An Automatic-Optimization Data Analysis Technique Based On Forbes' Second Method

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Abstract

There are many methods to reduce centrifuge data to obtain capillary pressure curves. One of the most accurate methods, according to a recent survey conducted by SCA, is Forbes' Second Method. A full implementation of this method requires the manipulation of the results by the analyst to minimize the error between the experimentally measured production versus rotational speed and the production versus rotational speed predicted using the calculated capillary curve. The final curve obtained using this method can therefore be dependent on the experience and talent of the analyst. Chen and Ruth have published a technique based on Forbes' Alpha Method; their technique implements an automatic-optimization procedure, rather than an operator dependent procedure, to minimize the difference between the two production histories. Forbes' method contains a parameter that has a fixed value based on the configuration of the centrifuge; in theory, this value is optimized. The Chen-Ruth technique treats this parameter as a variable and optimizes it to minimize the root-mean-square difference between the production histories. The present paper reports on a study where this idea was extended to Forbes' Second Method. Forbes's Second Method contains three parameters that are theoretically optimized. By making these parameters variable and optimizing them simultaneously, it is shown that very good predictions for the SCA Data Analysis Survey can be obtained.

Introduction

Ever since the publication of the Hassler-Brunner data reduction technique in 1945 for calculating capillary pressures from centrifuge experiments, researchers have actively sought improved methods. The publication by Forbes (1997) provides an exhaustive list of the published methods with brief descriptions of each. That publication also provided examples of data analyses by 13

laboratories using a total of 50 implementations of the various data reduction techniques. An inspection of these results determined that one of the best techniques is the “Forbes Second Method” in its original implementation, including the radial correction method documented in Forbes *et al.* (1994).

An essential part of any technique is to use the resulting capillary pressure curve to re-calculate (simulate) the original production data. Forbes Second Method (F2), in its original implementation, had one drawback: it included a step where the analyst did “hand-corrections” to the curve to improve simulation of the experimental results. Such a step is dependent on the talents of the analyst. An experienced analyst may obtain excellent results; however, an inexperienced analyst may not.

F2 is based on calculating two different solutions and then combining them. Mathematically these solutions are:

$$S_a \Big|_t = \langle S \rangle + \frac{1}{1+a} P_t \frac{d\langle S \rangle}{dP_t} \qquad where \qquad a = \frac{1-\sqrt{1-B}}{1+2\sqrt{1-B}}$$

$$S_b = (1+b)\int_{x=0}^{x=1} x^b S_{HB}(xP_t)\,dx \qquad where \quad b = 2\frac{1+2\sqrt{1-B}}{1-\sqrt{1-B}}$$

and

$$S_{ab} = \frac{B}{2} S_b + \left(1-\frac{B}{2}\right) S_a$$

In these equations, S is the saturation, $\langle S \rangle$ and P_t represent the mean saturation and the pressure at the top of the sample, the variables that make up the original data set, x is the ratio of the local capillary pressure to the capillary pressure at the top of the sample, and B is a geometric parameter,

$$B = 1 - \left(\frac{r_t}{r_b}\right)^2$$

where r_t is the distance from the center of rotation to the top of the sample and r_b is the distance from the center of rotation to the bottom of the sample. S_{HB} is the classic Hassler-Brunner solution.

Obviously, both a and β are well-defined physical parameters. However, they were arrived at by doing an approximate analysis, albeit a very accurate one.

The present work is based on assuming that these two parameters can be treated as fitting parameters. As such, we will use them to reduce the error between the simulated data and the experimental data. Furthermore, the equation used to combine the two solutions was also based on an approximation. By replacing B in this equation with a variable g , a three parameter method is obtained.

The implementation upon which the present work is based utilizes most of the original formulation described in Forbes (1994), obviously without hand-correction. A second difference is the method of consistency constraints. In the present work, a minimum of constraints were imposed: if the saturation points are out of order (that is, the saturation for a higher capillary pressure is greater than that for a lower capillary pressure) both values are set to the average value, then they are separated by a value of 0.0001. Combining the new method with other constraints, designed to exclude data points that are obviously corrupted, will be the subject of a future paper.

