

A NOVEL STUDY OF THE BROAD BAND COMPLEX CONDUCTIVITY OF VARIOUS POROUS ROCKS

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ABSTRACT

The frequency dependence of electrical rock properties in the frequency range from 10^{-3} to 10^9 Hertz has been analyzed with a Combined Conductivity and Permittivity Model (CCPM). Broad band measurements were carried out on 130 samples of different rock types in the frequency range from 1 mHz to 1 GHz. The CCPM model fits well the measured transfer function for all types of rock.

INTRODUCTION

The electrical transfer function of a rock sample can be characterized by the dependence of the conductivity and dielectric permittivity on frequency as well as on thermodynamic state parameters temperature, pressure and water content (DISSADO & HILL, 1984). Based on Maxwell's equations of electrodynamics, an effective current density J^* is defined as the response of a time varying electrical field E . For a sinusoidal time dependence of E the process is described with

$$J^*(\omega) = (\mathbf{s}_{eff} + i\omega\mathbf{e}_{eff})E(\omega) \quad (1)$$

where ω is the angular frequency. The term in brackets is the electrical transfer function. The quantities that describe the effective electrical or dielectrical properties are the electrical conductivity σ_{eff} and the dielectric permittivity ϵ_{eff} . The transfer function in equation (1) may be formulated in terms of a complex conductivity σ^* (OLHOEFT 1985):

$$\mathbf{s}_{eff} + i\omega\mathbf{e}_{eff} = \mathbf{s}^* = \mathbf{s}'(\omega) + i\mathbf{s}''(\omega) \quad (2)$$

Complex conductivity or permittivity data for a broad frequency range were published e.g. by VINEGAR & WAXMAN (1984), LOCKNER & BYERLEE (1985), DISSADO & HILL (1984).

BROAD BAND TRANSFER FUNCTION

In general the frequency dependent response of a water containing rock between some Millihertz and some Gigahertz may be characterized by the overlap of two ranges that are dominated by electrical conduction processes and dielectrical polarization processes (DISSADO & HILL, 1984). The new so-called Combined Conductivity and Permittivity Model (CCPM) is in terms of complex conductivity:

$$\mathbf{s}^*(\omega) = \mathbf{s}_{DC} + \mathbf{s}_{O,NF} \left(i\omega / \omega_N \right)^{1-p} + \omega_N \mathbf{e}_{O,HF} \left(i\omega / \omega_N \right)^n + i\omega\mathbf{e}_{\infty} \quad (3)$$

All parameters used in the CCPM (see tab. 1) may be obtained from one combined electrical and dielectrical measurement.

Tab. 1: Parameters of the CCPM

parameter	symbol	unit	comment
dc-component of conductivity	σ_{DC}	S/m	
amplitude factor of interface conductivity	$\sigma_{O,NF}$	S/m	$\hat{\mathbf{s}}_{O,NF} = \mathbf{s}_{DC} + \mathbf{s}_{O,NF}$
frequency exponent of interface conductivity	1-p	-	p determines slope of imag. part <10 kHz
high frequency dielectric permittivity	ϵ_{∞}	F/m	
amplitude factor of the interface permittivity	$\epsilon_{O,HF}$	F/m	
frequency exponent of interface permittivity	n	-	n determines slope of real part >100 MHz
normalizing angular frequency	ω_N	Hz	pragmatically $\omega_N=1$ Hz was used

The separation of a true dc-conduction σ_{DC} component in the low frequency range of the spectrum is impossible if $p > 0,95$. This is generally the case of high porous water saturated rocks with low interface conductivity. For this simple case $\hat{\mathbf{s}}_{O,NF}$ is the sum of σ_{DC} and interface conductivity $\sigma_{O,NF}$. Equation (3) becomes:

$$\mathbf{s}^*(\mathbf{w}) = \hat{\mathbf{s}}_{O,NF} (\mathbf{i}\mathbf{w} / \mathbf{w}_N)^{1-p} + \mathbf{w}_N \mathbf{e}_{O,HF} (\mathbf{i}\mathbf{w} / \mathbf{w}_N)^n + \mathbf{i}\mathbf{w}\mathbf{e}_{\infty} \quad (4)$$

The low frequency part of this equation may be used for quantitative parameter estimation from low frequency complex conductivity measurements (spectral IP) in high porous water bearing formations (SCHÖN, 1996). The model has only two free parameters:

$$\mathbf{s}^*(\mathbf{w}) = \hat{\mathbf{s}}_{O,NF} (\mathbf{i}\mathbf{w} / \mathbf{w}_N)^{1-p} \quad (5)$$

