

MODELING OF CAPILLARY PRESSURE FOR HETEROGENEOUS RESERVOIRS BY A MODIFIED J-FUNCTION

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ABSTRACT

Initial fluid distribution for heterogeneous reservoirs i.e. layered reservoirs with different permeability and porosity is often modeled by simplified capillary J-function for input to reservoir simulation. Leverett introduced the model in 1941 and showed that dimensionless capillary J (S_w)-function varies only with fluid saturation. However, theories of capillary pressure indicate that Leverett J-function should also be a function of tortuosity, irreducible liquid saturation and pore size distribution. As a result, scaling of capillary pressure by Leverett J-function should be limited to the formations whose pore size distribution and irreducible liquid saturation are equal.

The pore size distribution is likely to affect the shape of the drainage capillary pressure (P_c) - saturation curve. Therefore if pore size distribution is different in two formations, saturation exponent, a key parameter for the drainage P_c curve is likely to be different. The effect of tortuosity on J-function is not pronounced since its value does not change too much. However, if Leverett J-function is used to obtain P_c -saturation functions for different formations whose irreducible liquid saturation and the saturation exponent are different, then significant saturation discrepancy might arise in the scaled P_c -functions.

It is also observed that irreducible liquid saturation is a function of specific surface area, wettability and interfacial tension of the rock-fluid system. Therefore scaling of P_c by Leverett J-function may provide incorrect P_c -saturation value at low saturation range even for the similar rocks but whose fluid systems are different. In this paper, we define a modified J-function that includes tortuosity, irreducible liquid saturation and saturation exponent so that the modified J-function may be used for scaling of P_c -saturation functions for any formations. We also propose auxiliary equations for estimating tortuosity, irreducible liquid saturation and saturation exponent from permeability and porosity data that may be used to calculate the modified J-function.

INTRODUCTION

Fluid distributions in heterogeneous reservoirs depend on many factors. Lee¹ discussed the importance of temperature, fluid composition, phase pressures, geothermal gradient, fault, rock-fluid wettability as well as history of fluid trapping as potential elements for distributions of fluid in reservoirs. Alpak *et al*² theoretically showed how the capillary pressure might vary in porous media due to different interfacial area arises out of fluid saturation and pore structure. However, for all practical purpose it is assumed that drainage capillary pressure may define the initial fluid saturation for a given rock-fluids system. The variations of rock type and fluid system however, generate different capillary pressure saturation functions.

In 1941, Leverett³ proposed a dimensionless capillary J-function with an intention to eliminate the effect of rock heterogeneity i.e. effect of porosity and permeability variations from the capillary pressure-saturation function. Since then it is widely used as a scaling function for capillary pressure modeling. However, Melrose⁴ found that J-function could not scale capillary pressure at lower saturation typically below 20 percent even for Berea cores that are usually assumed homogeneous. Phelps⁵ also found lithology dependent J-function for heterogeneous reservoirs. El-khatib⁶ showed that Leverett J-function is not only a function of wetting fluid saturation but also a function of tortuosity, saturation exponent (a parameter for drainage capillary pressure related to pore size distribution of the porous media) and irreducible wetting fluid saturation. Lake⁷ also pointed that variation of pore size distribution will affect the J-function.

THE THEORY OF J-FUNCTION (LITERATURE REVIEW)

In 1941, Leverett investigated the effect of fluids interfacial tension (σ) and rock properties i.e. permeability (k) and porosity (ϕ) on capillary pressure. Based on experiments, he proposed following dimensionless group as a function of wetting fluid saturation (S_w) for capillary pressure (P_c) modeling

$$J(S_w) \equiv \frac{P_c}{s \cos \theta} \sqrt{\frac{k}{f}} = f(S_w) \dots (1)$$

Here, θ = contact angle, a property of rock-fluid system. $f(S_w)$ = a saturation function.

However, it has been reported in the literature^{1,2,5} that J-function often failed to scale P_c correctly for heterogeneous reservoirs. The possible reasons may be attributed to complex lithology, and the variation of fluid composition in the reservoirs.

Using simplified porous model such as bundle of capillary tubes, fundamental fluid flow equation for capillary tube (Hagen-Poiseuille) and Young-Laplace equation for capillary pressure, El-khatib⁶ showed that permeability (k) may be expressed in terms of porosity (ϕ), tortuosity (τ), contact angle (θ), wetting fluid saturation (S_w) and P_c .

$$k = \frac{f}{2t} (s \cos \theta)^2 \int_0^1 \frac{dS_w}{P_c^2} \dots (2)$$

For analytical purpose, we will assume that the following form of saturation function can model the drainage P_c . The sole purpose of assuming a P_c function is to show that Eq.1 is not a function of saturation alone. Therefore any other form of P_c -saturation function will reveal the same.

$$P_c = \infty, 0 \leq S_w \leq S_{wir}; \quad P_c = \frac{a}{\left(\frac{S_w - S_{wir}}{1 - S_{wir}} \right)^n} \quad S_{wir} < S_w \leq 1 \quad \dots (3)$$

