we calculated the initial *SSR* value. Figure 4 shows the results; “Delta k” is how much the permeability of C2 is higher than that of C1. The magnitude of permeability difference needed to exceed the expected value of *SSR*, is only between 3 and 4mD in this case. This represents a permeability difference of approximately 7%.

Experimental 1D Cases

Experiments on composite cores are often done to reduce end effects when interpreting the measured data. Although the reduced end effects make the JBN method more valid, serious error can be made by assuming a homogeneous core when it is actually heterogeneous. A 2cc/min unsteady state water flood experiment was performed at reservoir condition from S_{wi}, and measurements of pressure drop and oil production were taken. The experiment was done using a composite core which consisted of four individual cores, each with an individual absolute permeability. The porosities of the four cores were quite similar. Table 2 shows the permeabilities and the arrangement of the cores when conducting the experiment.

The composite core was modeled as an 1D heterogeneous core (using the individual permeabilities) as well as a homogeneous core with a measured average permeability of 560mD (harmonic average is 548mD, see Table 2). In both cases, the relative permeability and capillary pressure were estimated simultaneously utilizing the same experimental data.

The results are shown in Figure 5. Figure 5a) shows the comparison of the measured and simulated values for the first 20 minutes; for later times, only small differences between the two cases can be observed. As can be seen from Figure 5a), a better match of the breakthrough is experienced both for the production and for the pressure drop for the 1D heterogeneous approximation compared to assuming a homogeneous composite core.

More interesting is the estimated relative permeabilities. In Figure 5b) we see that a plateau in the oil relative permeability curve appears assuming a homogeneous composite core. Using a 1D heterogeneous core, the plateau vanishes and an accurate (narrow confidence intervals) oil relative permeability function with no plateau is obtained. No specific changes were seen on the capillary pressure function. Note that a Leverett j-function was not utilized in this case.

Synthetic 2D Cases

Constructing 2D cases, we first generate data using a selected permeability distribution. We then approximate this distribution using two approaches. A 1D heterogeneous distributions by calculating the vertical arithmetic average in each vertical grid column (denoted 1D heterogeneous cases), and a homogeneous approximation by calculating the harmonic average of the 1D heterogeneous cases.

Figure 6 shows the results of a 2D case, with a equal to 30° and permeability contrasts of 1:2 and 1:10. Modeling the 2D heterogeneity permeability pattern with a 1D heterogeneity permeability distribution, only minor changes are observed for the flow functions for the permeability contrast 1:2. Simplifying this pattern with a homogeneous distribution, we get a major compensation for the water relative permeability curve. Increasing the permeability contrast to 1:10, we notice that somewhat more compensation for the water relative permeability curve for the 1D heterogeneous case result. For the homogeneous case, this permeability contrast will lead to useless results.

Next we consider a rotation of the angle a. We generate simulated data for a 2D permeability distribution for $a = 30^{\circ}$, $a = \bar{q} + e$ and $a = \bar{q} - e$ (see Figure 1c), 1d) and 1e) respectively). The flow functions are estimating using a simplified 1D heterogeneous distribution pattern with permeability contrasts of 1:2 and 1:10. Figure 7 shows the results. Not much compensation in the flow functions is seen for the 1:2 permeability contrast. For the 1:10 permeability contrast, however, large compensation results on the estimated flow functions.

Figure 8 shows the influence of using one capillary pressure function instead of a Leverett j-function when estimating the flow functions from data generated with a 2D heterogeneity distribution pattern. Generating data with a 1:2 permeability contrast, no apparent difference is seen between using only one capillary pressure function instead of the Leverett j-function. Increasing the permeability contrast to 1:10, both the water relative permeability and the Leverett j-function deviate strongly from the true curves.

The last case considered is when the angle a is 0° (i.e., stratified medium). The flow function compensation is large for the 1:2 permeability contrast (see Figure 9), and gets dramatic when the permeability contrast is increased to 1:10. One new observation is that the oil relative permeability is shifted considerably upwards. Also, a very large compensation for the water relative permeability and the Leverett j-function is seen.

Conclusions

1. Significant modeling error exit for even moderate (1:2) heterogeneities.
2. Strong (1:10) heterogeneity gives useless results if not properly accounted for.
3. Using a simplified 1D homogeneous pattern gives more flow function compensation than a 1D heterogeneous approximation.
4. Estimation using an average capillary pressure function gives more flow function compensation than scaling each individual core with a Leverett j-function.