The advantage of this model is that all parameters may be obtained from one single complex measurement. For the analysis of high frequency measurements (>10 MHz, e.g. limestone) the following simplified transfer function may be applied (FECHNER, 2000):

$$\mathbf{s}^*(\mathbf{w}) = \hat{\mathbf{s}}_{O,NF} + \mathbf{w}_N \mathbf{e}_{O,HF} (\mathbf{i}\mathbf{w} / \mathbf{w}_N)^n + \mathbf{i}\mathbf{w}\mathbf{e}_{\infty} \quad (6)$$

EXPERIMENTAL

Combined complex conductivity and dielectric permittivity measurements were performed on 130 samples of different rock types. The rocks were classified in 6 lithological groups: Salt rocks, chemical sediments, consolidated and unconsolidated clastic sediments, crystalline rocks and ore-bearing rocks. The porosities range between 0,5 and 35% and the internal surface area between 0,08 and 40 m²/g. The salt samples were dry, the other samples were either fully or partially saturated with fresh water or brine. The experimental set up consists of two separated measurement systems: dielectric measurement system for the high frequency range and a conductivity measurement system for the low frequency range (see BÖRNER, 1997).

RESULTS

Fig. 1 shows electrical transfer functions of different kinds of rock measured with the system described above. It is obvious that all samples show a general frequency dependence of conductivity according to the CCPM. The CCPM can be used for all rock type used in this study. Furthermore, it can be seen that only a measurement over a

broad frequency range gives the opportunity to observe the characteristic features of the transfer function. The general shape of the spectra shows that small deviations (see e.g. the imaginary part in the low frequency range or the real part in the high frequency range) are of minor importance. These deviations are caused by electrochemical reactions or by measurement errors.

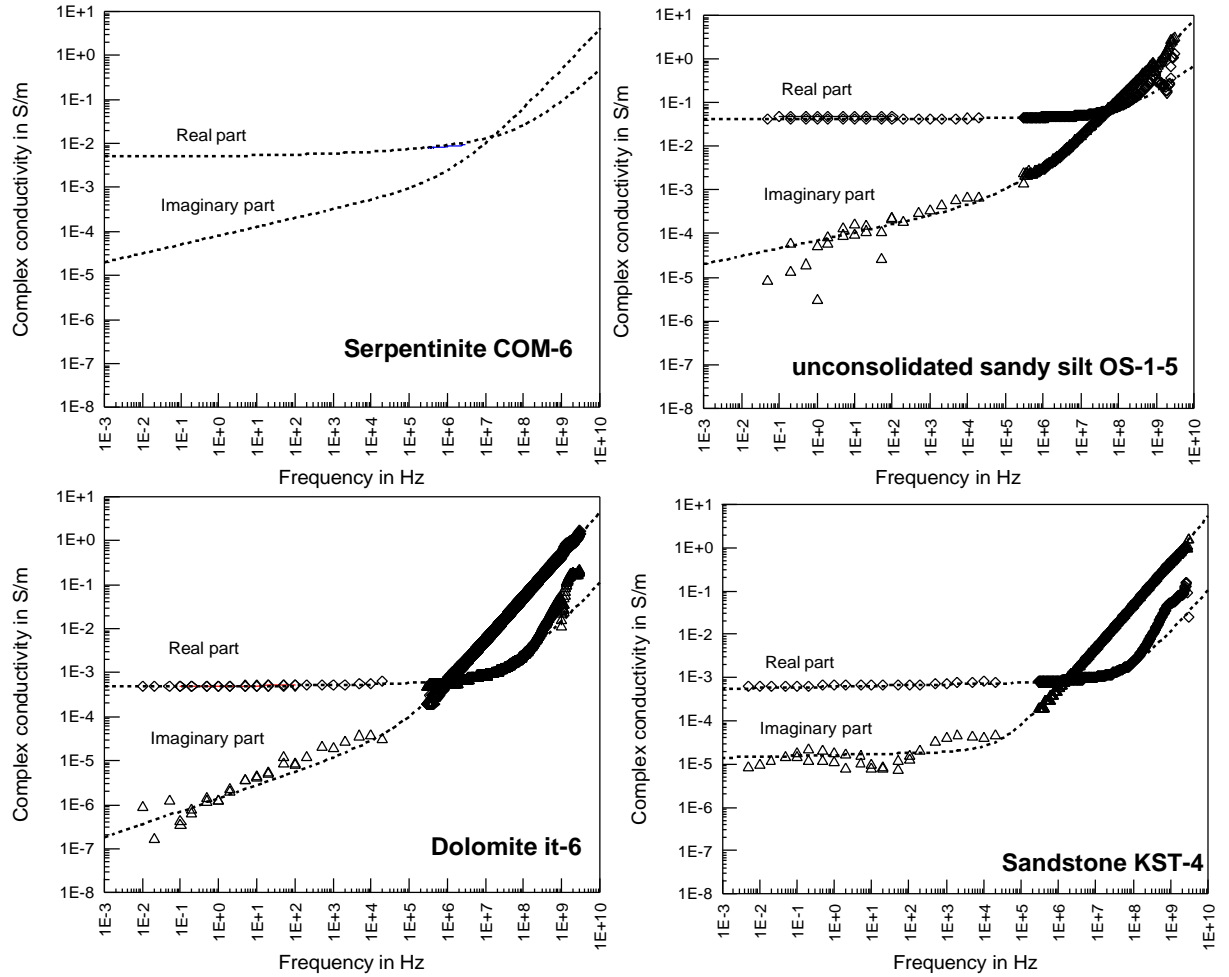


Fig.: 1: Complex conductivity spectra of rocks. Dashed lines are fits with the CCPM (tab.2).