Here, a , n and S_{wir} are matching parameters for a P_c -saturation function. However, each of these parameters indicates some properties of the porous media. For example parameter "a" indicates the

minimum required capillary entry pressure that in turn provide the maximum pore size of the given rock. S_{wir} , the irreducible wetting fluid saturation indicates the specific surface area of the given rock. The exponent "n" indicates the pore size distribution pattern. Now, substituting Eq.3 in Eq.2, one can express permeability (k) as:

$$k = \frac{f}{2t} (s \cos q)^2 \frac{1}{a^2 (2n+1)} (1 - S_{wi}) \dots (4)$$

Dividing Eq.3 by Eq.4 and using the identity of Eq.1, J-function can be expressed:

$$J = \frac{\sqrt{1 - S_{wir}}}{\sqrt{2t \cdot (2n+1)}} \frac{1}{\left(\frac{S_w - S_{wir}}{1 - S_{wir}} \right)^n} \dots (5)$$

Now if a modified J-function is defined such that J^* equal to,

$$J^* = J \cdot \sqrt{\frac{2t \cdot (2n+1)}{1 - S_{wir}}} \dots (6)$$

Then J^* may be written in normalized saturation, $S_w^* \equiv (S_w - S_{wir}) / (1 - S_{wir})$ as:

$$J^* = \frac{1}{(S_w^*)^n} \dots (7)$$

Eq.7 shows that J^* is a function of S_w^* and the saturation exponent "n". Now, if we know " τ ", "n" and S_{wir} , the scaling of Pc may be accomplished using Eq.1 and Eq.6. In the following section we would discuss a solution procedure for Eq.7.

SOLUTION PROCEDURES FOR MODIFIED J-FUNCTION

The objective of the modified J-function is to take into account the effect of tortuosity, irreducible liquid saturation and pore size distribution when Pc-saturation function is scaled from one formation to another. However, the unknown Pc-saturation function may be scaled only if k, ϕ and $(\sigma \cdot \cos \theta)$ are known in both formations. Note that in order to calculate J^* for a formation whose Pc-saturation function is known, one do not need to calculate τ, n and S_{wir} because n and S_{wir} can be estimated from Pc -function (Eq.3) and the right hand side of Eq.7 may then be used to calculate J^* . Using different n-values, family of J^* vs. S_w^* can be plotted (see Fig.1), which may then be used to equate J^* if n can be estimated for the unknown system. If S_{wir} can also be estimated for the unknown system, the actual saturation may be calculated from the following:

$$S_w = S_{wir} + (1 - S_{wir}) \cdot \left(\frac{1}{J^*} \right)^{1/n} \dots (8)$$

Now, one can solve Eq.6 for Pc if τ can also be estimated for the unknown system. By doing the above, one can obtain a pair of (Pc, Sw) for the unknown system. The calculation process can be repeated with different values of J* in order to obtain full Pc-saturation function for the unknown system. In the following section we would discuss how one may estimate n, S_{wir} and τ .

Irreducible water saturation (S_{wir})

Specific surface area (S_V) of the porous media is expected to be directly proportional to the irreducible water saturation. The S_V , which is defined as total surface area per unit flow volume, may be related to the characteristic size (C_S) and the porosity (ϕ) of the packing. The C_S may be defined as the ratio of volume to surface area of a single grain, for example the C_S equals to $D_p/6$ for a sphere of size D_p . If spheres of regular size constitute the porous media, the S_V may then be written for this type of packing as:

$$S_V = (1-f)/fC_S \quad \dots(9)$$

In real porous media, the S_V is difficult to calculate due to the presence of different sizes of grains. However, an equivalent constant C_L that may be considered as characteristic length for a particular porous media may be used to replace C_S in Eq.9. The irreducible water saturation S_{wir} may then be written as follows since S_V is proportional to S_{wir} .

$$S_{wir} = (1-f)/fC_L C_W \quad \dots(10)$$

Where, C_W is a proportional constant and may be calculated dividing S_V by S_{wir} . The term $(1-\phi)/(C_L.C_W)$ in Eq.10 may be replaced by another constant C_R that depends on the sedimentation process and the characteristic length of the porous media. In other words, C_R will depend on the permeability of the porous media. Considering the above arguments, S_{wir} may be written as:

$$S_{wir} = \frac{C_R}{f} \quad \dots(11)$$

Based on experimental observations from their own and from the other published sources, Sanyal¹² and Homes¹³ *et al.* also concluded the above relationship for S_{wir} . One apparent problem of Eq.11 comes from the fact that for many reservoirs, variation of ϕ is insignificant compare to S_{wir} . Therefore, in order to estimate S_{wir} from Eq.11, C_R must be estimated very accurately for different reservoir zones. Intuitively one may consider that value of C_R will increase if specific surface area S_V increased. In other words, if specific surface area changes, permeability is likely to change and C_R may then be estimated as a function of permeability:

$$C_R \equiv S_{wir} \cdot f = f(k) \quad \dots(12)$$

Where, $f(k)$ is a function of permeability. In order to obtain $f(k)$, we plot $(S_{wir} \cdot \phi)$ vs. k (see Fig.2). The most of the data were taken from literature⁹⁻¹¹ and the rest were taken from our own experiments. Note that we considered only air/N₂-water system and did not plot any data from carbonate samples. A hyperbolic decline function shown below was fitted to data (see Fig.2).

$$f(k) = \frac{a \cdot b}{b + k} \dots (13)$$

Where, α = constant, regressed value = 0.1683, β = constant, regressed value = 96.93

Now, S_{wir} may be calculated from Eq.12 replacing $C_R = f(k)$.