The Results

To test the new method, the 10 cases analyzed in Forbes (1997) were considered. The Hassler-Brunner original method was used as a reference method. F2 was also calculated, without hand-correction but using the new constraint. The differences between the experimental and simulated data are plotted. Also, the difference between the resulting curves and the known curves (the curves used to generate the “experimental” production data) were calculated.

The attached graphs show the results of the analysis. The two columns of figures show the differences between predicted cumulative production (the figures labelled “a”) and the error between the calculated curve and the true curve (the figures labelled “b”). The squares are for the original Hassler-Brunner method, the filled-circles are for F2, and the triangles are for the new method (RF). In all methods, the data has been corrected for radial effects. The bold horizontal lines (red when in colour) give the error limits based on the errors-of-observation quoted in the original report.

In all cases, the new method leads to an improved prediction of the experimental data, albeit sometimes only marginally. In most cases, the original capillary pressure curve is also better predicted, but not always. This is because the new method optimizes on production. The model data sets were “corrupted” by adding in random errors. Errors in the data can drive the resulting curve away from the original because the method is designed to find the curve that yields the given data set, not the original curve. The method therefore is seeking a different

curve. Because the true curve is never known in practice, optimizing methods have no choice but to seek this “other” curve.

Table 1 shows values for the fitting parameters. The values for all three parameters were constrained to satisfy the ranges allowed by their original definitions, that is:

$$0 \le a \le 1$$
$$2 \le b \le \infty$$
$$0 \le g \le 1$$

It is observed that these constraints are exercised in Cases 5 and 8 for α, Case 3 for β, and Cases 6,7,9, and 10 for γ. In the last cases, the new method reduces to the Chen-Ruth method (1993) because the Beta Solution is not considered (hence β has been left out of the table for these cases).

Table 1 - A Summary of the Calculated Fitting Parameters

Case	B	α for F2	α for RF	β for F2	β for RF	γ for RF
1	0.3253	0.0676	0.0700	29.600	15.052	0.2704
2	0.9086	0.4348	0.4117	4.600	4.051	0.8400
3	0.6597	0.1923	0.0936	10.400	2.000	0.1897
4	0.8264	0.3182	0.3174	6.286	5.355	0.8756
5	0.4375	0.1000	0.0000	20.000	60.783	0.1491
6	0.4969	0.1202	0.1691	16.640	---	0.0000
7	0.6735	0.2000	0.1723	10.000	---	0.0000
8	0.2344	0.0455	0.0000	44.000	12.872	0.3802
9	0.4375	0.1000	0.0006	20.000	---	0.0000
10	0.5452	0.1386	0.0196	14.429	---	0.0000

Table 2 summarizes the root-mean-squared differences between simulated and experimental production (V), and between the calculated and the given capillary pressure curves (C). It is observed that all cases show a reduction in the difference between experimental and simulated production; often this reduction is very significant. However, agreement between the real curves and the experimental curves is not significantly improved except for Case 3. What is happening is that the curve is simply moving around to accommodate the differences in production and is not improving the match with the original curve.

Table 2 - A Summary of the Quality of the Fit

Case	RMS (V) F2	RMS (V) RF	RMS(C) F2	RMS (C) RF
1	0.087	0.023	0.011	0.010
2	0.034	0.030	0.015	0.014
3	0.166	0.015	0.016	0.007
4	0.025	0.016	0.008	0.011
5	0.123	0.049	0.266	0.266
6	0.023	0.015	0.054	0.050
7	0.050	0.017	0.037	0.037
8	0.099	0.053	0.038	0.032
9	0.092	0.062	0.089	0.089
10	0.106	0.048	0.044	0.044

