The effect of heterogeneity on unsteady-state displacements

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Abstract

In this paper, we discuss the effect of heterogeneity on unsteady-state core-level displacements. We show that if the results are interpreted as if the core were homogeneous, significant errors in calculation of both the relative permeability (Kr) and capillary pressure (Pc) can occur. We describe a series of high-resolution numerical simulations performed on heterogeneous permeability fields of various correlation lengths. Pressures and fractional flows were calculated at the ends of the core, as well as saturations along the length of the core, mimicking in-situ measurements. For each grid block in the high-resolution simulation, simple Corey exponent Kr curves were used and Pc was set to zero. First, the results are interpreted as if the sample were homogeneous using JBN analysis. With this approach, it is impossible to fit the saturation profiles within the core. This is due to the effects of heterogeneity and the inability for a single Kr curve to capture the smearing of the invading front at all length scales. The saturation profiles can be fitted by history matching only when a pseudo-Pc is included, even though at the local scale Pc=0. Surprisingly, this numerical pseudo-Pc has the same shape as a real laboratory capillary pressure curve. Finally, we show how tracer tests on core samples can be used to quantify the effect of heterogeneity and decouple the effects of heterogeneity from the effects of petrophysical properties.

Introduction

The unsteady-state method is probably the most popular method used to determine relative permeability (Kr). However, unlike the steady-state method, relative permeability using the unsteady-state method is calculated indirectly. Typically, one injects water into a core saturated with oil and measures the fluid production and the pressure drop across the core. With this data, the Kr of both the water and oil can be determined using the method presented in Johnson et al.[1] (JBN). However, in order to calculate Kr, JBN assumes that the flow is one-dimensional, in a homogeneous core, dominated by viscous forces (i.e. capillary and gravity forces are negligible). These assumptions are easily challenged when we consider reservoir rocks and reservoir displacement conditions. Heterogeneity is quite common in reservoir rocks, even at the scale of a core plug (see [2] and [3] among others). Huang et al. [2] measured the permeability of a laminated sandstone and found variations of order 100. Graue and Pederson [3] found correlation lengths of permeability near the length of the core. Capillary forces are often quite significant at the core scale. In addition, when the viscosity ratio between the injected and produced fluids is high, viscous fingering may occur. These factors can influence the calculation of relative permeability.

In this paper we focus on the effect of heterogeneity on unsteady-state displacements. This subject has been investigated previously by several authors. Huppler [4] performed a numerical investigation of two-dimensional, two-phase displacements. Huppler studied either layered media or fields with high or low permeability lenses. Relative permeability curves were calculated from pressure drop and fluid production data using JBN analysis. Differences between calculated relative permeability curves, and input relative permeability curves, indicated the influence of heterogeneity. Huppler found that when the permeability contrast was not high between layers or when lenses were sufficiently small and spaced apart, the JBN relative permeability did not significantly deviate from the input Kr. However, for large lenses or layered systems with high-permeability contrast, the resulting calculated Kr differed significantly from the input Kr, and were found to be dependant upon the length of the system, the flow rate, and wetting properties.

Chang & Mohanty [5] used the same procedure as Huppler to calculate Kr. In addition, Chang & Mohanty monitored the saturation profiles within the grid at different times. This information was used to determine whether the calculated Kr could adequately model the displacement within the grid. They found that when the contrast in permeability values was sufficiently weak, the Kr from the JBN analysis was very similar to the input Kr and the assumption of homogeneity was adequate. However, when the permeability constrast was strong, the JBN Kr differed from the input Kr for all values of correlation length. Pande et al. [6] examined purely layered systems (infinite correlation length) and found identical results. Note that instead of using a numerical simulator, Pande et al. employed analytical solutions. We will refer again to the work of Chang & Mohanty and Pande et al. below.

Mohanty & Miller [7] performed very detailed two-phase unsteady-state experiments using JBN analysis to calculate relative permeability curves. Cat scans were used to monitor the saturation profile during the waterflood. Mohanty & Miller found that even though the heterogeneity of the core was not obvious when examining the porosity map, cat scans during tracer experiments showed significant layering quite clearly. The saturation profiles during the waterflood were never found to be uniform within the core. Mohanty & Miller concluded that the unsteady relative permeability calculated by the JBN method was affected by fingering and heterogeneity until approximately one pore volume of fluid was injected. In the later part of the flood, the relative permeability calculated corresponded better with the assumptions behind JBN.