Irreducible liquid saturation vs. Interfacial tension (S)

To address the effect of interfacial tension on irreducible liquid saturation, we conducted experiments on four Berea samples (B1, B2, B3 and B12). First the water-saturated samples were centrifuged to determine S_{wir} then Pc-saturation functions were determined by directly monitoring the saturation^{14,15} for the same samples using a different fluid system (Air-C₁₉). From the Pc-saturation curve, we determine irreducible liquid (C₁₉) saturation for each sample. The irreducible liquid (C₁₉) saturation was scaled by a factor = $\sigma(\text{air-water: } 72 \text{ mN/m}) / \sigma(\text{air- } C_{19} : 22.3 \text{ mN/m})$ and then it was plotted against the corresponding S_{wir} (see Fig.3). We found almost a diagonal relationship between the two. Therefore, we conclude irreducible liquid saturation for one fluid system may be scaled from another fluid system by the ratio of interfacial tension.

Tortuosity (t)

The tortuosity (τ) may be expressed in terms of the product of porosity and formation factor^{11, 16}. Formation factor F may be defined as ratio of electrical resistivity of water saturated porous medium (R_o) to that of the water alone (R_w). Thus τ may be written as:

$$t = \frac{R_o}{R_w} \cdot f = F \cdot f \dots (14)$$

The development of Eq.14 is based on simplified tortuous path; therefore it is expected that Eq.14 needs modification for its application to the porous media. Amyx¹¹ *et al* reported a correction for Eq.14 based on experimental works of others, which may be written as:

$$t = (F \cdot f)^{1.2} \dots (15)$$

There are several electrical resistivity models reported in the literature¹⁷ for various formations type. In order to calculate τ from Eq.15, one can either directly measure F or select a model depending on the rock type. For simplicity, we would consider Archie's resistivity model¹⁸ so that F may be written as:

$$F = c \cdot f^{-m} \dots (16)$$

Where, m= cementation exponent and c= constant. The value of c is considered to be 1 from the fact that when $\phi=1$, then $F=1$, therefore $c=1$. However, a lower value of "c"(say 0.81) is also reported for some formation⁸. Thus, considering $c=1$, τ may be expressed:

$$t = f^{-1.2 \cdot (m-1)} \dots (17)$$

Dullien¹⁶ and Sanyal¹² reported a range ($1.3 \leq m \leq 2.5$) for "m" therefore, one can calculate τ from Eq.17 if cementing property of the formation can be estimated qualitatively. For an approximate estimation of "m", Fig.4 was constructed based on the experimental "m" values¹¹. Note that cementing ranking of the formation is arbitrary i.e. we refer reservoir rank1 unconsolidated while rank10 may be considered highly cemented.

Saturation exponent "n"

For a given lithology, the product of irreducible liquid saturation (S_{wir}) and the saturation exponent "n" may be expressed as function of permeability (k) and porosity (ϕ). At a given k and ϕ , the effect of interfacial tension on "n.S_{wir}" is likely to be insignificant since it is expected that when irreducible liquid saturation increases, the saturation exponent "n" decreases proportionally.

In order to determine "n.S_{wir}" with respect to k and ϕ , we conducted experiments in macro and micro scale to obtain necessary data. In macro scale, Pc-saturation functions (oil-brine, σ :36 mN/m) were determined for 11 samples (v2k/1-v2k/11; see Table1 for basic data) by porous plate method. The samples were taken from various depth of a North Sea field. In micro scale, Pc-saturation functions (C₁₉-air, σ :22.3 mN/m) were determined by saturation monitoring¹⁵ for three more samples (v128, v129 and v130) from the same North Sea field but from different formations (see Table2 for basic data). Some Pc data from various fluid systems were also taken from literature^{8-11,14} to validate the assumption that effect of interfacial tension is insignificant with respect to n.S_{wir} vs. k, ϕ relation.

The porous plate and reported literature data are plotted in Fig5-6 while micro scale experimental data are plotted in Fig7-9. Note that n.S_{wir} are plotted against the so called reservoir quality index i.e. $(k/\phi)^{0.5}$. An exponential decay function of the following form was fitted to each group of data.

$$n.S_{wir} = I.e^{-\lambda \left(\frac{k}{\phi}\right)^{0.5}} \dots (18)$$

Note that the parameters λ and γ (Eq.18) are not constant (Fig5-9). Therefore, we suggest when Pc scaling is conducted in accordance with Eq.6-7, at least couple of full Pc-saturation functions are determined so that λ and γ may be estimated satisfying the known pair of "n, S_{wir}, k and ϕ ".

EXPERIMENTAL PROCEDURES: MICRO SCALE

Very fine (mm) scale porosity (ϕ) was measured (see Fig.10) for samples v128, v129 and v130 by gamma absorption (see *Appendix: Micro scale porosity*). Initially an attempt was also made to measure permeability distribution along the length of the samples by a mini permeability meter¹⁹. Failing to design an appropriate air injection nozzle, we could not use the instrument for permeability measurement in mm scale. However, the mm scale permeability (k) was determined from ϕ -k correlation obtained by collecting large number of field porosity-permeability data (see Fig.11).

Capillary pressure-saturation functions were determined for these samples in two different methods- *Hassler-Brunner*²⁰ and *saturation monitoring*^{14,15}. Note that in the *Hassler-Brunner method*, each sample was cut into two pieces (1"x1" approximately) before they were put into the centrifuge (Beckman L8-55M/P). The data set {*centrifuge speed, average liquid saturation*} from each piece (total six; each sample consisting two pieces) was interpreted with radial correction²¹.