Table 3 shows a re-analysis of the data based on the “additional error” defined in Forbes (1997). By definition additional error indicates the amount by which the difference between the experimental and true saturations exceeds the error-of-observation in the experimental mean saturation. The table includes the two highest plus values and the two lowest minus values of the additional error for each case. Also included are the number of high values, the number of low values and the number of values with no additional errors. Forbes (1997) suggests that the best methods should have additional errors of less than plus or minus 3-saturation units (0.03) 90% of the time when constraints designed to account for corrupted data are imposed. From the table it is observed that the 3-saturation unit criteria is meet 100% of the time for Cases 1,2,3 and 4. Cases 5, 8, 9 and 10 each has 1 point that fails, while Cases 6 and 7 perform poorly. However, of the 115 points included over the 10 cases, only 12 points fail the criteria, and 8 of those are accounted for by two cases (6 and 7). Therefore, even without fully constraining the data, the new method performs very well. This suggests that with full constraints, the new method should perform extremely well.

Conclusions

The results given above support the following conclusions:

1. The new method provides accurate results without the necessity of hand-correction. This means that Forbes’ Second Method can be generalized into a solution method that is independent of the talents of the analyst.
2. Overall, the new method meets Forbes’ 3-saturation units criteria 90% of the time.
3. The optimal values of the fitting parameters can deviate significantly from the theoretical values.

4. Full constraints should be implemented in this solution method should lead to even better results.

Table 3 - An Analysis of Additional Errors

Case	First +	Second +	First -	Second -	Highs	Lows	Zeros
1	0.0181	0.0053	0.0	0.0	6	0	1
2	0.0169	0.0134	-0.0106	-0.0068	4	5	1
3	0.0	0.0	-0.0047	-0.0042	0	2	6
4	0.0109	0.0064	0.0	0.0	3	0	7
5	0.5809	0.0	-0.0051	-0.0044	1	4	0
6	0.0636	0.0312	-0.0983	-0.0633	6	4	2
7	0.0568	0.0113	-0.0727	-0.0624	4	3	10
8	0.0282	0.0	-0.0800	0.0	1	1	12
9	0.2291	0.0	-0.0135	-0.0068	1	3	4
10	0.1746	0.0	-0.0266	-0.0019	1	2	21

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Forbes, P., 1994, “Simple and Accurate Methods for Converting Centrifuge Data into Drainage and Imbibition Capillary Pressure Curves,” The Log Analyst, July-August, pp.31-53.

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Forbes,P., Z.A.Chen and D.W.Ruth, 1994, “Quantitative Analysis of Radial Effects on Centrifuge Capillary Pressure Curves,” SPE 28182, SPE Fall Meeting, New Orleans.