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Nomenclature

$\vec{b}$	Vector of parameters
C_i	Core piece, i=1 or 2
$\vec{F}$	Model function
G	Constraint matrix
J	Objective function
W	Weighting matrix
$\vec{Y}$	Vector of data points

Subscript / superscript

con	Constraint
m	Measured data
s	Simulated data
T	Transposed

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Appendix: Estimation Procedure

The basic methodology used for estimating the relative permeability and capillary pressure functions is an implicit approach in which the physical process is represented by an adequate mathematical model [16]. The relative permeabilities and capillary pressure are saturation dependent functions and can not be measured directly. These properties can be estimated through an inverse procedure where simulated data are compared to experimental data. The simulated data are generated by the mathematical model. It is essential that the flow functions representation within this model has sufficient degrees of freedom to represent the true although unknown functions. It has been shown [10] that B-spline functions [14] can provide the necessary flexibility to represent both the relative permeability and capillary pressure functions. The coefficients in the B-spline representation are estimated from data from some displacement experiment, through solution of the non-linear least-squares problem. The basic idea is that the simulated data should reconcile those actually measured. The non-linear least-squares problem is defined by

$$J(\vec{b}) = \left[\vec{Y}_m - \vec{F}_s(\vec{b})\right]^T \mathbf{W}\left[\vec{Y}_m - \vec{F}_s(\vec{b})\right]. \qquad \text{(A-1)}$$

Here $\vec{Y}_m$ and $\vec{F}_S(\vec{b})$ are the measured and simulated data, respectively. **W** is a weighting matrix, and $\vec{b}$ contains the coefficients or parameters to be estimated. $\vec{F}_S(\vec{b})$ is calculated using the fully implicit, two dimensional black oil, core flood simulator CENDRA [15]. CENDRA is tailor made for core analyses application, including initial boundary conditions adequate for modeling flooding, porous plate/micro membrane and centrifuge experiments in 1D and 2D. In the cases considered in this paper, P_c=0 at the outflow end and a constant injection rate at the inflow end are utilized. The least-squares problem in equation (A-1) is minimized subjected to the linear inequality constraints $\mathbf{G}\vec{b} \geq \vec{b}^{con}$, to ensure monotonic behavior of the flow functions. $\mathbf{G}$ and $\vec{b}^{con}$ are constraint matrix and vector, respectively. The minimization problem is solved by using a trust-region implementation of the Levenberg-Marquardt optimization algorithm.

Tables and Figures

Table 1: Core and fluid properties used.

Core length [cm]	6
No. blocks in x-direction	60
Core width [cm]	3,25
No. blocks in y-direction (2D)	10
Thickness [cm]	3,25
Absolute permeability [md]	50,100,500
Porosity [%]	30,0
Water viscosity [cP]	0,35
Oil viscosity [cP]	0,85
Oil injection rate [cc/min.]	0,1 and 1,0
Initial Water Saturation [frac.]	1

Table 2: Physical parameters and plug arrangement of the composite core.

Plug arrangement	Length [cm]	Ko(Swi) [mD]
1	6,85	401
2	7,31	498
3	6,80	625
4	7,22	819

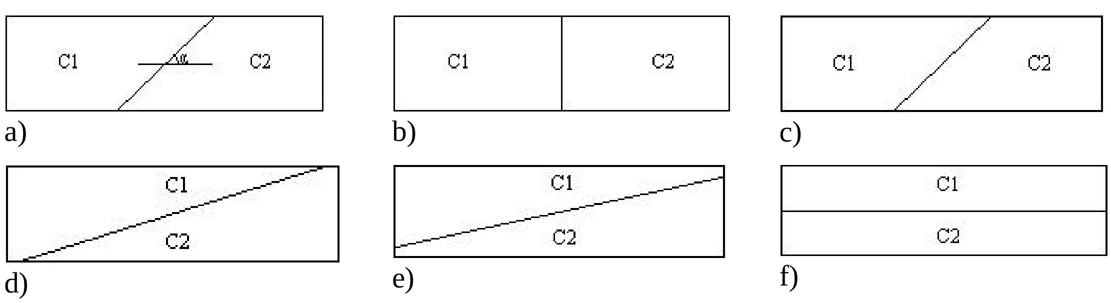


Figure 1: Illustration of the cases used in this work. a) General case, b) Composite core example, $a = 90^\circ$, c) $a = 30^\circ$, d) $a = \bar{q} + e$, e) $a = \bar{q} - e$, f) $a = 0^\circ$.