From *Hassler-Brunner* type experiment, average irreducible liquid saturation and average saturation exponent "n" was determined for each sample by regression (see Fig.12a, 13a and 14a) while effect of mm scale heterogeneity (k - ϕ variations) on the P_c -saturation functions were captured by saturation monitoring method (see Fig.12b, 13b and 14b). For details of saturation monitoring method see *Appendix: Capillary pressure-saturation function by saturation monitoring*. From Fig.12a-b, 13a-b and 14a-b, it can be seen that at a given P_c value, there is significant difference in saturation values between the *Hassler-Brunner* and *saturation monitoring* methods. The reason for these differences may be attributed to the local heterogeneity in mm scale.

For each of the P_c -saturation functions shown in Fig.12b-14b, Eq.7 was solved for saturation exponent "n" using the local P_c , k , ϕ , S_{wir} , τ and σ data. Note that some local P_c data points were discarded due to no solution for Eq.7.

CONCLUSIONS

1. The limitation of Leverett J-function is investigated and it has been shown that Leverett J-function is suitable for capillary pressure-saturation (P_c - S_w) function scaling from one formation to another if lithology (i.e. pore size distribution, irreducible liquid saturation and tortuosity) of the two formations is same.
2. To overcome the limitation of Leverett J-function, a modified J-function is defined which includes pore size distribution effect in the form of saturation exponent, irreducible liquid saturation and tortuosity.
3. Solution procedures for estimating saturation exponent, irreducible liquid saturation and tortuosity have been discussed those may be used to calculate modified J-function.
4. When lithology of the two formations are different, scaling of P_c -function by modified J-function is expected to provide reliable saturation values for the scaled P_c -function than the Leverett J-function.

NOMENCLATURE

a = Capillary entry pressure (kPa), constant for a given system (Eq.3)
 c = Constant (Eq.12)
 C_R = a function of permeability (md) (Eq.15)
 F = Formation factor (Eq.10)
 J = Leverett J-function (Eq.1)
 J* = Modified J-function (Eq.6)
 k = Permeability (md)
 m = Cementation exponent (Eq.12)
 n = Saturation exponent (Eq.3)

P_c = Capillary Pressure (kPa)
 R = Resistivity (ohm), subscript -o: Formation, w= water
 S = Saturation, subscript-w: wetting phase, wir =wetting phase irreducible
 α = Constant, β = constant (Eq.16)
 σ = Interfacial tension (mN/m)
 θ = Contact angle
 τ = Tortuosity
 ϕ = Porosity

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Table-1: Basic data for v2k/1-v2k/11

k	ϕ
188	0.246
259	0.258
41.2	0.217
1630	0.262
1220	0.256
1710	0.262
0.532	0.154
2201	0.287
33.9	0.211
840	0.239
44.8	0.246

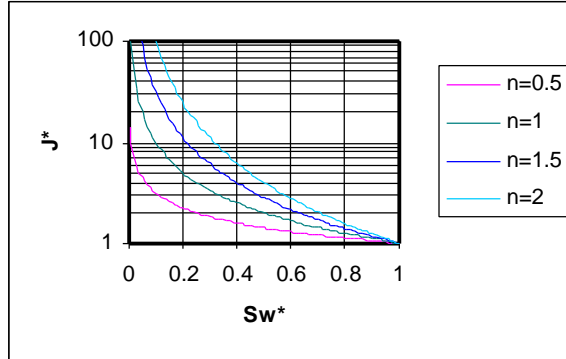


Figure1: Modified J-function vs. normalized saturation for different n-values

Table-2: Basic data for v128-v130

k	ϕ
16	25.2
35	25.6
18	25.3

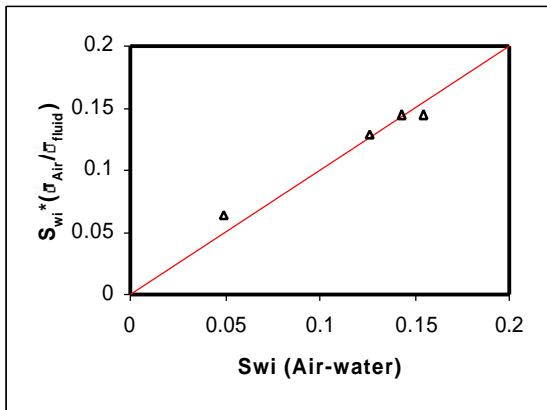


Figure3: Effect of interfacial tension on irreducible liquid saturation.

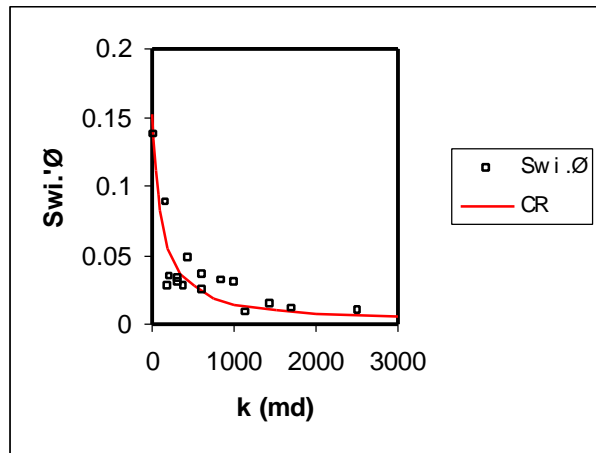


Figure2: Irreducible water saturation vs. permeability. Data is taken from literature9-11 and from our own experiments. A hyperbolic function was fit to data

