A LOOK AT BREAK-THROUGH AND END-POINT DATA FROM REPEATED WATER-FLOOD EXPERIMENTS IN GLASS BEAD-PACKS AND SAND-PACKS

J.T. Bartley and D.W. Ruth

Department of Mechanical and Industrial Engineering, University of Manitoba,
Winnipeg, Manitoba, Canada R3T 5V6

ABSTRACT

Experimental data from a multi-variate set of water-flood experiments in sand-packs and glass bead-packs are presented and discussed. Previously [1], the JBN relative permeability curves were shown and a statistical analysis performed to infer significant effects on the interpreted curves. The present paper examines the uninterpreted data, i.e., the capillary number and oil productions (average saturation of water) at break-through of the water phase and at the end-point of each experiment. Over 90 experiments were performed in vertically-orientated sand-packs and glass bead-packs. The experimental variables examined were the pressure drop across the media-pack, which was maintained constant during each experiment, the viscosity of the mineral oil displaced, and the absolute permeability of the medium, where the wettabilities of the glass beads and silica sand were assumed to be similar. The Rapoport and Leas criterion for a stabilized water-flood was calculated at the point of break-through and a trend with saturation was not seen. The dimensionless Lake stability criterion was also calculated from the experimental data but no evident trend in the results was seen to indicate stability in the water-floods. The Peters and Flock stability number, *N*s, was calculated and it showed that all of the water-floods conducted were above the critical value of 13.56, which indicated, according to their theory, that viscous fingering (defined as unstable conditions) occurred in all of the experiments. A plot of saturation at break-through versus capillary number showed evidence of several physical phenomena: saturation (dimensionless oil production) tends to decrease with increases in oil viscosity; the absolute permeability affects the range of saturations observed; and the level of pressure applied across the column of media affects the capillary number differently for each permeability. The end-point saturation of water versus capillary number data delineated a trend that spanned several decades of capillary number. In general there was an increasing trend in the data with increases in the permeability of the media and with greater applied pressures. The use of a lower viscosity oil tended to increase the end-point capillary number and the saturation of water for a given medium. A plot of residual saturation of oil and the end-point relative permeability to water was in qualitative agreement with a previous work when the data were coupled with a pore geometry factor.

INTRODUCTION

The nature of immiscible displacement of oil by water injection in sand-packs and consolidated porous media has been investigated by authors such as Uren and Fahmy [2], Everett, Gooch and Calhoun [3], and in following decades by various researchers. The emphasis in many of the published works was to test or verify a novel theoretical idea; e.g., Buckley and Leverett [4]. Much of the early work utilized the steady-state method in experiments, e.g., Wyckoff and Botset [5] and Leverett [6]; but the motivation for the present study was to generate a large data-set from repeated experiments from the control of three main parameters: the value of absolute permeability, K, the viscosity of the oil, μ_o, and the overall pressure drop across the test column, ΔP. Several experimental studies have shown the viscosity ratio of oil to water to affect the recovery of oil at break-through, or ultimate recovery, but the number of experiments performed in the studies were limited to a few water-floods and with no repetition of experiments (e.g., Everett, Gooch and Calhoun [3]; Engelberts and Klinkenberg [7]; Croes and Schwarz [8]; Singhal, Mukherjee and Somerton [9]). Also, the effect of flow rate on break-through recovery has been investigated by Jones-Parra, Stahl and Calhoun [10] in glass bead-packs; the increase or decrease in recovery of oil depended on the injection rate of water. Beginning with a partial differential equation describing two-phase flow, Rapoport and Leas [11] identified a group of variables whose value, it was argued, indicates when a flow is stabilized, i.e., becomes independent of capillary pressure gradient effects. The dimensional group they identified is $Lv\mu$, where v is the speed of the injected phase and μ is its viscosity. The Rapoport and Leas (RL) criterion group was applied to the break-through data from the present work to observe if any trend was noticed and if it was comparable to RL's work (the authors observed a plateau in the break-through recovery data when plotted against their group of terms, $Lv\mu$). Lake [12] improved upon the criterion of RL to include the geometrical parameters of the porous medium, K and ϕ, the interfacial tension, σ, and make the expression dimensionless. Lake's dimensionless group was also applied to the present data to observe the groupings of data at break-through in the context of saturation versus Lake's number.