Procedure

Flow is considered in a cross-sectional, heterogeneous domain with an injector at one end and a producer at another to represent the inlet and outlet face of the core. Water is injected at a constant rate into a domain containing oil. Both fluids are considered incompressible.

Flow is performed on a series of heterogeneous permeability fields generated on a 250x100 size two-dimensional grid using sequential Gaussian simulation [8]. Two different correlation lengths were considered: λ=0.01, and 1.0 (lengths are normalized by the length of the grid in the flow direction). Figure 1 demonstrates how the fields differ between λ=0.01, and λ=1.0. The correlation length in the transverse direction was kept constant at 0.01. We present results primarily when the core has very strong contrasts in permeability, where the standard deviation σ^2 (lnK)=1. Similar studies were performed for and σ^2 (lnK)=0.1, and will be mentioned briefly.

In the simulations presented in this paper there is no capillary pressure. The relative permeabilities are Corey expressions using rescaled saturations, with no connate or residual saturations. Thus,

$$Krw = Sw^{\alpha}, \text{ and } \quad Kro = So^{\alpha}.$$

We show two cases, α=1 and α=2. The case of α=1 can be considered as a tracer case with no molecular diffusion or microscopic dispersion. The tracer case will be studied extensively as it is often a simple and efficient way to measure the heterogeneity of the permeability field. Results for an oil/water viscosity ratio (M) of 1 are presented. Runs were also made for M=5, and will be mentioned briefly below. In this paper, we will refer to the input Kr curves as the local Kr.

Numerical simulations were performed using 3DSL[1], a highly fast and accurate streamline simulator developed at Stanford University [9]. Simulation speed and precision were important for the results presented below. Often it was necessary to inject over 100 pore volumes in some cases, which would have been time-prohibitive using a standard reservoir simulator. Results from 3DSL have been shown to contain minimal effects from numerical diffusion which when using standard simulators were found to impact significantly the results.

We calculated Kr by measuring the flux at the output and the pressure drop between the injector and producer and employing the JBN method. Saturation profiles along the length of the core were generated by averaging the saturation along the transverse axis. In this paper, we will define the Kr calculated using the JBN procedure the effective Kr, or <Kr>. We subsequently substituted <Kr> into a simple 1D simulator using a discrete form of the Buckley-Leverett equation,

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[1] 3DSL is a registered trademark of StreamSim Technologies, Inc.

$$\frac{\partial S_w}{\partial t} + \frac{\partial f_w}{\partial x} = 0\,, \tag{1}$$

where *t* is pore volumes injected, *x* is dimensionless distance, *Sw* is the water saturation, and *fw* is the fractional flow of water (which is a function of Kr and the viscosities of the two fluids). From the 1D simulation, we stored the flux and saturation profile information. The fractional flow at the outlet and saturation profiles along the core were compared. Note that JBN analysis uses the Buckley-Leverett equation to calculate <Kr>. Since the 1D simulator uses the exact same equation to move the fluids, the resulting fractional flow must be reproduced. Any differences in the outlet fractional flow between the 2D simulation and the 1D simulation using <Kr> are due to errors in calculating <Kr> as well as numerical diffusion.

Results

Low Correlation Length

We first present results for λ=0.01. Figure 2(a) and (b) compare the local Kr and <Kr> for σ^2 (lnK)= 1, with α=1 and α=2, respectively. Note that the points, representing the calculations of Kr using JBN, do not correspond with the input Kr curves given to 3DSL. This is due to the heterogeneity of the permeability field. The effect of heterogeneity is also evident even for weak contrasts in K (σ^2 (lnK)= 0.1), shown in Figure 3(a) and (b).

Figure 4(a) and (b) compare the production of water at the outlet face for σ^2 (lnK)= 1. The results from the 2D simulations and 1D simulations with <Kr> are virtually identical, as expected. However, when we compare the saturation profiles within the core at different times (pore volumes injected – see Figure 5), the 1D and 2D simulations do not match. At later times, the curves appear to coincide. Yet at early times before breakthrough, the 2D profile shows greater spreading of the invasion front.

Figure 6(a) presents the results for σ^2 (lnK)= 1, α=2, and M=5. Note that <Kr> is much closer to the local Kr, compared to the same case with M=1. This result is consistent with other simulations with M=5.