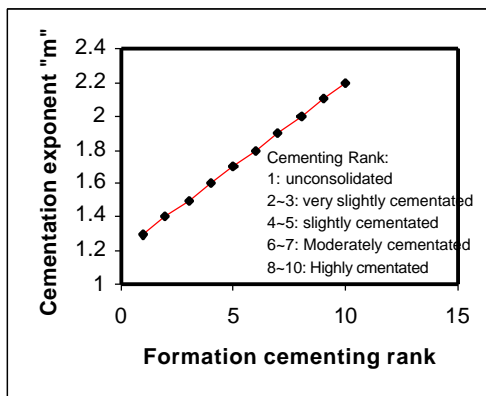


Figure4: Cementation exponent "m" is plotted against formation cementing Rank¹¹

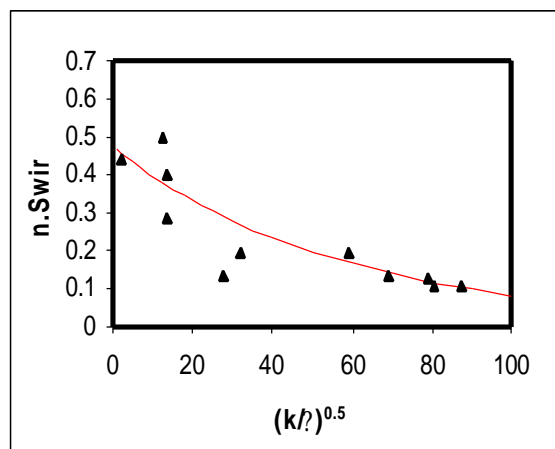


Figure5: n.Swir vs. $(k/\phi)^{0.5}$ for samples v2k/1-v2k/11

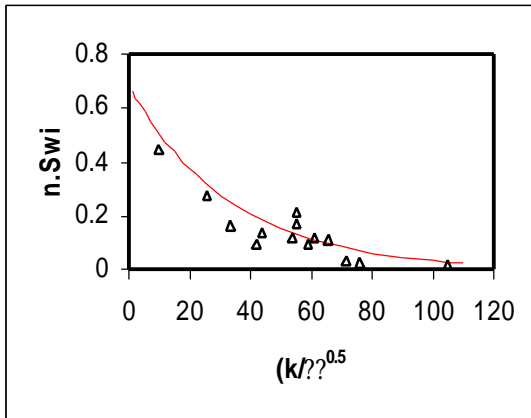


Figure6: $n.S_{wir}$ vs. $(k/\phi)^{0.5}$ from literature⁸⁻¹¹

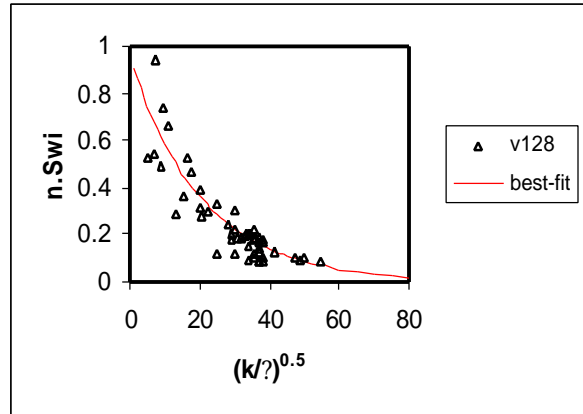


Figure7: $n.S_{wir}$ vs. $(k/\phi)^{0.5}$ from micro scale data: sample v128

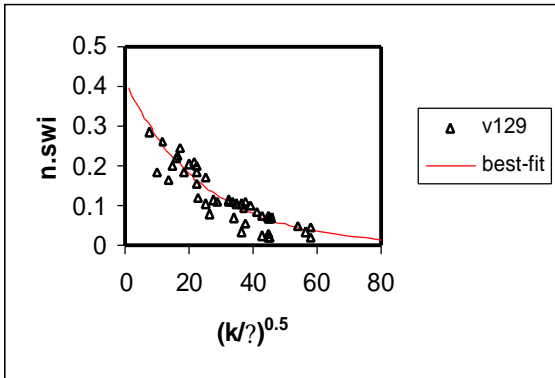


Figure8: $n.S_{wir}$ vs. $(k/\phi)^{0.5}$ from micro scale data: sample v129

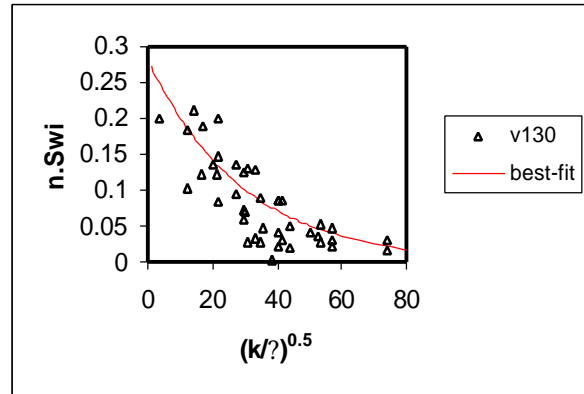


Figure9: $n.S_{wir}$ vs. $(k/\phi)^{0.5}$ from micro scale data: sample v130

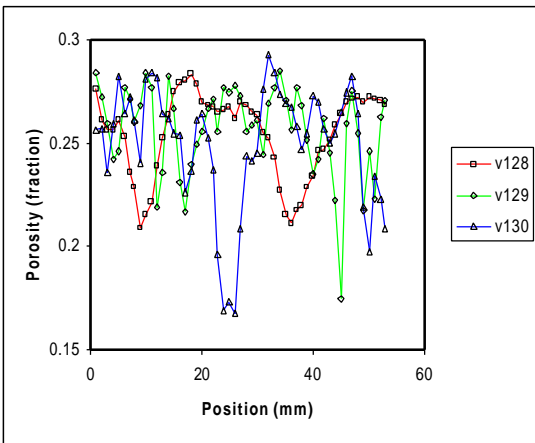


Figure10: variation of local porosity for the heterogeneous samples

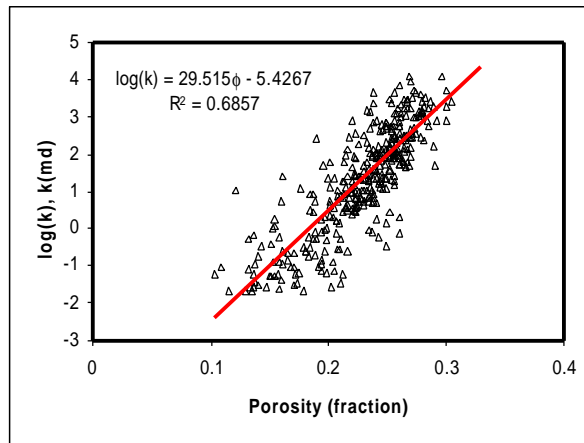


Figure11: Field permeability-porosity (k - ϕ) correlation