Peters and Flock [13] performed a stability analysis of the equations of two-phase flow to determine under what conditions viscous fingers tend to grow and propagate through a porous medium during immiscible flow. The dimensionless groups of Peters and Flock were applied to the set of data from the present study to determine if the experiments were stable (no viscous fingering) or unstable (viscous fingering occurs). This work of Peters and Flock is based on the earlier theoretical work of Chuoke et al. [14]. Values of the Peters and Flock number Ns that are less than 13.56 indicate stable displacements and values of Ns greater than 13.56 indicate unstable displacements (viscous fingering occurs). The Peters and Flock stability number depends on the mobility ratio M, a wettability number N_w (assumed a value of 4.44 for experiments conducted with no initial water saturation: Peters and Khataniar [15]), and a calculated value for a characteristic velocity v_c. In the context of Peters' and Flock's work, ‘stable’ means that extensive viscous fingers do not form during a water-flood, or they become damped out; ‘unstable’ means that viscous fingers continue to grow and propagate through the porous medium.

One of the main purposes for doing the present experimental study was to generate a large data-set that included the systematic variation of the main parameters of two-phase flow and to repeat experiments for each combination of parameters. Repeating the experiments permitted observation of the degree of variability in the outcome. The data were examined by calculating the groups discussed above and comparison with data from other researchers. In addition, the capillary number at break-through and at the end of each experiment was plotted with the corresponding saturation. The large data-set permitted several trends to be observed, along with the experimental uncertainty for each combination of parameters. The experiments conducted in the present study were designed to be fundamental, using distilled water, mineral oils and common media, and were conducted at typical room conditions. The authors propose that more basic studies of this kind are needed to fill the gap between fundamental knowledge of the physics of two-phase flow and the complexities of an oil reservoir.

EXPERIMENTAL PARAMETERS AND PROCEDURE

Details of the experimental method that was used can be found in Bartley and Ruth [1]; however a brief recap of the major elements of the methods is given here for clarity of presentation. Three experimental parameters were varied systematically throughout the suite of water-floods performed; they were the absolute permeability of the porous medium, K, the viscosity of the oil that was displaced from the bead- or sand-pack, μ_o, and the pressure drop, ΔH, across the vertical cylinder containing the media. The experiments reported in this work were done using a constant pressure-drop boundary condition. For a given combination of experimental parameters, water-floods were conducted at least three times to provide a statistical range in the data and to demonstrate repeatability of the data (a total of 92 water-floods were done). Note that for each successive water-flood experiment, a new packing of granular media was created from a random sample of the bulk supply of each porous medium. The three porous media used that gave three distinct magnitudes of absolute permeability, K, are referred to as Glass Beads (GB), Silica Medium sand (medium grain-sized sand, SM), and Silica Fine sand (fine-grained sand, SF). The permeabilities, K, were measured to be around 207 μm^2 for GB, 47 μm^2 for SM and 11 μm^2 for SF. The porosities, ϕ, were found to be 0.32 for GB, 0.29 for SM and 0.30 for SF. Three mineral oils provided viscosity ratios with respect to water of 29 (LV), 54 (MV) and 171 (HV). The pressure drop across the test section was varied by changing the difference in the elevation head of water between the inlet and the outlet as measured through the water phase (ΔZ = 0.25 m, 0.5 m and 1 m). The cumulative volumes of water and oil were collected in burettes and were recorded over time by a computer; the flow rates were then calculated by difference.