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Acknowledgements

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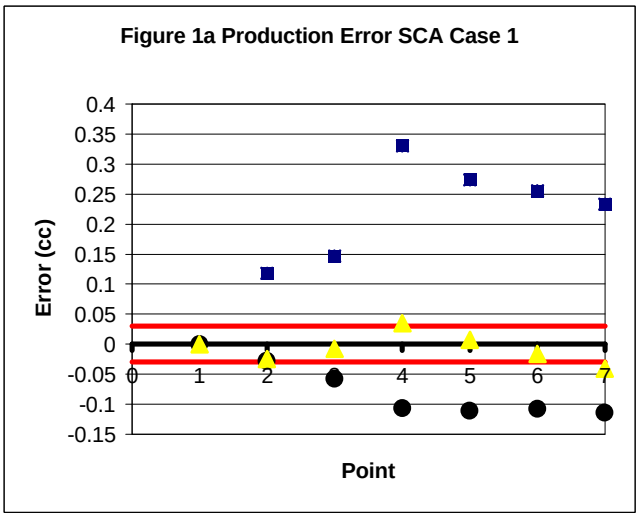
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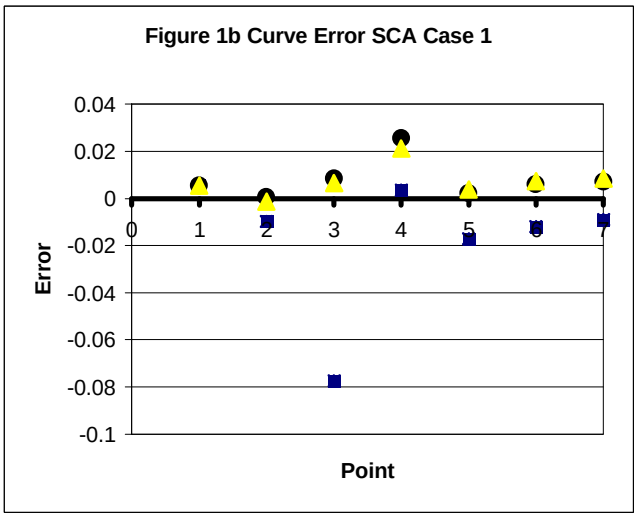
Figure 1b Curve Error SCA Case 1
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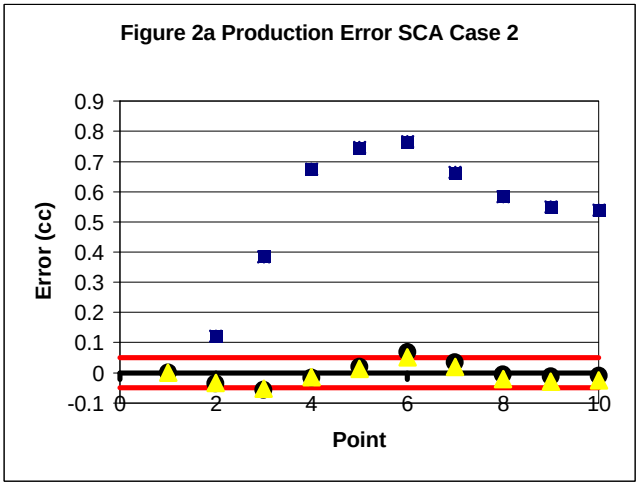
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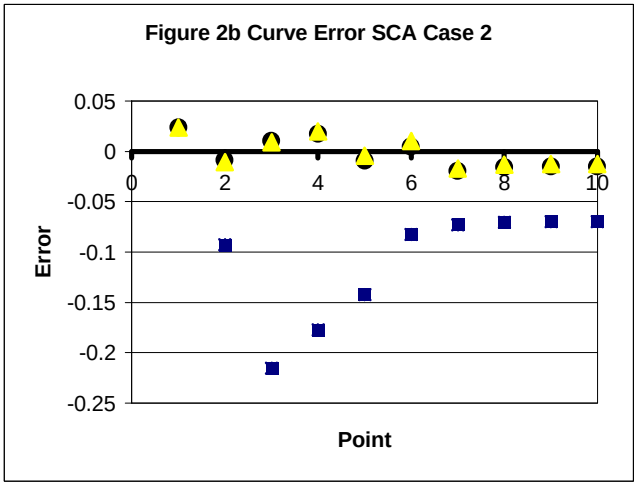