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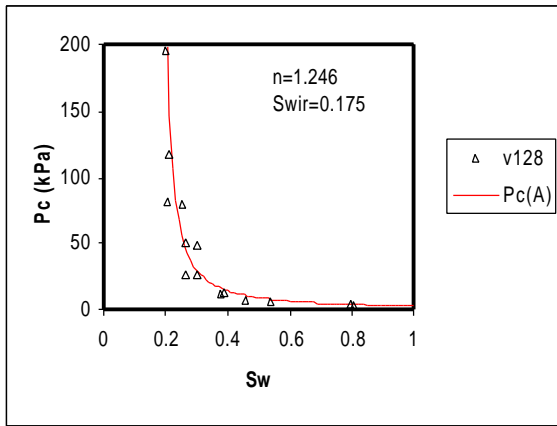


Figure12a: Capillary pressure is determined by Hassler-Brunner²⁰ method for sample-v128. Pc (A) is the best-fit analytical Pc function.

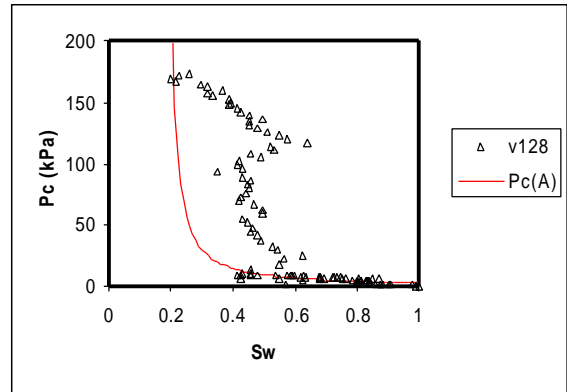


Figure12b: Capillary pressure is determined by saturation monitoring¹⁵ method for sample-v128. Pc (A) is the analytical Pc that is redrawn from Fig.12-a. Significant saturation discrepancy is observed between these two methods for this heterogeneous sample.

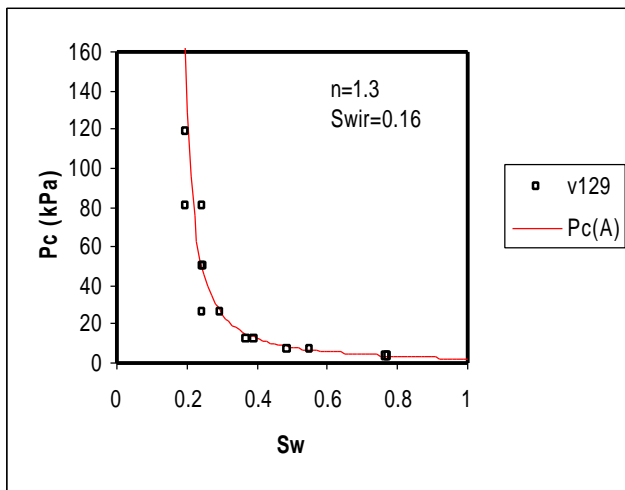


Figure13a: Capillary pressure is determined by Hassler-Brunner²⁰ method for sample-v129. Pc (A) is the best-fit analytical Pc function.

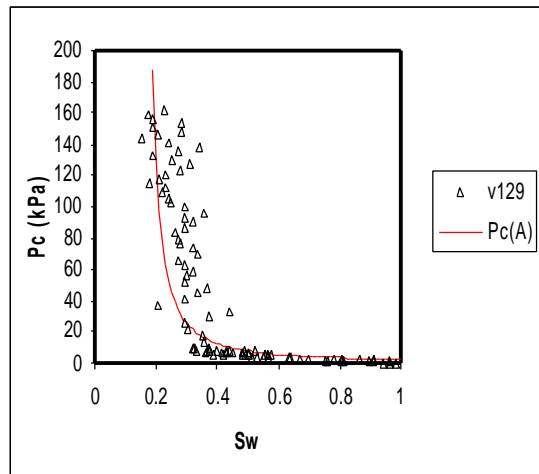


Figure13b: Capillary pressure is determined by saturation monitoring¹⁵ method for sample-v129. Pc (A) is the analytical Pc that is redrawn from Fig.13-a. Saturation discrepancy is however not observed like the sample v128.

