SCA-9825

A VISCOUS COUPLING MODEL FOR RELATIVE PERMEABILITIES IN A FRACTURE

M. FOURAR
Ecole des Mines de Nancy
Laboratoire d'Energétique et de Mécanique Théorique et Appliquée
Parc de Saurupt - 54042 Nancy Cedex, France
Email : fourar@mines.u-nancy.fr

R. LENORMAND
Institut Français du Pétrole
1 et 4, avenue de Bois Préau, 92852 Rueil Malmaison Cedex, France

Abstract

Two-phase flow properties are required for modeling fluid flows and pollutant transport through fractured media. Several laboratory experiments have been reported in the literature but so far there is no general model to predict the two-phase conductivity of a fracture for a given pair of flowing fluids. In this paper we propose a simple model based on viscous coupling between two fluids flowing simultaneously in a single fracture. This viscous coupling model leads to simple analytical relationships between the relative permeabilities (Kr) and either saturation or fluid velocities. This model also explains the dependence of the non-wetting fluid Kr on the viscosity ratio. Results obtained with this model were compared to several series of data available in the literature with air and water. The fit between predicted and measured Kr is good when the ratio of flow rates is used as variable (Lockhart-Martinelli parameter). For use in numerical simulations, the saturation can be derived from the flow rates using the same model. The proposed model can improve the relative permeability curves that are used in the numerical simulations.

Introduction

Multiphase flows in fractured porous media occur in several domains: in petroleum engineering (gas-oil-water flow), in geothermal energy (water-vapor) and in protection of the environment (NAPL, gas migration from nuclear repositories, etc.). When inertial effects are negligible, two-phase flow in porous media and fractures are described by the generalized Darcy's equations. In one-directional flow with no gravity effects (horizontal fracture), these equations are the following:

$$V_L = -Kr_L \frac{K}{m_L} \frac{dP_L}{dx} \qquad (1)$$

$$V_G = -Kr_G \frac{K}{m_G} \frac{dP_G}{dx} \qquad (2)$$

where subscripts L and G stand for liquid and gas respectively ; V is the superficial velocity (flow rate/surface area of the sample) ; dP/dx the pressure gradient in the flow direction, and μ the viscosity. K is the permeability (single phase flow) of the fracture related to the mean aperture h by $K=h^2/12$ [Witherspoon et al., 1980]. The relative permeability Kr accounts for the reduction of permeability to one fluid in the presence of the other fluid. When capillary forces are negligible (compared to viscous forces), the pressures in both phases are equal and therefore $dP_L/dx = dP_G/dx = dP/dx$.

It is generally assumed that the relative permeability of each phase is equal to its saturation ("X" shape Kr curves). This assumption is based on the work of Romm [1966]. Mahoney et al. [1997] also showed that relative permeabilities were equal to fluid saturations. This result could be attributed to the punctual injection of fluids that causes a channeling flow. However, This result was not confirmed by other authors (Merrill [1975], Kouamé [1989], Fourar et al. [1993, 1995], Persoff and Pruess [1995]). A more complete review can be found in Fourar and Bories [1995]. All these authors concluded that, due to the interaction between phases, the sum of Kr is less than 1. However, there is no available method to predict the relative permeabilities. In this paper, we propose a simple model based on the viscous coupling between the fluids flowing together inside the fracture.

Viscous coupling model for relative permeability

Visual observations of two-phase flow in fractures have shown a mixing at small scale and therefore a strong interaction between the fluid. For instance, Kouamé [1989] and Fourar and Bories [1993, 1995] have identified several fluid configurations for air-water flows in an artificial fracture : bubbles, unstable bubbles, film flow, etc. As a consequence of these observations, any physical model should account for this coupling between the two-fluids flowing simultaneously. The real mechanisms are very difficult to model since the real geometry of the interface between the fluids is unknown and that one of the phases is generally discontinuous. We will assume that the complexity of the real flow can be modeled, in a first approximation, by viscous coupling between the fluids. This mechanism has already be theoretically proposed for two-phase flow in porous media [De Gennes, 1983 ; Kalaydjian and Legait, 1987 ; Zarcone and Lenormand, 1994]. In this paper, we will show that viscous coupling can also explain the shape of the Kr curves.