High Correlation Length

When λ=1.0, the effect of heterogeneity is significant. The injected water channels through the core in the high permeability layers and breaks through rapidly at the outlet face. Figure 7(a) and (b) show <Kr> and the local Kr for σ^2 (lnK)= 1. The JBN curves show the effect of the channeling. Using the expressions of <Kr> within the 1D simulator, we reproduce the flow at the outlet, shown in Figure 8(a) and (b). When we compare the saturation profiles in Figure 9(a) and (b), the 2D simulations exhibit greater spreading of the invasion, similar to the cases for λ=0.01.

Figure 6(b) presents the results for σ^2 (lnK)= 1, α=2, and M=5. Although there is still a noticeable difference, <Kr> is closer to the local Kr compared to the case when M=1, seen in Figure 7(b).

Discussion

There is continuing discussion about the meaning of relative permeability for flow in heterogeneous porous media. Chang & Mohanty judged the "adequacy" of the relative permeability formulation by analyzing plots of water saturation and *x/t*, where *x* is distance of the displacement and *t* is time. Buckley-Leverett theory shows that Sw scales as *x/t* and thus plots of Sw vs *x/t* will fall on the same line. Chang & Mohanty found that except for fractal permeability distributions with high variance, the solutions scaled nearly as *x/t*, and were judged "Kr-formulation adequate." Figure 10 presents the same plots for the results found here. In general, the curves are similar. However, there is a systematic deviation of the curves for the 2D simulations with respect to the 1D simulations at different times. At early times, Sw shows greater spreading, which as time increases converges towards the profile for the 1D simulation. This result is consistent with the experimental results of Mohanty & Miller, who found that at early times heterogeneity and viscous forces affected their results more than at later times.

Using the JBN method, we have calculated the relative permeabilities for several unsteady-state displacements. Using <Kr> in a 1D simulation, we were able to reproduce the flux history at the outlet end of the core. However, we notice that the 1D simulation has not reproduced the saturation history within the core. This is due to the known problem of scaling of flow in heterogeneous porous media [10]. Heterogeneity can be identified as having two effects on the spreading of an invasion front: convective and diffusive. Hewett & Behrens [10] analyzed scaling in heterogeneous porous media for both miscible and immiscible flow. They demonstrated that neither a frontal advance equation (such as Buckley-Leverett) nor the convection-diffusion equation can correctly model flow in heterogeneous porous media at all scales. Pande et al. demonstrated that unless the flow was "strictly convective", frontal advance equations can only model the fractional flow and not the "local details" of the fluid composition. Flow was shown to be strictly convective only for homogeneous fields with Pc=0 or tracer flow in perfectly layered systems. Gelhar and coworkers [11] have shown that when the λ is small and σ^2 (lnK) ~1 or less, flow of tracer can be modeled using the convection-diffusion equation with an effective diffusion coefficient. However, Furtado and Pereira [12] concluded that, in practice, multiphase flow in heterogeneous permeability is neither purely diffusive nor strictly convective, but between the two. Thus, relative permeabilities that are calculated using JBN (or any other method) are only valid at the scale of the measurements.

Few approaches have been proposed to resolve this problem. Taggert et al. [13] have proposed a length-dependant pseudofunction technique based upon the Buckley-Leverett equation. They proposed a methodology to modify the effective Kr along the length of the displacement. This approach may be difficult in complex 2D or 3D flow. Another approach is to recognize that Darcy's law is no longer valid for scale-dependant flows. Lenormand & Fenwick [14] and Lenormand [15] have proposed a new formulation for the fractional flow of an injected fluid,

$$F = F_c - F_d \frac{\partial S}{\partial x}, \qquad (2)$$

where *Fc* and *Fd* are functions of saturation, heterogeneity, and fluid properties, but not on the scale of the displacement. *Fc* and *Fd* represent the convective and diffusive terms found in the standard transport equation. Glimm et al. [16] proposed a similar expression, but it does not have as general an expression as Eq. 2. Note that the Buckley-Leverett equation is found when $Fc=fw$ and $Fd=0$ (no diffusive term). Using Eq. 2, the saturation profile within the core as well as the flux at the outlet can be reproduced. Lenormand [14] derived semi-analytical expressions for *Fc* and *Fd* for tracer flow within various types of heterogeneous permeability fields. In the context of multiphase flow, *Fd* can be shown to be equivalent to a pseudo capillary pressure, even though for tracer flow Pc=0. Figure 11 presents pseudo-Pc curves for tracer experiments for various types of heterogeneity.