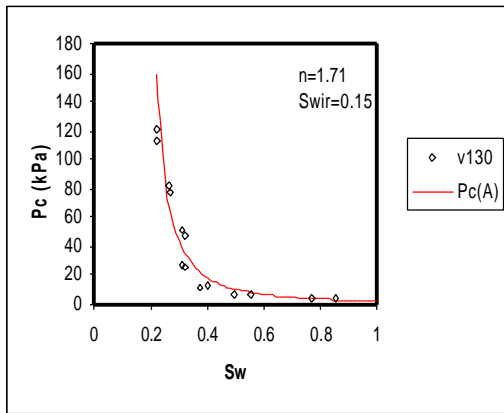


Figure 14a: Capillary pressure is determined by Hassler-Brunner²⁰ method for sample-v130. Pc (A) is the best-fit analytical function.

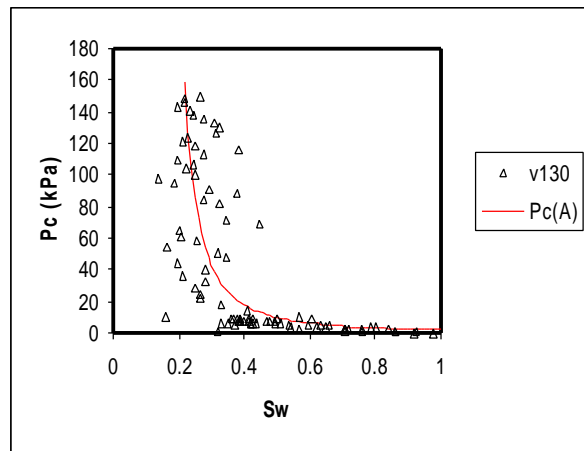


Figure 14b: Capillary pressure is determined by saturation monitoring¹⁵ method for sample-v130. Pc (A) is the analytical Pc that is redrawn from Fig. 13-a. Saturation discrepancy is however unlike the sample v128.

APPENDIX

Micro scale porosity

A radioactive scanner (source: ²⁴¹Am, half-life: 433 years, energy 59.5 KeV; detector: NaI-Tl) was used to capture the core scale porosity variation in millimeter scale (see Fig.A1 for set up of the instrument). The governing equation for estimating porosity (ϕ) at any location of the sample can be written by the following equation:

$$I_C = I^* e^{-[(1-f)m_s + fm_g + m_b]} \dots (A-1)$$

Where, I_C = Number of gamma photons counted in the detector per unit time, I^* = Number of incident gamma photons per unit time. μ_s , μ_g and μ_B = absorption coefficients for matrix (solid), gas (air) and the background. Note that μ_s , μ_g are not defined as linear attenuation coefficients rather they may be regarded as lumped attenuation coefficients whose magnitude depend on the samples' geometry, orientation and its position within source and detector (see Fig.A-1).

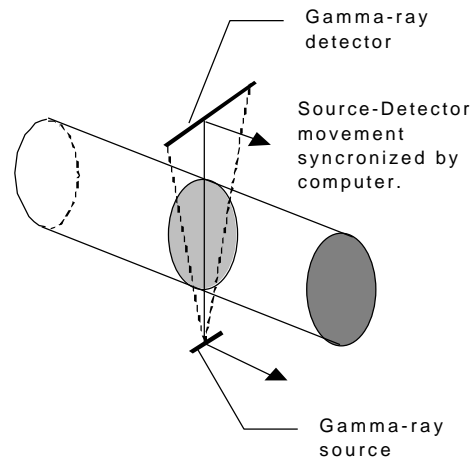


Figure A-1: Schematic of saturation monitoring by γ -absorption

In order to determine porosity (ϕ) from Eq.A-1, it is necessary to know the absorption coefficients (μ_s , μ_g) and the intensity of the gamma source. The I^* may be written in terms

of background scanning counts (I_B) i.e. without placing any sample in between source and detector.

$$I_B = I^* e^{-(m_b + m_g)} \dots (A-2)$$

Dividing Eq.A-2 by Eq.A-1 and rearranging the terms, one can write following:

$$\ln\left(\frac{I_B}{I_C}\right) = (m_s - m_g)(1-f) \dots (A-3)$$

For a given cylindrical rock, μ_s and μ_g are constant. The difference of the coefficients i.e. ($\mu_s - \mu_g$) may be determined if we take few dry samples so that experimental values of $\ln(I_B/I_{C,avg})$ vs. ϕ_{avg} may be fitted to a linear function that passes through origin. The slope provides the value ($\mu_s - \mu_g$) while the local porosity variations for similar samples may then be calculated by Eq.A-3 if value of I_C is known at that location. Here, the term $I_{C,avg}$ is

defined as $\frac{1}{N} \sum_{j=1}^N I_{C,j}$, where, j = an index assigned to identify specific location (x-section) of a given sample, N = maximum number of locations where radioactive scanning is performed. ϕ_{avg} is defined as sample's average porosity measured by helium porosity meter.