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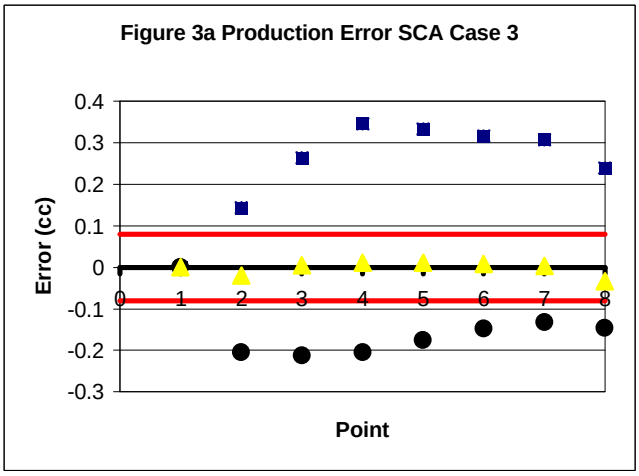
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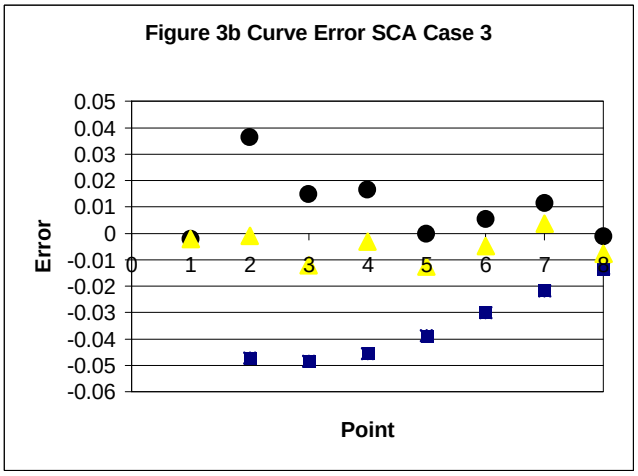
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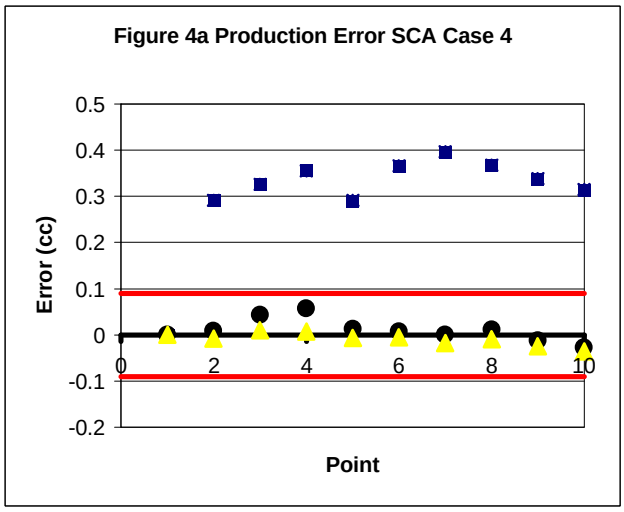
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Error (cc)
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0
2
4
6
8
10
Point