Relative Permeability in Heterogeneous Porous Media

We have noted that even for small correlation lengths and small variance of permeability, <Kr> differs from the local Kr (see Figure 3). For non-tracer displacements, the effective water relative permeability curves all show similar characteristics: at low water saturation <Kr> deviates from the local Kr and is approximately equal to <Kr> when $\alpha=1$, but at high water saturation we find <Kr>~Kr (see Figure 12). Based upon this visual observation, we propose a new model of Kr for flow in heterogeneous porous media. For the injected fluid (water), we propose

$$\langle K_{rw} \rangle = K_{rw}^{tracer}(1 - S_w) + K_{rw}^{local}(S_w). \qquad (3)$$

The effective oil relative permeability curves are similar to Corey exponent curves. Thus, we propose a similar expression,

$$\langle K_{ro} \rangle = K_{ro}^{\left(\alpha_t - 1 + \alpha_l\right)} \qquad (4)$$

where α_t is the exponent that best matches <Kro> from the tracer experiment, and α_l is the exponent of the local Kr. Figure 13 compares the prediction of <Kr> with those calculated using JBN. The predicted curves are not exact, but match well the trends. Perhaps the best use of Eqs. 3 and 4 is to obtain an approximate expression for the local Kr. The tracer test can thus be used in a quantitative fashion to measure the effect of heterogeneity and (in conjunction with a multiphase core flood) back-calculate the local Kr. Thus, the effects of heterogeneity can be decoupled from the effects of petrophysical properties.

Conclusions

In this paper, we have shown several fine-grid simulation results of unsteady-state displacements in heterogeneous porous media. The results clearly show that even low correlation length and small variances in the permeability field can have an effect on calculated relative permeabilities. If the calculated relative permeabilities are used in a one-dimensional numerical simulator employing the Buckley-Leverett equation, we show that the results are strictly valid only at the length at which the measurements were obtained. In order to model the flow at all length scales using the standard multiphase flow formulation, either the relative permeability must be a function of distance, or a pseudo-capillary pressure must be introduced.

For tracer flow, a general expression is proposed that can model the flow at different length scales. From these solutions, a pseudo-Pc is presented that resembles standard Pc curves. Note that the pseudo-Pc is non-zero, even when at the local scale Pc=0.

We have observed that the difference between the effective Kr and the local Kr diminishes when we increased the mobility ratio. Although this increases the possibility of viscous instabilities that cause fingering, this approach merits further investigation as a way to determine the local Kr in a heterogeneous core.

A new expression of Kr is presented for flow in heterogeneous permeability fields. This form is based upon visual observations of the simulation results. The new expression uses tracer experiments quantitatively to decouple the effects of heterogeneity and from the effects of petrophysical properties.

Figures

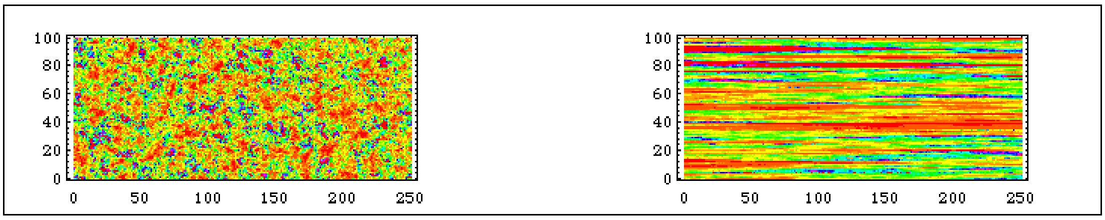

Figure 1 : Examples of the two correlation lengths used in this study : (a) λ=0.01, (b) λ=1.0.