The fracture is modeled by two parallel planes with a small aperture. The two fluids are flowing simultaneously and the interface is assumed to be a plane. Fluid L is considered as the wetting fluid and therefore is in contact with the walls, and fluid G (non-wetting) flows in between. The viscous coupling between the fluids is derived by writing Stokes' equation in each fluid with a no-slip condition at the interface. The established equations are the following :

$$V_L = -\frac{S_L^2}{2}(3 - S_L)\frac{K}{m_L}\frac{dP}{dx} \tag{3}$$

$$V_G = -\left[(1 - S_L)^3 + \frac{3}{2} m\, S_L (1 - S_L)(2 - S_L)\right]\frac{K}{m_G}\frac{dP}{dx} \tag{4}$$

where $\mu = \mu_G/\mu_L$ is the viscosity ratio. Similar equations were obtained by Bacri et al. [1992] in the case where capillary effects are not negligible.

Identification of equations (3) and (4) with Darcy's law leads to the relative permeabilities :

$$Kr_L = \frac{S_L^2}{2}(3 - S_L) \tag{5}$$

$$Kr_G = (1 - S_L)^3 + \frac{3}{2} m S_L (1 - S_L)(2 - S_L) \tag{6}$$

These equations show that the relative permeability of the non-wetting fluid depends on the viscosity ratio μ. Kr_G can be larger than unity when $\mu > 1$ (lubrication effects). However, the experiments have only been performed with $\mu < 1$ and we have no evidence of such lubrication effects in fractures. For $\mu = 1$, the sum of Kr is equal to 1. However, the Kr is not equal to saturation.

In two-phase flow in pipes, one uses the Lockhart-Martinelli parameter defined by $\chi = (\mu_L Q_L)/(\mu_G Q_G)$ [Lockhart and Martinelli, 1949]. This parameter can be derived using equations (5) and (6) :

$$c = \frac{S_L^2 (3 - S_L)}{2(1 - S_L)^3 + 3 m S_L (1 - S_L)(2 - S_L)} \tag{7}$$

In petroleum engineering, the relative permeabilities are often approximated as power law functions of saturation (Corey functions):

$$Kr_L = S_L^{a_L} \text{ and } Kr_G = (1 - S_L)^{a_G} \tag{8}$$

Such functions can be derived from the polynomial equations (5) and (6) using a root mean square fit. The exponent is determined by minimizing the squared difference with the exact function over the interval [0-1] for saturation as follows :

$$\frac{d}{d\alpha_i}\left[\int_0^1 \left(S_i^{\alpha_i} - Kr_i\,(S_i)\right)^2 dS_i\right] = 0 \qquad (i = L,G) \qquad (9)$$

The exponent α_L for the wetting fluid is independent of μ and found to be equal to 1.66. For the non-wetting fluid, the exponent α_G is a function of the viscosity ratio μ (Fig. 1). This function can be approximated by :

$$\alpha_G = 3\exp(-1.6\,\mu) \qquad (10)$$

In the case of air/water flow (μ = 0.018), we found α_G = 2.91. As shown in Fig. 2, the approximation by a power law is quite good, at least for μ <1.

Results

We used the results presented in Fourar and Bories [1995] to test the validity of the coupling model for Kr(S), Kr(χ) and S(χ).

Fourar and Bories [1995] have studied the air-water two-phase flow in a fracture constituted of two parallel glass plates (1m x 0.5 m) with an opening equal to 1 mm. The injector consisted of 500 stainless steel tubes of 1 mm outside diameter and 0.66 mm inside diameter. Air and water were injected through alternating capillary tubes to achieve uniform distribution of the inlet flow. For all experiments, air was injected at a constant pressure and its volumetric flow rate was measured by a rotameter and corrected to the standard pressure. Water was injected by a calibrated pump. At the outlet of the fracture, the gas escaped to the atmosphere and the water was collected.

The fracture was initially saturated with water which was injected at a constant flow rate for each experiment. Air injection was then started and increased stepwise. When the steady state was reached for each flow rate, the pressure drop and liquid volume fraction (saturation) were measured. The pressure drop was measured by a transducer and the saturation was measured by using a balance method. Then, the fracture was re-saturated with water and the experiment was repeated several times at different liquid flow rates.

This study has been extended to fractures constituted by bricks made of baked clay (30 cm x 14 cm) with different apertures (0.54 mm, 0.40 mm and 0.18 mm).

For relative permeabilities vs. saturation, the comparison between the coupling model (Eq. 5 and 6) and experimental results from Fourar and Bories [1995] are presented in Fig. 3. There is a significant difference between predicted and measured relative permeabilities. The agreement is much better when the Lockhart and Martinelli parameter χ is used instead of saturation (Fig. 4).

In addition to the smooth fracture, Fourar and Bories also presented the results for a fracture made of two permeable bricks (Fig. 5). For these experiments, the saturation was not measured (the volume was too small), but the χ parameter was well determined. For the three sizes of fractures, the viscous model predicts correctly the experimental results.