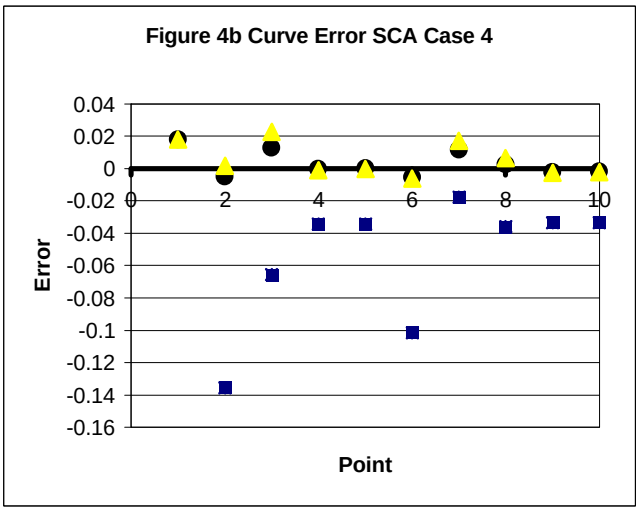
Figure 4b Curve Error SCA Case 4
Error
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
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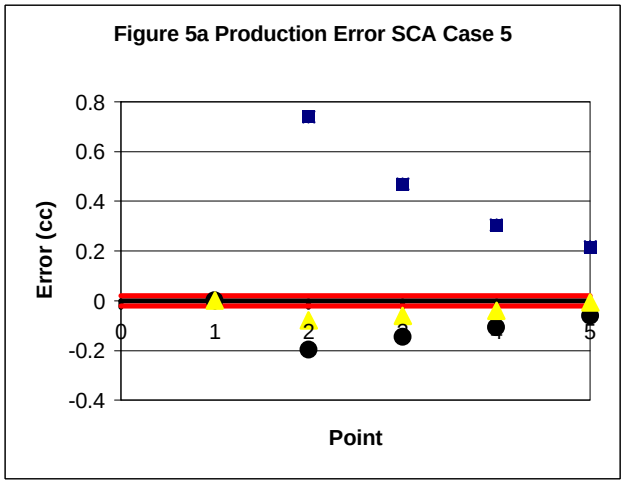
6
8
10
Point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Error (cc)
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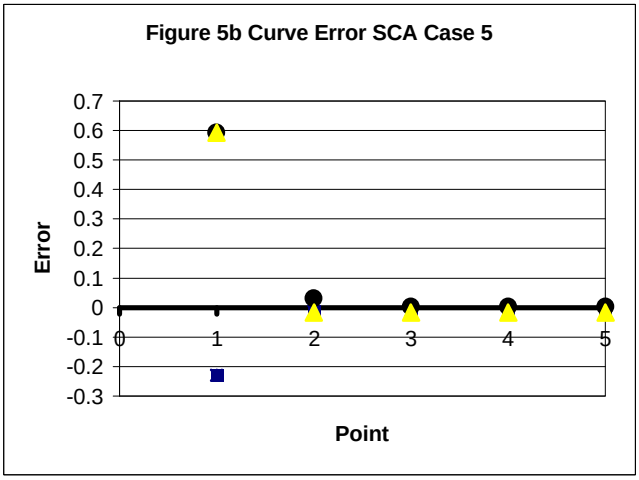
0.6
0.4
0.2
0
-0.2
-0.4
0
1
2
5
Point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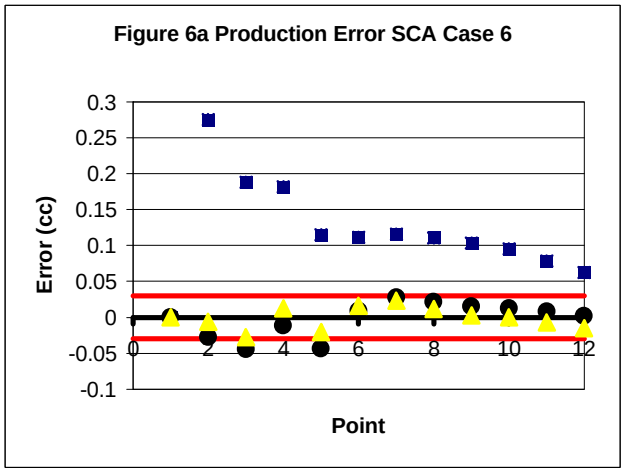
Error
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
1
2
3
4
5
Point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Error (cc)
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
0
2
4
6
8
10
12
Point