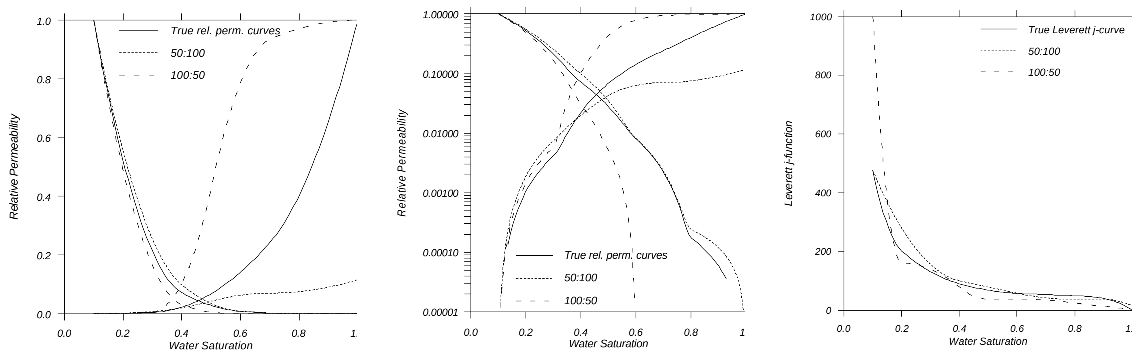


Figure 2: Flow function compensation for modeling error for the permeability contrasts 2:1 and 1:2 and $a = 90^{\circ}$.

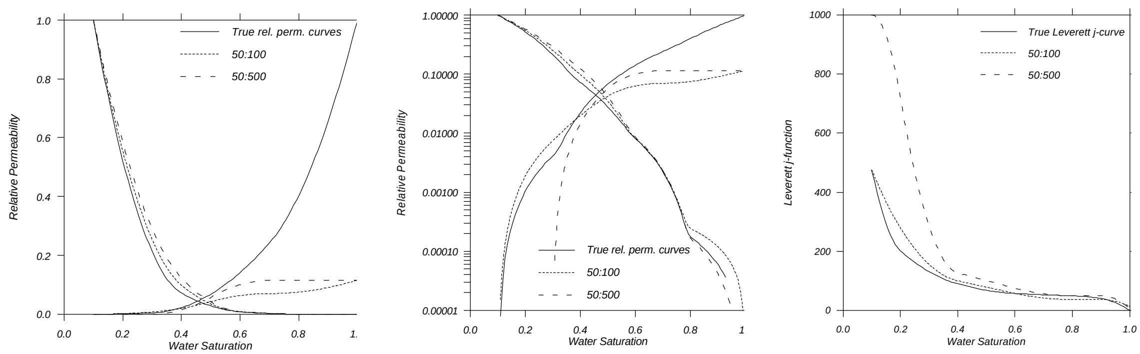
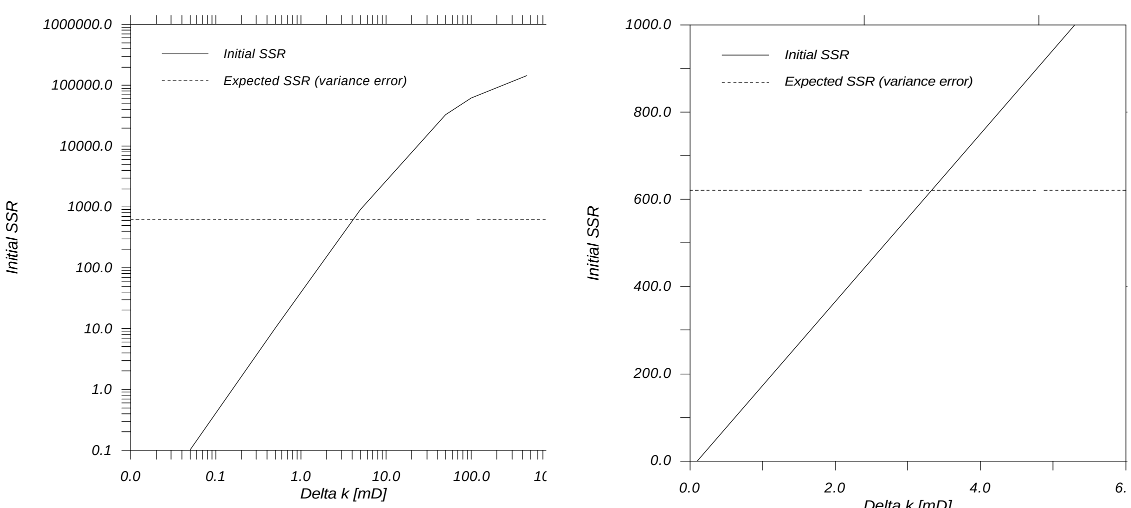