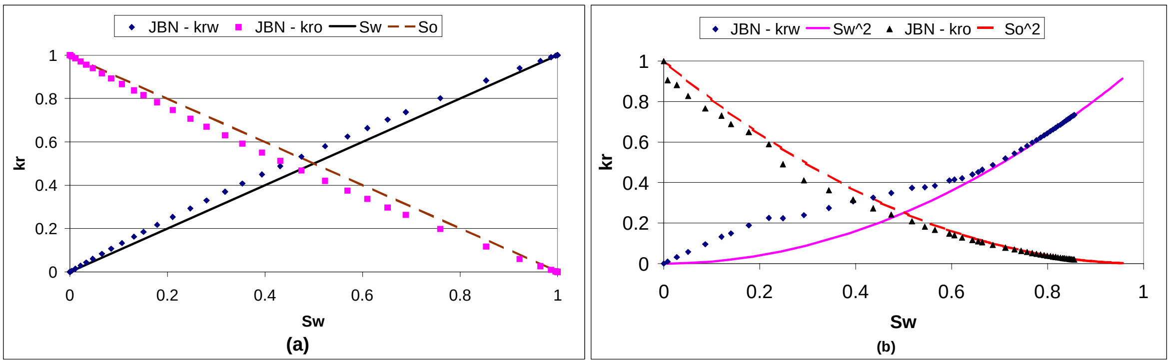

Figure 2: Local and effective Kr for λ=0.01 and $\sigma^2(\ln K)=1.0$. (a) Tracer case, (b) α=2.

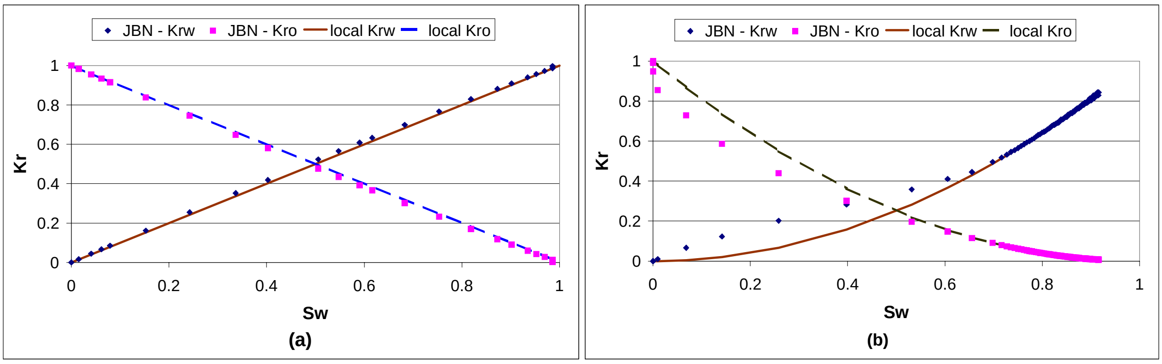

Figure 3: Local and effective Kr for λ=0.01 and $\sigma^2(\ln K)=0.1$. (a) Tracer case, (b) α=2. Note the effect of heterogeneity even for small permeability variance.

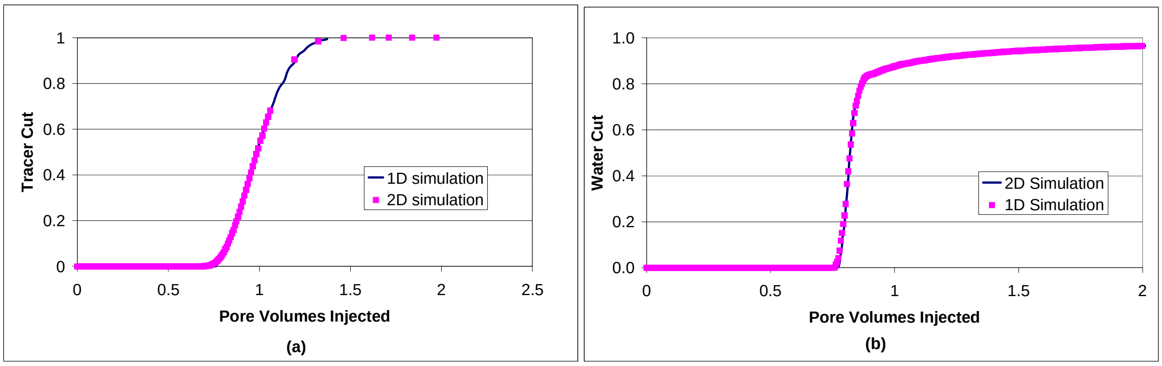

Figure 4: Flux for λ=0.01 and $\sigma^2(\ln K)=1.0$. Comparison between 2D simulations on heterogeneous porous media using local Kr and 1D simulations with homogeneous permeability using Kr from JBN: (a) Tracer case, (b) α=2.