Plots of saturation as function of parameter χ are presented in Fig. 6. The experimental saturation is underestimated. That may be due to experimental inaccuracy, as it was mentioned in Fourar and Bories [1995]. From these comparisons, we can conclude that our coupling model is validated by these experimental data in terms of Kr against parameter χ. This χ parameter is directly related to the flow rates that are the main parameters for flow in fractures. Contrary to porous media, the volume of fractures is usually very small and an accurate determination of saturation is not generally useful. However, most of numerical simulators need saturation as variable instead of flow rates. For this purpose, we can use the relationship given in Eq. 7, to convert flow rates into saturations.

Conclusions

A theoretical model for describing two-phase flow in fractures was presented. It is based on a simple calculation of viscous coupling of two fluids flowing simultaneously in a single fracture. This viscous coupling model leads to simple analytical relationships between the relative permeabilities and either the saturation or the fluid velocities. This model explains the dependence of the non-wetting fluid Kr on the viscosity ratio.

The model was compared to experimental data obtained by Fourar and Bories [1995], but with only one pair of fluids (air and water). The study support the following conclusions :

1) The calculation of Kr as function of saturation is not in good agreement with experiments. That may be due to the difficulty of measuring the saturation in a fracture.

2) The fit between predicted and measured values is very good when the Kr is plotted vs. the Lockhart-Martinelli parameter that is directly related to the ratio of fluid flow rates.

3) The viscous coupling model allows the determination of saturation as a function of flow rates. This calculation can be used in numerical simulators which needs saturation as a variable.

The proposed model can improve the relative permeability curves that are used in numerical simulations.

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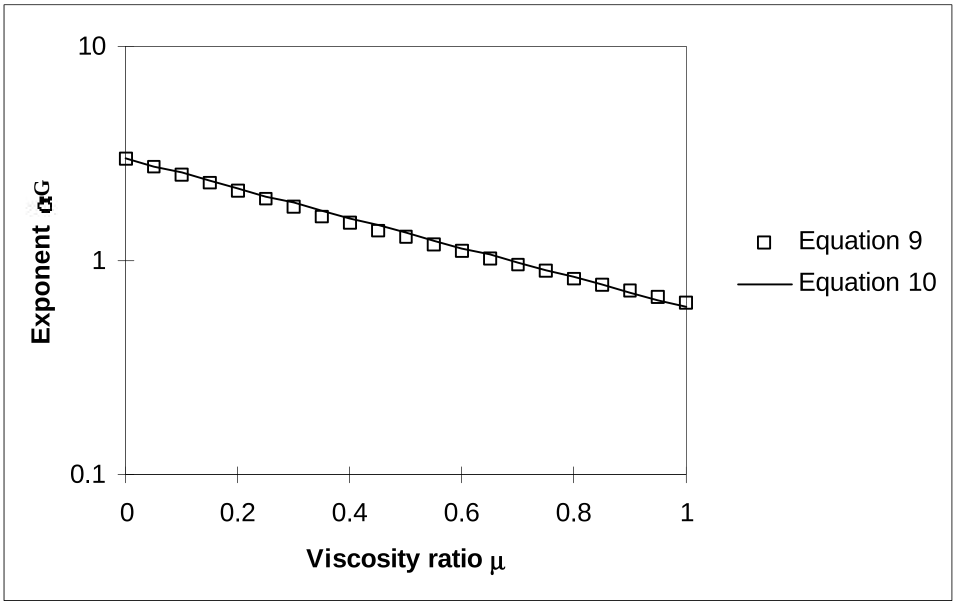