Figure 3: Flow function compensation for modeling error for the permeability contrasts 1:2 and 1:10 and $a = 90^{\circ}$.



a) b)

Figure 4: Initial SSR as a function of permeability difference when C1=50mD, C2=50+Δk mD and $a = 90^{\circ}$; a) log-log plot, b) magnified lin-lin plot.

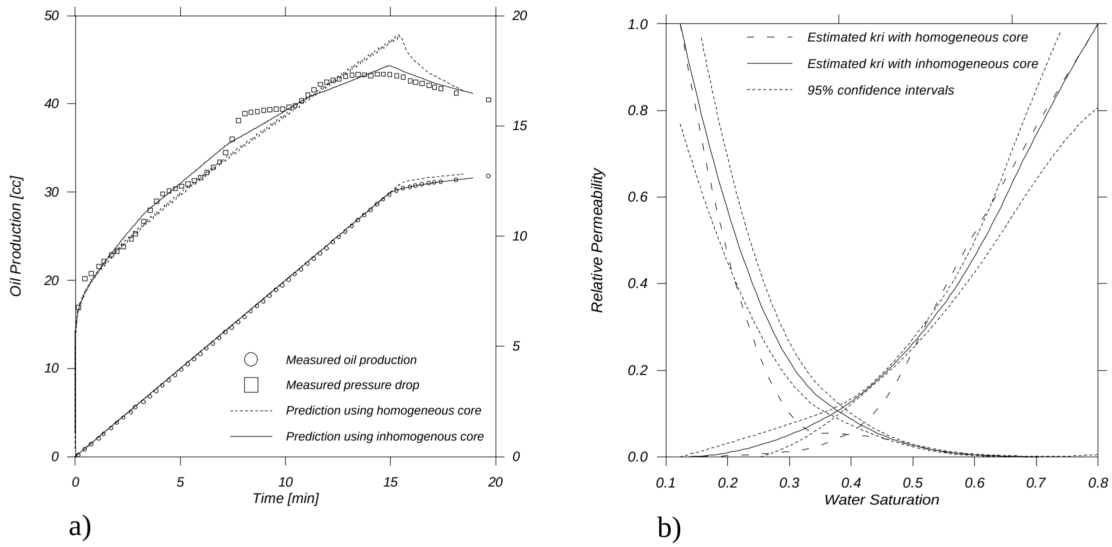


Figure 5: Experimental case; a) measured and predicted data, b) estimated relative permeability curves using homogeneous and 1D heterogeneous cores