EXPERIMENTAL DATA AND RESULTS

After performing at least three water-flood experiments for a given combination of experimental parameters, it was observed graphically that good repeatability of the total production data and the oil production data was achieved (please see Bartley and Ruth [1]). Variability in the data between experiments was noted more for the total production data than for the oil production data and this was attributed to small differences in the absolute permeability of the media-packs between successive packings. The use of a high viscosity oil and a low pressure drop also added to the disparity between data sets for repeated experiments, especially for the smallest-grained media. In such cases, viscous and capillary pressure effects were dominant influences on the flow (a heavier viscosity oil increased the resistance to flow, and capillary pressures aid in the displacement of the oil). The viscosity of the oil was a factor in the duration of each of the experiments. The number of pore volumes of water that were required to complete a water-flood increased with increases in the viscosity of the oil. This observation is reasonable because, after break-through of the water phase, an oil of heavy viscosity (e.g., HV), which remained in the form of stranded oil ganglia, required more time to navigate through the porous medium while the flow of water through the medium continued.

Examination of The End-Point Data

Two important quantities from the end of each experiment are the volume of oil produced, V_{oe}, and the end-point flow rate of water, Q_{we}. The volume of oil produced can be recast as the end-point saturation of water, $S_{we} = V_{oe} / V_p$, where V_p is the pore volume. The flow rate of water was used to calculate an end-point effective permeability for each packing and consequently the end-point relative permeability to water (please see Bartley and Ruth [1] for more details); the capillary number corresponding to the end-point of each experiment was also calculated, and the following form of *Ca* was adopted:

$$Ca = \frac{Q_{we}\,\mu_w}{\phi\,A_b\,\sigma}\ , \qquad (1)$$

where σ is the interfacial tension between the oil and water phases and $\phi\, A_b$ is the cross-sectional area open to flow. A plot of the end-point saturation of water versus capillary number for all experiments conducted is shown in Figure 1; a general increasing trend in the data is evident. (Note that LV refers to the light viscosity oil, MV to the medium viscosity oil and HV to the heavy viscosity oil.) By indicating a trend-line through the data it is not postulated that the end-point data from *any* laboratory water-flood would fall on or close to this trend-line; the trend from the current experiments may be limited to cases where the geometry of the cylindrical test column is similar to that used in the present work (16.3 mm diameter by 275 mm length).

The wettability of the porous media and oil/water system can be classified as being weakly to moderately water-wet because the mineral oils used in the experiments were not cleaned. That is, the polar impurities in the oils were not removed prior to their use in the water-flood experiments (Morrow, [16]); this is consistent with the values of interfacial tensions that were measured with the oils using the pendant-drop method (σ = 23.0, 27.3 and 34.7 dyne/cm).

Several features of Figure 1 suggest discussion. Notice that most of the Glass Beads data are located near the top right segment of the plot and that most of the Silica Fine data are located near the bottom left segment of the plot. This indicates that the water-flood process was more effective at displacing oil from the pore spaces in the Glass Beads pack than from the finer-grained silica sand-pack. The data for the Silica Medium sand are clustered around the mid-region of the plot. The Glass Beads data were seen to fall mostly in the range of S_{we} of 0.88 to 0.94, which indicated that increasing the applied pressure gradient had little effect on the total recovery of oil for the Glass Beads. The effect of increasing the pressure gradient was seen to be more pronounced with the Silica Fine and Silica Medium sands; as pressure increased, the amount of oil displaced also tended to increase. This was especially evident in the Silica Fine data where distinct regions corresponding to each pressure level were noticed. The higher degree of scatter in the Silica Fine data at the low pressure level (lower-left sector of the plot) indicates a greater degree of randomness in the experimental outcome, which was probably due to slight differences in the permeability of each sand-packing, combined with the dominant capillary-pressure effects on the flow in the fine-grained sand. The overall trends in the data showed the expected increase in capillary number with increases in the applied pressure drop. With respect to changes in the viscosity of oil that were used in the experiments, it was noticed that, to some extent, the use of a lower viscosity oil tended to increase the end-point capillary number and the end-point saturation of water. In other words, using a lighter viscosity oil tended to allow for a greater amount of oil recovery; and because less oil was present in the pore spaces at the end of an experiment, water was able to flow through the sand-pack or bead-pack with less resistance (therefore the end-point flow rate increased).