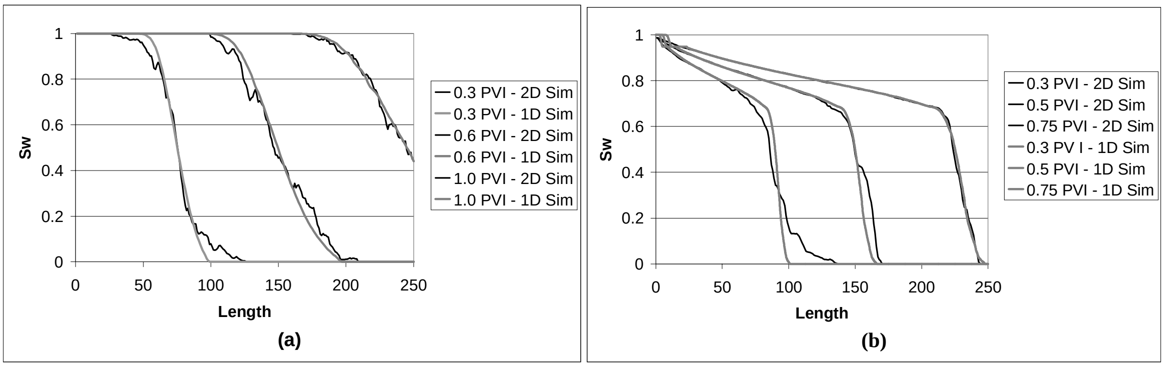

Figure 5 : Saturation profiles for λ=0.01 and $\sigma^2(\ln K)=1.0$. Comparison between 2D simulations on heterogeneous porous media and 1D simulations with homogeneous permeability using Kr from JBN: (a) Tracer case, (b) α=2. The 2D curves show greater spreading of the water front until breakthrough (at distance 250).

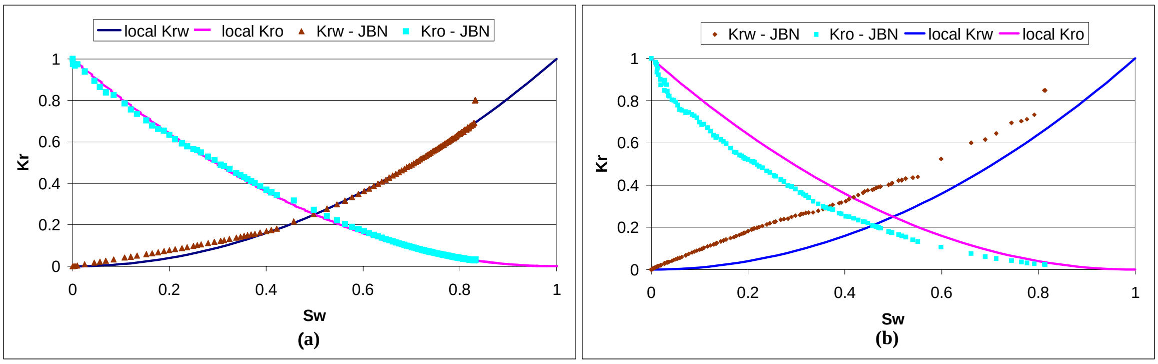

Figure 6 : Local and effective Kr for λ=0.01 (a) and λ=1.0 (b) and $\sigma^2(\ln K)=1.0$. The viscosity ratio (M) is 5, and the Corey exponent is 2. The JBN Kr curves are closer to the local curves when M=5.

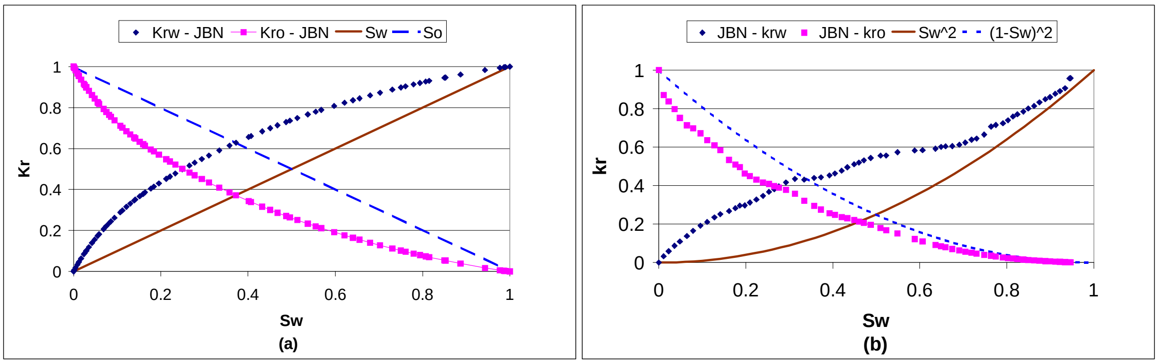

Figure 7 : Local and effective Kr for λ=1.0 and $\sigma^2(\ln K)=1.0$. (a) Tracer case, (b) α=2. The correlated heterogeneity significantly affects the Kr curves calculated by JBN.