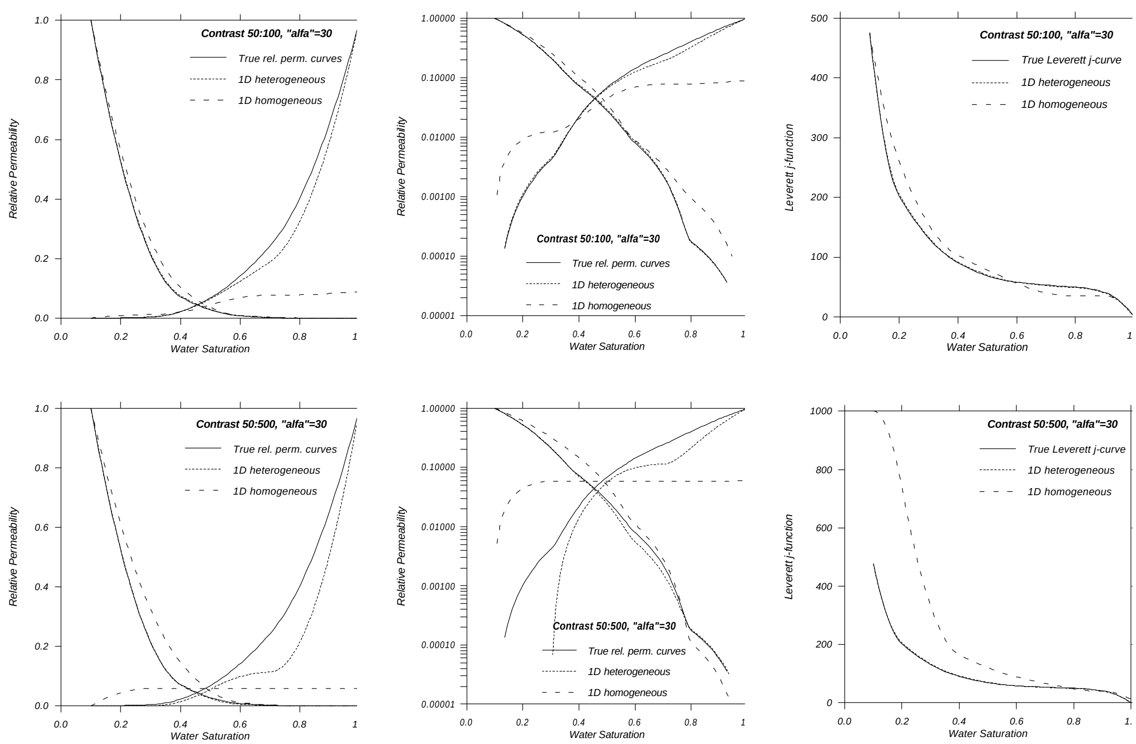


Figure 6: Flow function compensation for different permeability contrasts and simplified heterogeneity patterns.

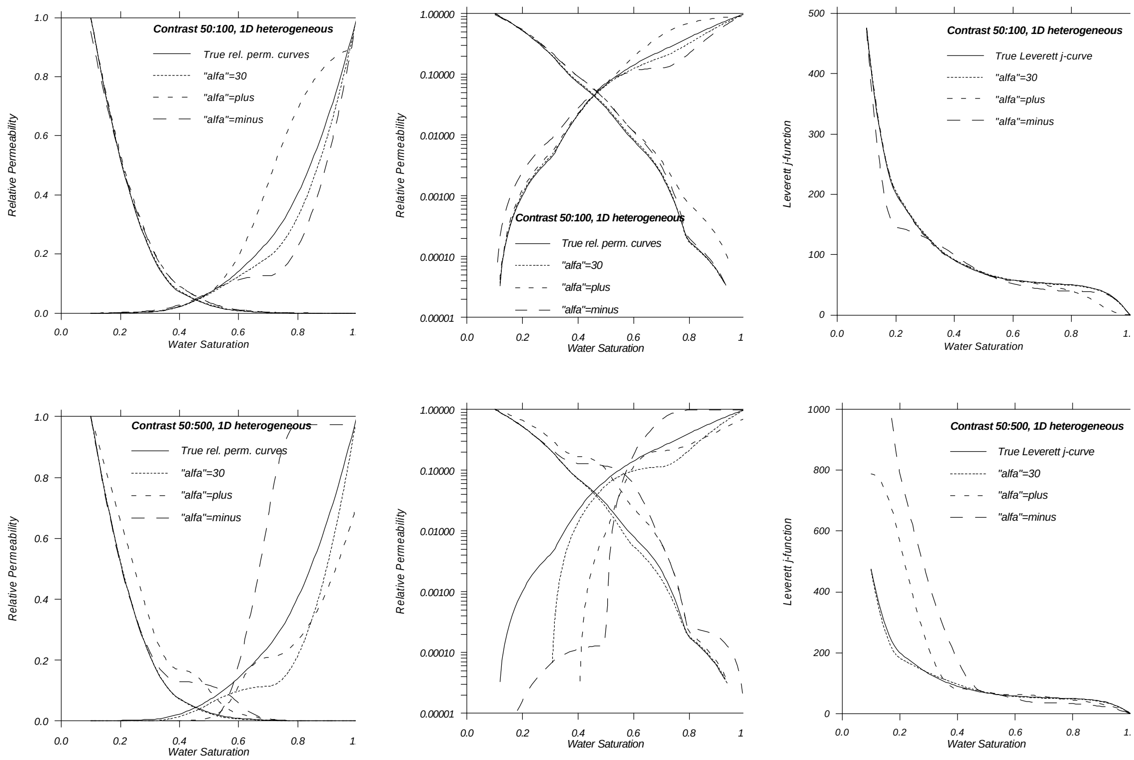


Figure 7: Flow function compensation for different permeability contrasts and rotation of the angle a for 1D heterogeneities.