Archer [17] proposed that the end-point relative permeability to water and the residual oil saturation data should be coupled together with a third parameter, a pore geometry measurement, because he observed an interdependence of these quantities and showed their mutual relationship on a three-dimensional surface plot. Archer made use of the term $\sqrt{K/\phi}$ to characterize the pore geometry of a core and contended that this term, along with S_{or} and k_{rwe} should be included as input to reservoir simulators and that the pore geometry term should accompany the end-point relative permeability data when plotted. Archer's three-dimensional plot showed that at small values of $\sqrt{K/\phi}$ the residual oil saturation values tended to be slightly greater than those with a larger value of pore geometry factor. The residual oil saturation decreased with increases in the end-point relative permeabilities to water regardless of the magnitude of the pore geometry factor. The data from the present experimental study were also plotted in the same manner as that of Archer [17] except all the data are shown on a two-dimensional plot (Figure 2). Three values of the pore geometry factor were calculated as $\sqrt{K/\phi}$ = 25.4 μm for GB, 12.7 μm for SM sand and 6.1 μm for SF sand. Figure 2 shows the residual oil saturation (S_{or}) on the ordinate axis (which is 1 - S_{we} assuming 100% oil saturation at the start of each water-flood) and end-point relative permeability to water (k_{rwe}) on the abscissa. Although there is a degree of scatter among the data, the data show trends that agree qualitatively with those of Archer (Archer's data were limited to $\sqrt{K/\phi}$ of 15). The data for Glass Beads (squares) show a very gradual decreasing slope of the residual oil saturation as end-point relative permeability increased. The range of k_{rwe} occupied by the GB data is the widest of the three permeabilities used. The data for the Silica Medium sand (triangles) occupy a smaller range of relative permeability but the slope with residual oil saturation is greater (steeper). The Silica Fine sand data (circles) fall roughly around those for the SM sand but extend slightly higher on the S_{or} scale and they occupy the narrowest range on the k_{rwe} scale (from 0 to 0.25). Note that, although distinction is made in the figure for the viscosity of oil, no special distinction is made to show the pressure level, but all of the end-point data from the experiments are plotted. The overall quality of the trends with the present data-set is similar to that of Archer [17] except that he did not report any obvious reduction in the slope of the S_{or} versus the k_{rwe} data with increases in the pore geometry factor. This is likely due to that fact that Archer's data was limited to values of $\sqrt{K/\phi}$ = 15 µm and the present data for Glass Beads extend the empirical range of $\sqrt{K/\phi}$ to 25.4 µm.

Examination of the Break-Through Data

The Rapoport and Leas [11] criterion for a stabilized water-flood was calculated as $Lv\mu_w$ where v is the bulk speed of flow of the injected phase and is calculated as Q_t/A_b. Rapoport and Leas (RL) suggested that their criterion value be somewhere between 1 and 10 cm^2·cp/min for stabilized conditions to prevail during a water-flood. Data from the current set of experiments were plotted in the form of saturation of water versus the RL scaling criterion (Figure 3). Most of the data exceeded the criterion value of 1 cm^2·cp/min except for the Silica Fine sand data at the lowest pressure level and at the middle pressure level using the heavy viscosity oil. Contrary to the findings of Rapoport and Leas, a plateau did not form in the data from the current set of experiments for criterion values greater than 1 to 10 cm^2·cp/min; in fact the data appeared quite disperse. Two other published studies that also applied the Rapoport and Leas criterion to their own data were not able to obtain a plateau and a well-defined trend with the data. Jones-Parra, Stahl and Calhoun [10] tried to achieve a general scaling of their displacement data from experiments conducted using glass bead-packs, but a correlation with the RL criterion was not seen. Maguss and Flock [18] conducted experiments with sand-packs and a limestone core and plotted the oil recovery data against the RL criterion. The data of Maguss and Flock showed decreased recovery with increases in the flow rate. Most of the change in oil recovery at break-through occurred for $Lv\mu_w$ values of less than about 5 cm^2·cp/min; and a plateau in the data, like that of RL, was not seen. These additional negative findings confirm the trends seen with the data from the present set of experiments, when plotted in the manner suggested by Rapoport and Leas [11].