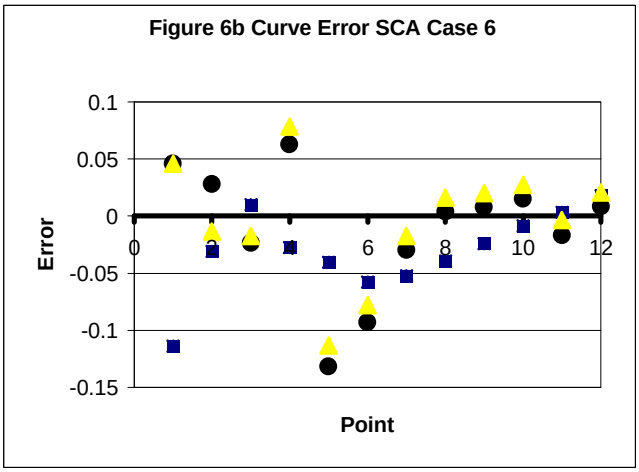
Figure 6b Curve Error SCA Case 6
Error
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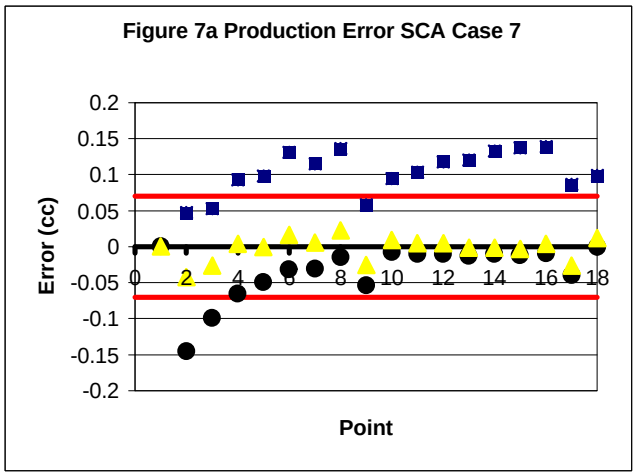
0.05
0
-0.05
-0.1
-0.15
0
6
8
10
12
Point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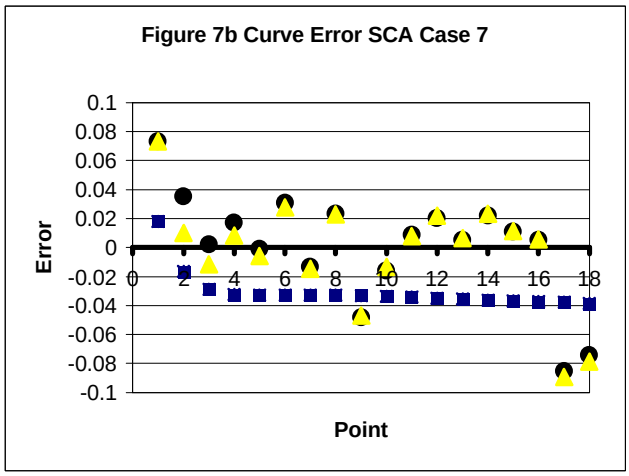
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Error
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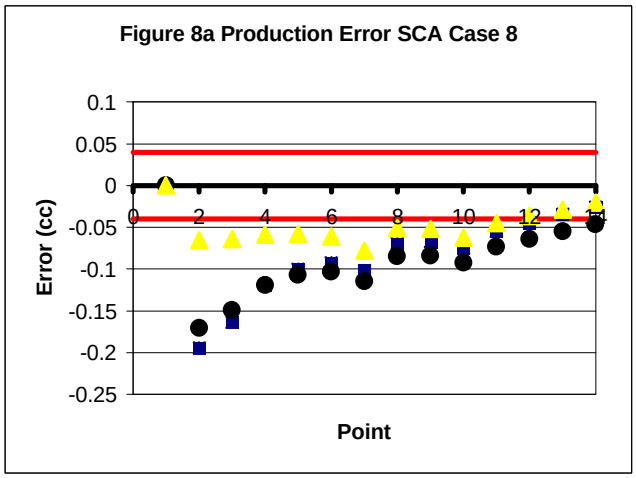
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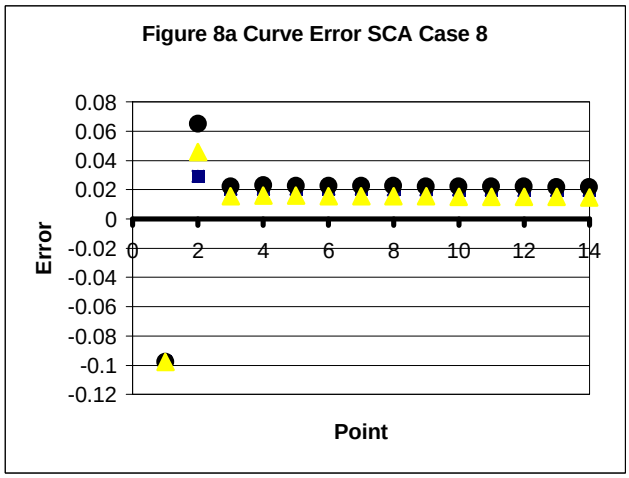
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Error