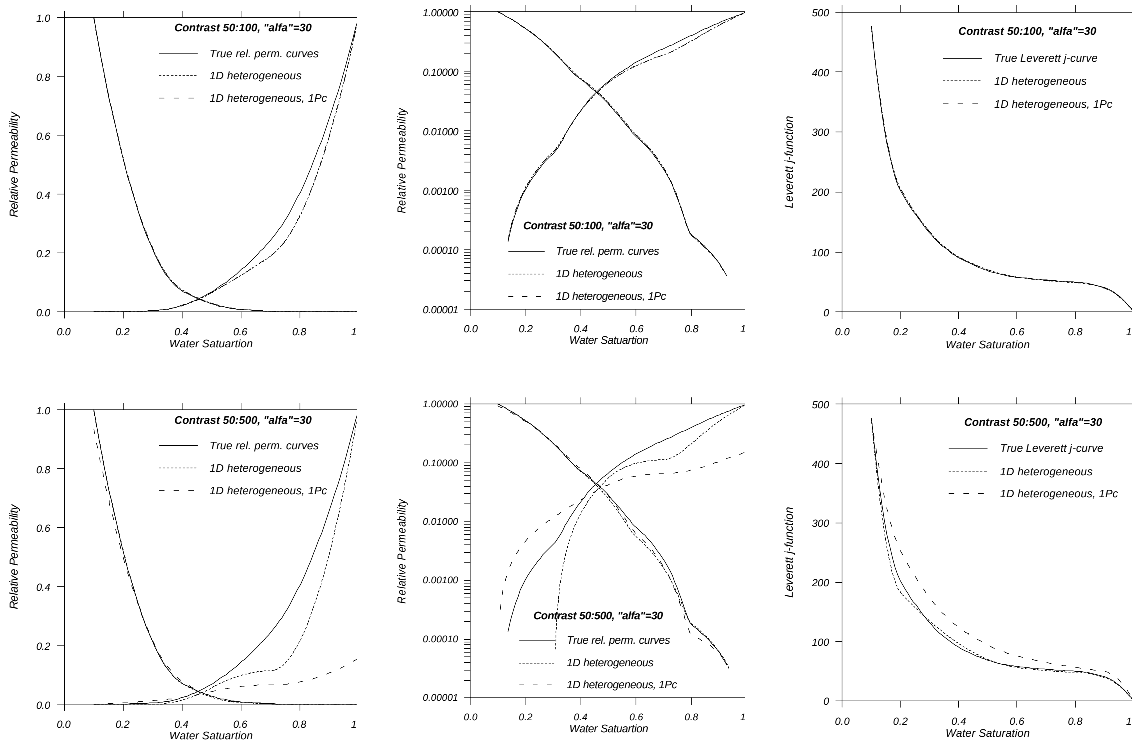


Figure 8: Flow function compensation for different permeability contrasts and the use of several or one capillary pressure function.

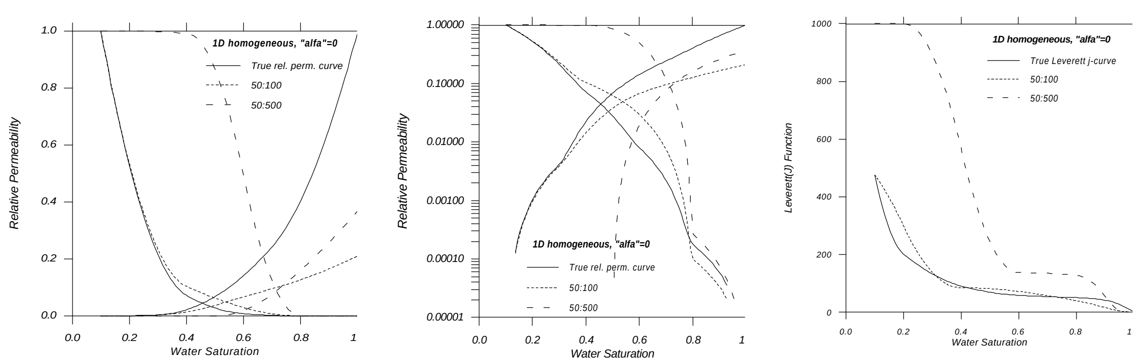


Figure 9: Flow function compensation for different permeability contrasts when $a = 0^{\circ}$.