Fig. 1. Exponent α_G versus viscosity ratio μ

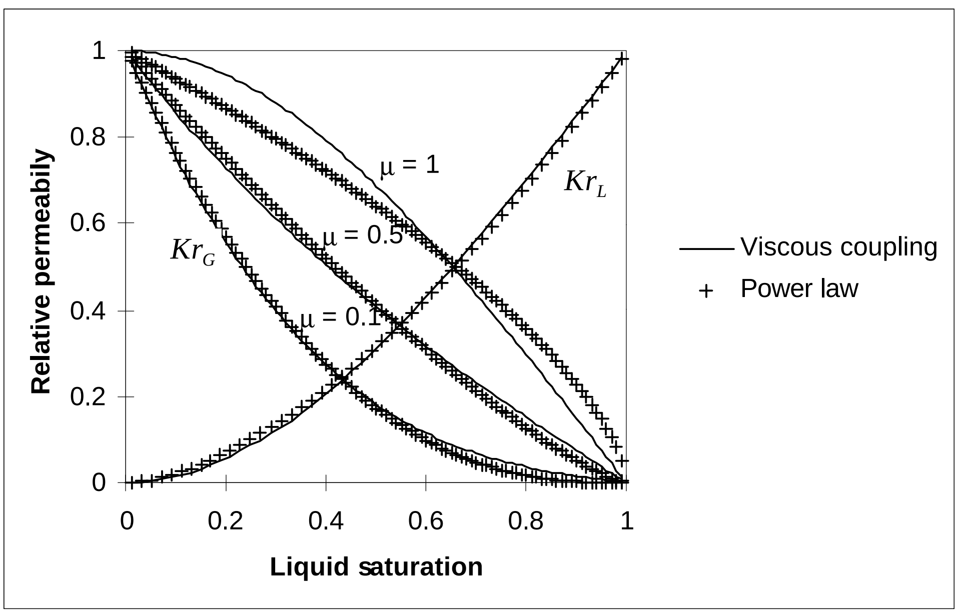

Fig. 2. Relative permeability versus saturation using the coupling model and the corresponding power law approximation for μ= 1; 0.5 ; 0.1

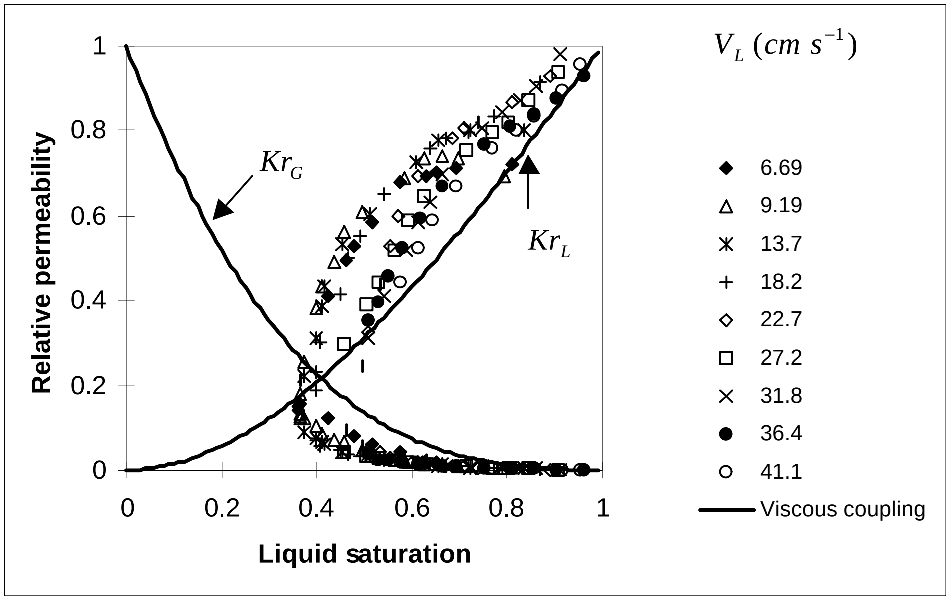

Fig. 3. Relative permeabilities versus liquid saturation
Data from Fourar and Bories [1995] (smooth fracture)

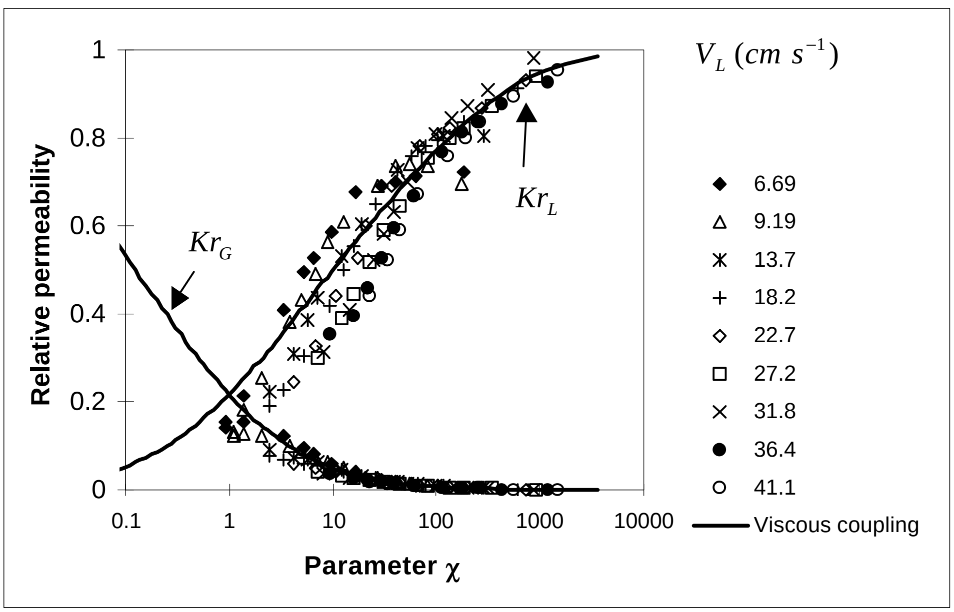

Fig. 4. Relative permeabilities versus parameter χ
Data from Fourar and Bories [1995] (smooth fracture)