The analysis of transfer functions for different rock samples results in the prediction of electric and dielectric parameter ranges for special rock types. Table 3 gives ranges obtained for the parameters of the CCPM-model of the transfer function.

Tab. 2: CCPM-parameters for special rock samples shown in fig. 1.

Sample	σ_{DC} [S/m]	$\epsilon_{\infty}/\epsilon_0^{**}$ [-]	$\sigma_{0,NF}$ [S/m]	$\epsilon_{0,HF}$ [F/m]	1-p [-]	n [-]
COM 6	5,0E-03	4,1	5,0E-04	4,0E-08	0,10	0,71
OS-1-5	4,2E-02	12	1,8E-04	5,0E-08	0,18	0,69
IT-6	5,0E-04	6,5	1,8E-06	1,0E-10	0,30	0,92
KST-4	1,0E-04	6,5	5,0E-04	7,0E-11	0,02	0,96

Tab. 3: CCPM-parameter ranges of all rock samples from this study.

Rock type	σ_{DC} [S/m]	$\epsilon_{\infty}/\epsilon_0^{**}$ [-]	$\sigma_{0,NF}$ [S/m]	$\epsilon_{0,HF}$ [F/m]	1-p [-]	n [-]
Rock salt	-	4,0 - 4,5	(1,0 - 2,0)E-05	(3,0 - 18)E-11	0	0,85 - 0,90
Limestone	(1,1 - 16)E-04	5,6 - 6,5	(4 - 1000)E-07	(3 - 15000)E-11	0,10 - 0,40	0,58 - 0,95
Anhydrite	(2,0 - 18)E-03	4,5 - 5,0	(1,0 - 50)E-05	(2,5 - 20)E-08	0,025 - 0,18	0,58 - 0,85
Chalk*		28 - 30	(1,2 - 1,3)E-02	5,0E-08	0,0008	0,65
Sandstone	(1,0 - 600)E-04	5,5 - 6,0	(5,0 - 8,0)E-04	(7,0 - 160)E-11	0,008 - 0,04	0,82 - 0,96
Silt	(1,0 - 4,7)E-02	6,0 - 12	(1,2 - 4,0)E-04	(2,5 - 10)E-08	0,13 - 0,20	0,66 - 0,71
Clay	(3,5 - 12)E-02	7,5 - 13	(1,5 - 3,0)E-04	(4,0 - 5,0)E-07	0,18 - 0,20	0,57 - 0,59
Weathered r.	(2,0 - 28)E-02	6,0 - 20	(5,0 - 15)E-03	(4,0 - 40)E-07	0,10 - 0,15	0,54 - 0,61
Marble	(1,0 - 5,0)E-03	4,1 - 6,5	(1,0 - 50)E-05	(1,5 - 400)E-10	0,05 - 0,10	0,71 - 0,90
Gneiss/granite	(9,0 - 290)E-05	3,6 - 5,0	(1,0 - 70)E-06	(5,0 - 4000)E-11	0,08 - 0,25	0,65 - 0,96
Basalt/ gabbro	(2,1 - 23)E-04	3,0 - 6,6	(4 - 300)E-07	(6,0 - 1500)E-11	0,05 - 0,32	0,62 - 0,92
Amphibolite	(2,0 - 140)E-04	5,0 - 7,0	(1,0 - 500)E-05	(6,0 - 1500)E-09	0,09 - 0,22	0,58 - 0,72
Serpentinite	(3,0 - 10)E-04	4,5 - 6,5	(6,0 - 400)E-06	(1,5 - 120)E-09	0,13 - 0,22	0,66 - 0,96

* $\hat{\sigma}_{0,NF}$; ** - $\epsilon_0=8,854E-12$ F/m

CONCLUSIONS

The following conclusions were drawn from the actual study: The CCPM model fits well the measured transfer function between 1 mHz and 1 GHz for all types of water wet rock. The CCPM-model and its simplifications can be used immediately for quantitative rock evaluation. Different rock types are characterized by significant different sets of CCPM-parameters. The exploitation of the results is focused on a better integration and interpretation of electrical and dielectrical logging and laboratory measurements.

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