Lake [12] defined a dimensionless group that included the RL criterion, $Lv\mu_w$, interfacial tension, contact angle, end-point relative permeability to the water phase and geometrical parameters of the porous medium. The Lake number, N_{L}, is defined as

$$N_L = \left(\frac{\phi}{K}\right)^{1/2} \frac{\mu_w\, v\, L}{k_{rwe}\, \sigma\, \cos\theta}\,. \tag{2}$$

When this dimensionless group was applied to break-through data, no trend or general meaning was evident with the data when plotted as saturation of water at break-through versus N_L. The appeal of the dimensionless Lake number is that it incorporates many of the important parameters pertaining to immiscible two-phase flow, including the pore geometry factor, $\sqrt{K/\phi}$ (it employs the inverse of this factor). Its apparent lack of success with the present set of data may be due to the use of a constant pressure drop boundary condition instead of constant flow rate conditions. Typical values of N_L in the current results for GB ranged from about 0.1 to unity; values of N_L for SM sand ranged from about 0.20 to 1.4; and N_L for SF sand ranged from about 0.1 to 0.5.

The Peters and Flock stability number was applied to the break-through data of the current study and, although a certain degree of scatter exists in the plot of the data (Figure 4), an overall decreasing trend in S_{wbt} with increasing Ns is evident. This observation concurs with the findings of Peters and Flock [13] where they reported a marked decrease in break-through recovery as instability increased. The present calculated Ns values were greater than the critical value of 13.56 which therefore indicated that all of the experiments were unstable (viscous fingering occurred). The degree of instability intensified as the viscosity of the oil increased. It is interesting to note that the data in the plot point toward higher break-through saturations as Ns decreases; this trend makes physical sense because as displacements become more stable (Ns $\rightarrow$ 13.56) viscous fingering diminishes and hence more oil is produced before break-through of the water phase occurs.

The capillary number Ca based on the total flow rate Q_t was calculated for the time of break-through of water, along with the average saturation of water. A plot of saturation versus capillary number is shown in Figure 5 and some interesting features of this figure are worth noting. First, with respect to changes in the oil viscosity, it was noticed that the saturation values generally decreased with increases in viscosity. In addition, values of the capillary number also decreased, due to increases in the viscosity of oil, when comparing data for the same pressure level. The trends mentioned here are evident at all three pressure levels used in the current study. The decrease in capillary number can be attributed to increases in the viscous resistance in the flow as the viscosity of the oil increased. It is postulated that the decrease in the saturation of water at break-through is due to extensive viscous fingering. The observation of a decrease in saturation (oil recovery) due to increases in viscosity was also reported by Everett et al. [3], Engelberts and Klinkenburg [7], Jones-Parra, Stahl and Calhoun [10], and Croes and Schwarz [8].

From Figure 5 it is noticed that the data for Glass Beads are spread over a wide range of saturation (0.1 to 0.8) compared to the data for the Silica Medium and Silica Fine sands. Overall, as the permeability of the porous medium decreased, the range of saturation occupied by the data also decreased. It is postulated that this is due to viscous fingering. The Peters and Flock stability number calculations showed that the water-floods were unstable. The work of Stokes et al. [19] reported empirical evidence on the width of the viscous fingers decreasing with decreases in the pore size and also with increases in the rate of injection. Therefore, for the present data, the viscous fingers at break-through for the Glass Beads data were relatively large for low pressure experiments and became smaller in width in the high-pressure experiments. It is postulated that large-sized viscous fingers are characterized by high values of saturation at break-through; and conversely, smaller-sized fingers are characterized by lower values of saturation. The relatively wide range of saturation values seen in the Glass Beads data is indicative of changes in the width of the viscous fingers with changes in the pressure level; however, very little change occurs in the capillary number. It is contended that, as the pressure level increased, smaller viscous fingers formed that lowered the effective permeability of the water flowing into the porous medium, compared to flow in larger viscous fingers at low pressures, hence lowering the flow rate at the outlet, which is reflected in similar values of *Ca*. The data for the Silica Medium sand show a slightly different trend; changes in saturation occur with increases in the pressure level, although the spread in the data is not as apparent as that of the Glass Beads, and more disparity in the capillary number values is seen. Considerable variation in *Ca* is seen with the Silica Fine sand data and with only a marginal decrease in saturation with increases in the pressure level. This trend suggests that there may not have been much difference in the number or width of the viscous fingers for the Silica Fine sand. Regarding the trend of decreases in saturation with increases in the pressure level, several earlier works also reported observing decreases in saturation (oil recovery) at break-through with increases in the flow rates, when performing constant flow rate experiments. Such works include those of Engelberts and Klinkenburg [7], Jones-Parra, Stahl and Calhoun [10], and Peters and Flock [13].