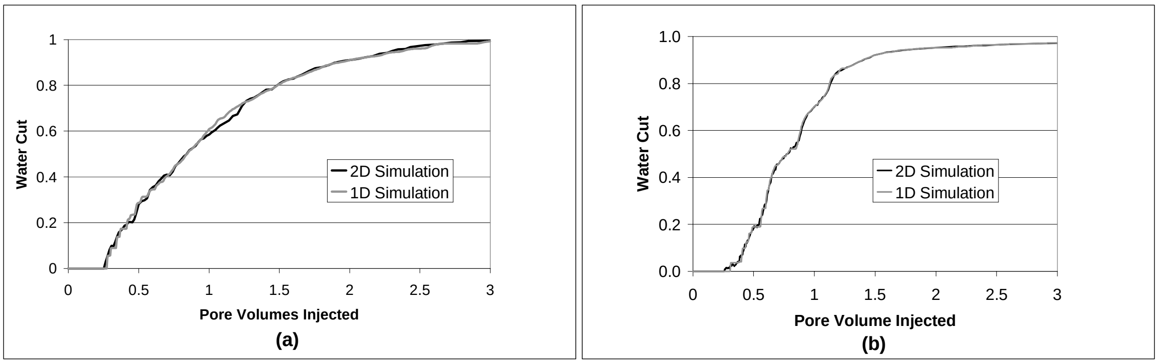

Figure 8: Flux for λ=1.0 and σ^2(lnK)=1.0. Comparison between 2D simulations on heterogeneous porous media using local Kr and 1D simulations with homogeneous permeability using Kr from JBN: (a) Tracer case, (b) α=2.

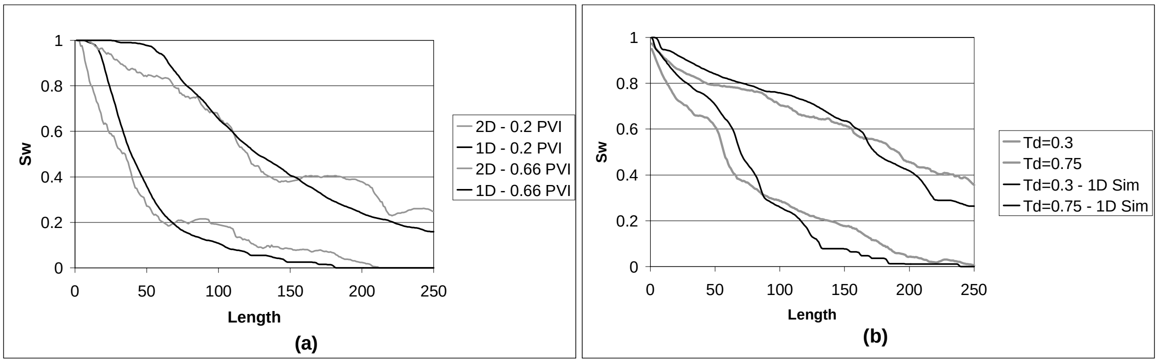

Figure 9 : Saturation profiles for λ=1.0 and σ^2(lnK)=1.0. Comparison between 2D simulations on heterogeneous porous media and 1D simulations with homogeneous permeability using Kr from JBN: (a) Tracer case, (b) α=2. The 2D curves show greater spreading of the invasion front.