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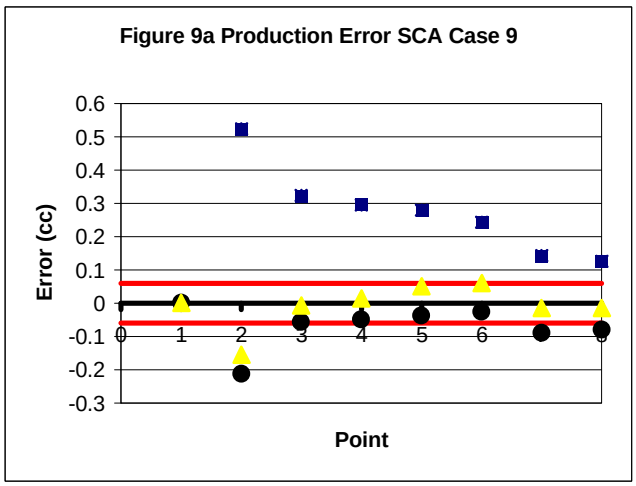
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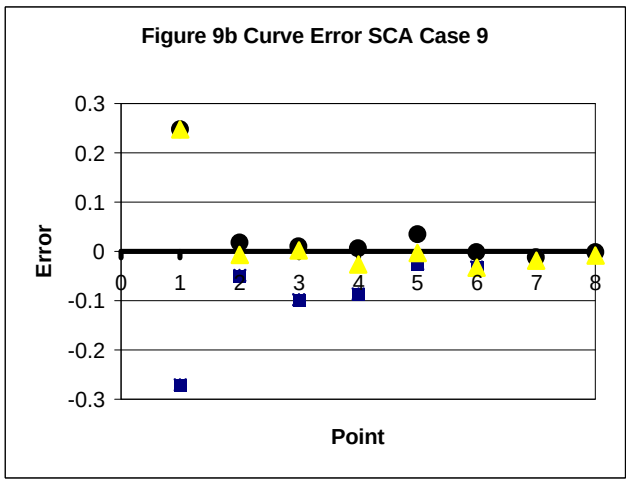
Figure 9b Curve Error SCA Case 9
Error
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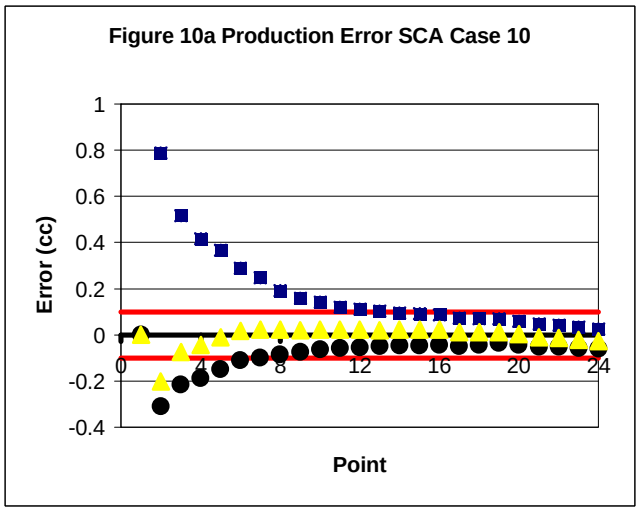
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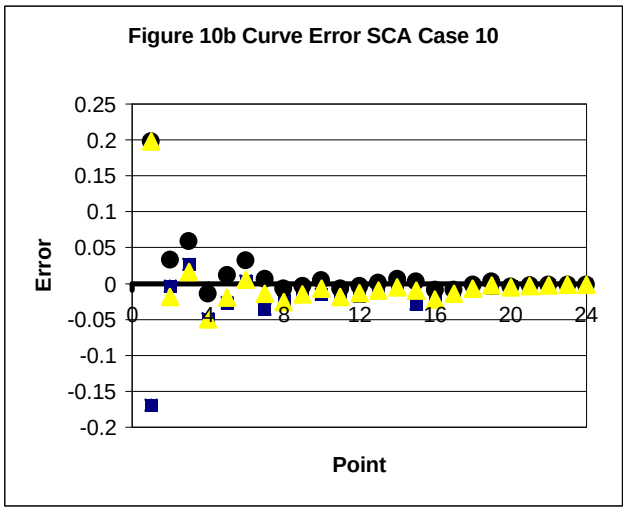
Error (cc)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
4
8
12
16
20
24
Point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Error
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0
4
8
12
16
20
24
Point