It is speculated that there may also be a boundary effect on the flow that inflences the size and number of sustainable viscous fingers. The present set of experiments were conducted in a fixed-diameter cylindrical test section. The average grain size for each porous medium was different (600 μm for Glass Beads, 400 μm for Silica Medium sand, and 200 μm for Silica Fine sand) and it is not known what effect the variation in the number of grains that occupy the width and length of the test section has on the viscous fingering phenomenon, although it has been observed that the largest viscous finger is limited by the dimensions of the test section (Stokes et al. [19]). Future experimental work should scale the test-section geometry so that approximately the same number of grains occupy the width and length of the test column for different grain sizes.

CONCLUSIONS

1. The end-point data-set revealed a general increasing trend of the saturation of water with capillary number. The three magnitudes of absolute permeability were seen to occupy three general overlapping regions of a plot of saturation and capillary number. The data showed that displacement of the oil was most effective in the Glass Beads and less effective in smaller-grained media (SM and SF).
2. The end-point data were plotted in the form of residual saturation of oil, S_{or}, versus end-point relative permeability to water, k_{rwe}, and distinction was made between three

absolute permeabilities using the value of a pore geometry factor, $\sqrt{K/\phi}$. The data plotted in this manner showed qualitative agreement with a previous work. Three regions in the figure were identified that correspond to the three pore geometry factors for GB, SM, and SF. The present set of experimental data includes a value of pore geometry factor for Glass Beads (25.4 μm) that is considerably higher than that of most sand-packs and consolidated cores.

3. Application of the Rapoport and Leas [11] scaling criterion for a stabilized water-flood to the present data at break-through did not show any trend to suggest when capillary pressure effects can be neglected. Also, no meaningful trends in the data were noticed when the dimensionless number by Lake [12] was applied. Application of the Peters and Flock [13] stability criterion to the break-through data indicated that all of the experiments in the current study were unstable (viscous fingering was prevalent).
4. Certain trends in the break-through data were evident when the saturation of water was plotted versus the capillary number. In general, saturation and capillary number decreased with increases in the viscosity of oil, with all other parameters remaining constant. The permeability of the medium affected the range of saturation values observed at break-through and it is postulated that this effect was due to differences in the viscous fingering phenomenon and to the magnitude of absolute permeability. The effect on the capillary number due to changes in the pressure level was also more apparent as the permeability decreased (fine-grained sand).

REFERENCES

1. Bartley, J.T. and D.W. Ruth, "Experimental Investigation of Unsteady-State Relative Permeability in Sand-Packs," 2001 SCA Symposium, Edinburgh, paper 22.
2. Uren, L.C. and E.H. Fahmy, "Factors Influencing the Recovery of Petroleum from Unconsolidated Sands by Water-flooding," *Petroleum Development and Technology*, (1928), 318–335.
3. Everett, J.P., F.W. Gooch and J.C. Calhoun, "Liquid-Liquid Displacement in Porous Media as Affected by the Liquid-Liquid Viscosity Ratio and Liquid-Liquid Miscibility," *Trans. AIME*, (1950) **189**, 215–224.
4. Buckley, S.E. and M.C. Leverett, "Mechanism of Fluid Displacement in Sands," *Trans. AIME*, (1942) **146**, 107–116.
5. Wyckoff, R.D. and H.G. Botset, "The Flow of Gas-Liquid Mixtures Through Unconsolidated Sands," *Physics*, (1936) **7**, 325–345.
6. Leverett, M.C., "Flow of Oil-Water Mixtures Through Unconsolidated Sands," *Trans. AIME*, (1939) **132**, 149–171.
7. Engelberts, W.F. and L.J. Klinkenberg, "Laboratory Experiments on the Displacement of Oil by Water from Packs of Granular Material," in: Proc. of the 3rd World Petroleum Congress, Section II, (1951) The Hague, pp.544–554.
8. Croes, G.A. and N. Schwarz, "Dimensionally Scaled Experiments and the Theories on the Water-Drive Process," *Trans. AIME*, (1955) **204**, 35–42.