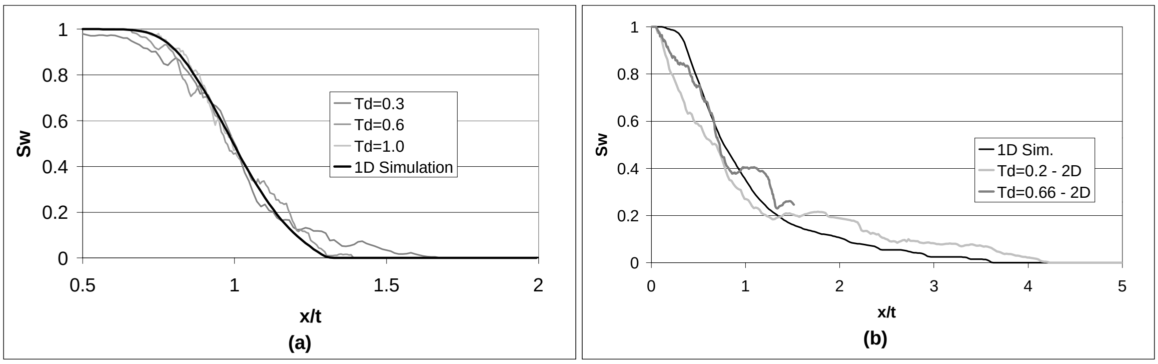

Figure 10: Water saturation as a function of x/t for σ^2=1.0 : (a) λ=0.01, (b) λ=1.0. The effect of heterogeneity is clear for λ=1.0. Note that at early times the saturation front shows greater spreading, whereas at later times the results are more consistent with the 1D simulation.

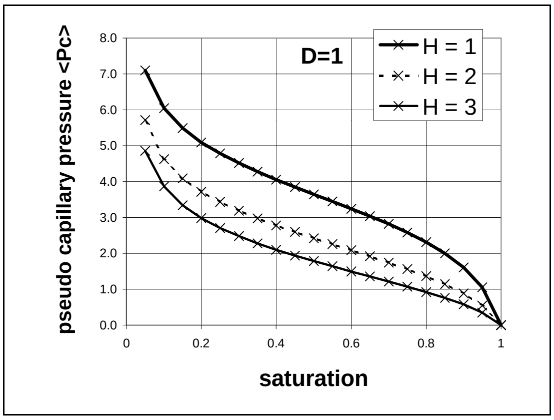

Figure 11 : Pseudo capillary pressure determined by Lenormand [14] for tracer flow under various types of heterogeneity. When H=1, the heterogeneity is uncorrelated. When H=3, the heterogeneity is strongly correlated.

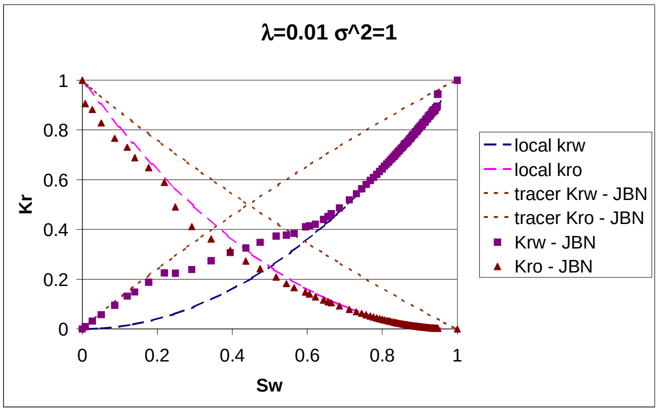

Figure 12 : Comparison of Kr curves for λ=0.01 and σ^2(lnK)=1.0. Note how at low Sw the Krw from JBN follows the Krw curve from the tracer experiment, but at high Sw the JBN Krw is approaches the local Krw.

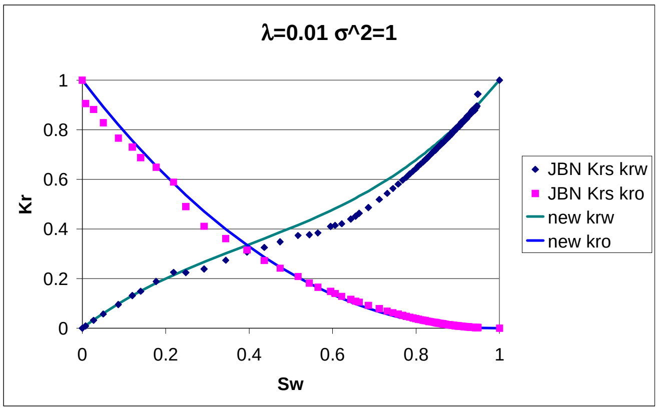

Figure 13 : Prediction of <Kr> for λ=0.01 and σ^2(lnK)=1.0 with α=2 using Eqs. 3 and 4.

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