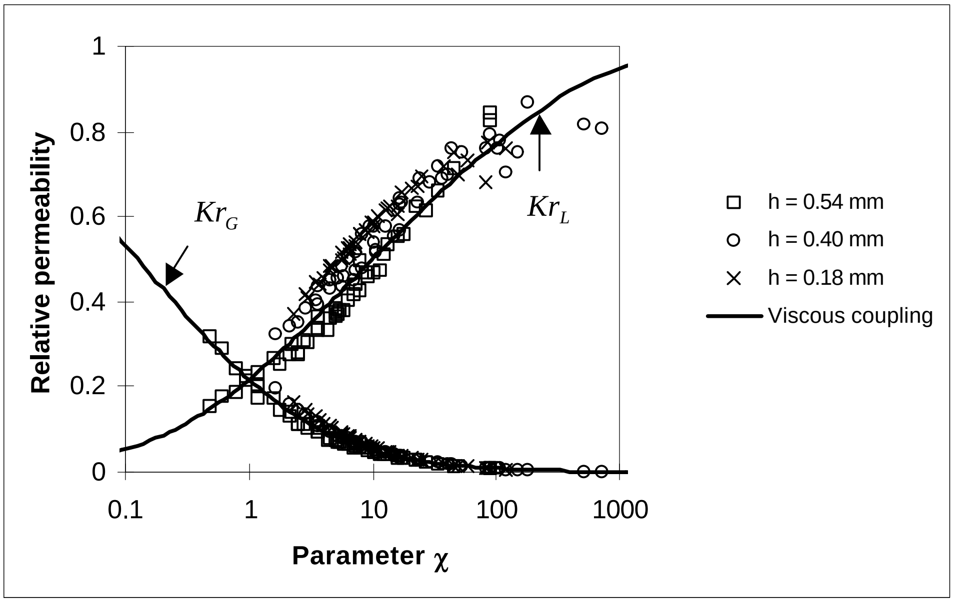

Fig. 5. Relative permeabilities versus parameter χ
Data from Fourar and Bories [1995] (bricks)

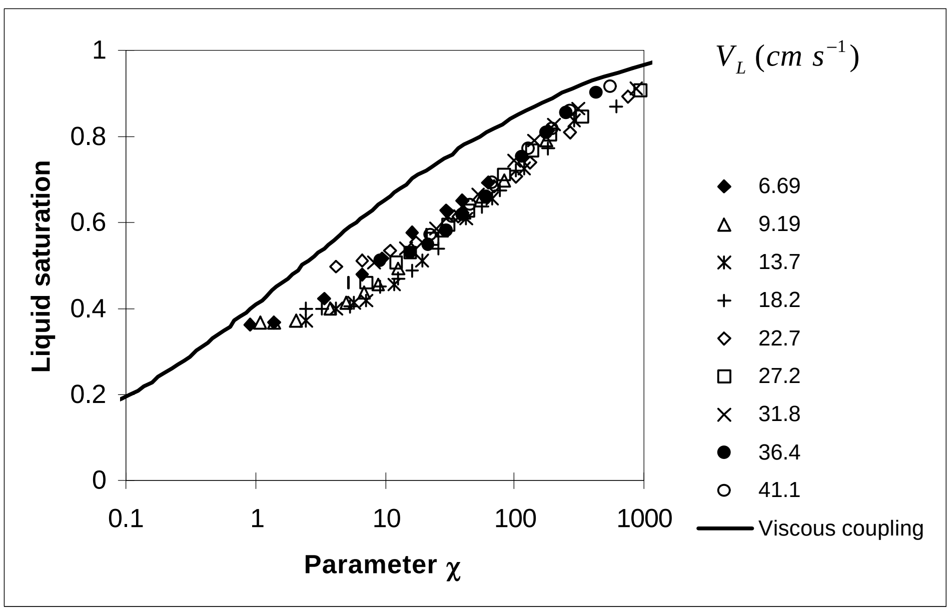

Fig. 6. Liquid saturation versus parameter χ
Data from Fourar and Bories [1995] (smooth fracture)