Capillary pressure-saturation function by saturation monitoring

Capillary pressure and fluid saturation relationship for reservoir cores may be determined by saturation monitoring. The method requires that the samples are cleaned and dried. The samples may then be wrapped very tightly for example by heat-shrinking sleeve so that only end faces remain open for flow. At this stage the samples are placed in between a γ -ray source and a detector (see Fig.A1). The γ -source and detector slots (openings) are set in such a way that the

instrument counts only a fan of attenuated γ -rays from a thin x-section of a sample. For a given location j , the recorded count rate, I_{Cj} may then be written like the Eq.A-1.

$$I_{C,j} = I_j e^{-m_g f_j} \dots (A-4)$$

Where, $I_j = I^* \cdot e^{-(\mu_B + \mu_s(1-\phi))}$. ϕ_j = porosity at location j .

After finishing the radioactive scanning all along the dried samples, it is required that the samples are saturated fully. The samples are then scanned again exactly on the same locations ($j=1, N$) like the dry samples. At location j , the recorded count rate, I_{Sj} for saturated samples may then be written as

$$I_{S,j} = I_j e^{-m f_j} \dots (A-5)$$

Where, μ_l = γ -ray absorption coefficient for saturating liquid.

For capillary pressure measurement, the saturated samples are then rotated at a fixed speed so that a saturation profile is established within the sample due to equilibrium between capillary and centrifugal force. If we choose a saturating liquid whose melting point is higher than the room temperature then the equilibrium saturation profile can be locked lowering the system temperature to room temperature. The samples may then be scanned like the other two scans and for location j , we can write recorded count rate, I_{Tj} :

$$I_{T,j} = I_j e^{-(m f_j S_{o,j} + m_g f_j S_{g,j})} \dots (A-6)$$

Where, $S_{o,j}$ = average liquid saturation at location j , $S_{g,j}$ = average gas saturation at location j . Using the constrain equation $S_{o,j} + S_{g,j} = 1$ and Eq. A-4, A-5 and A-6, $S_{o,j}$ can be solved as:

$$S_{o,j} = \frac{\ln(I_{C,j}) - \ln(I_{T,j})}{\ln(I_{C,j}) - \ln(I_{S,j})} \dots (A-7)$$

In order to obtain capillary pressure-saturation function, average capillary pressure at location j may be related to $S_{o,j}$ since $S_{o,j}$ represents average liquid saturation at this location. The average capillary pressure (P_c) at location j may be obtained integrating the P_c over the entire x -section at j and dividing it by the x -sectional area at that location. The average P_c at location j may then be calculated from the following function²²:

$$\overline{P_{C,j}} = \frac{1}{2} \Delta r \omega^2 \left(r_2^2 - r_j^2 + \frac{3}{4} R^2 \right) + \frac{4}{3p} \Delta r g R \dots (A-8)$$

Where, $\overline{P_{C,j}}$ = average P_c at location j , $\Delta \rho$ = density difference between liquid and gas, ω = centrifuge speed, g = acceleration of gravity, R = sample radius, r_2 = distance between rotation axis and center of the sample at outer end or at gas-oil contact. r_j = distance between rotation axis and center of the sample at location j . The P_c -saturation function may then be obtained from the following data set $\{ S_{o,j}, \overline{P_{C,j}} \ j=1, N \}$

The scanning interval length i.e. $(r_{j+1} - r_j)$ may be set for example 1mm in order for better resolution. However, in order to set counting time for j -locations, one must consider statistical uncertainties involved with radioactive measurements.

All radioactive counting follows Poissonian statistics²³. Therefore, standard error associated to a radioactive measurement may be calculated from the square root of the magnitude of the measurement. If counting time increased, the magnitude of the counts/events also increased, as a result, the standard error associated to an average quantity derived from the radioactive measurement is

decreased²⁴. However, in order to reduce the standard error, one cannot simply set the counting time infinite rather it must be optimized according to an accuracy one wish to achieve from the measurement.

The process of setting the counting time for a dependent random variable (for example $S_{o,j}$) may be determined if system's standard error could be described as a function time. If a function, u can be formulated by a set of independent random variables $(x_1 \dots x_k)$ such that $u = u(x_1 \dots x_k)$ then the standard error for u may be written as

$$s_u^2(t) = \sum_{i=1}^k \left(\frac{\partial u}{\partial x_i} \right)^2 s_{x_i}^2(t) \dots (A-9)$$

Where, $s_u(t)$ and $s_{x_i}(t)$, $i=1, k$ are standard errors for u and x_i which are functions of time t . Note that if $s_{x_i}(t) \neq$ a function time t then $s_u(t) =$ constant.

Now, we are interested to solve Eq.A-9 for $S_{o,j}$ so that the standard error associated with it can be optimized. Replacing $u = S_{o,j}$ and x_i 's by $\ln(I_{C,j})$, $\ln(I_{S,j})$ and $\ln(I_{T,j})$, we can write standard error for $S_{o,j}$ approximately as²²:

$$s_{S_{o,j}} \approx a \frac{1}{\sqrt{t}} \dots (A-10)$$

Where, $\sigma_{S_{o,j}}$ = standard error for $S_{o,j}$. a is defined in the following equation and it may be treated as constant for a given rock-fluid and γ -source.

$$a = \frac{\sqrt{3}}{\sqrt{I_{S,j}} \ln \frac{I_{C,j}}{I_{S,j}}} \dots (A-11)$$

Setting $\sigma_{S_{o,j}} = 0.01$, the optimum counting time $t=20$ minutes has been determined for our system which has then been used at all scanning locations i.e. $j=1, N$.