9. Singhal, A.K., D.P. Mukherjee and W.H. Somerton, "Effect of Heterogeneous Wettability on Flow of Fluids Through Porous Media," *J. Can. Petrol. Tech.*, (1976) **15**, 3, 63–70.

10. Jones-Parra, J., C.D. Stahl and J.C. Calhoun, "A Theoretical and Experimental Study of Constant Rate Displacements in Water Wet Systems," *Producers Monthly*, (1954) **18**, January, 18–26.

11. Rapoport, L.A. and W.J. Leas, "Properties of Linear Waterfloods," *Trans. AIME*, (1953) **198**, 139–148.

12. Lake, L.W., *Enhanced Oil Recovery*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, (1989).

13. Peters, E.J. and D.L. Flock, "The Onset of Instability During Two-Phase Immiscible Displacement in Porous Media," *Soc. Petrol. Eng. J.*, (1981), April, 249–258.

14. Chuoke, R.L., P. van Meurs and C. van der Poel, "The Instability of Slow, Immiscible, Viscous Liquid-Liquid Displacements in Permeable Media," *Trans. AIME*, (1959) **216**, 188–194.

15. Peters, E.J. and S. Khataniar, "The Effect of Instability on Relative Permeability Curves Obtained by the Dynamic-Displacement Method," *SPE Form. Eval.*, (1987), December, 469–474.

16. Morrow, N., private communication, (September, 2001).

17. Archer, J.S., "Some Factors Influencing Fluid Flow in Petroleum Reservoir Models," in: North Sea Oil and Gas Reservoirs, eds. J. Kleppe et al., The Norwegian Institute of Technology, Graham and Trotman, London (1987), pp.327–335.

18. Maguss, H. and D.L. Flock, "Laboratory Model Flow Tests," *J. Can. Petrol. Tech.*, (1962), Spring, 31–34.

19. Stokes, J.P., D.A. Weitz, J.P. Gollub, A. Dougherty, M.O. Robbins, P.M. Chaikin and H.M. Lindsay, "Interfacial Stability of Immiscible Displacement in a Porous Medium," *Phys. Rev. Lett.*, (1986) **57**, 14, 1718–1721.

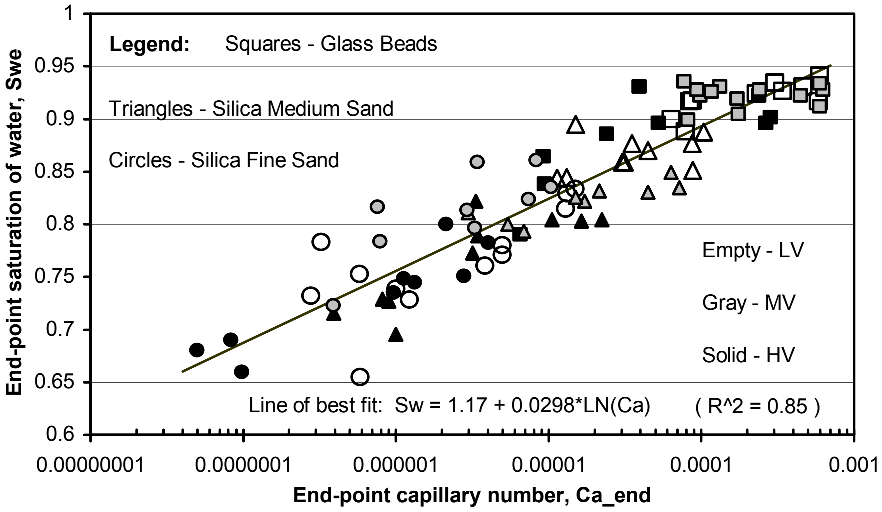

Figure 1. End-point saturation of water versus capillary number.

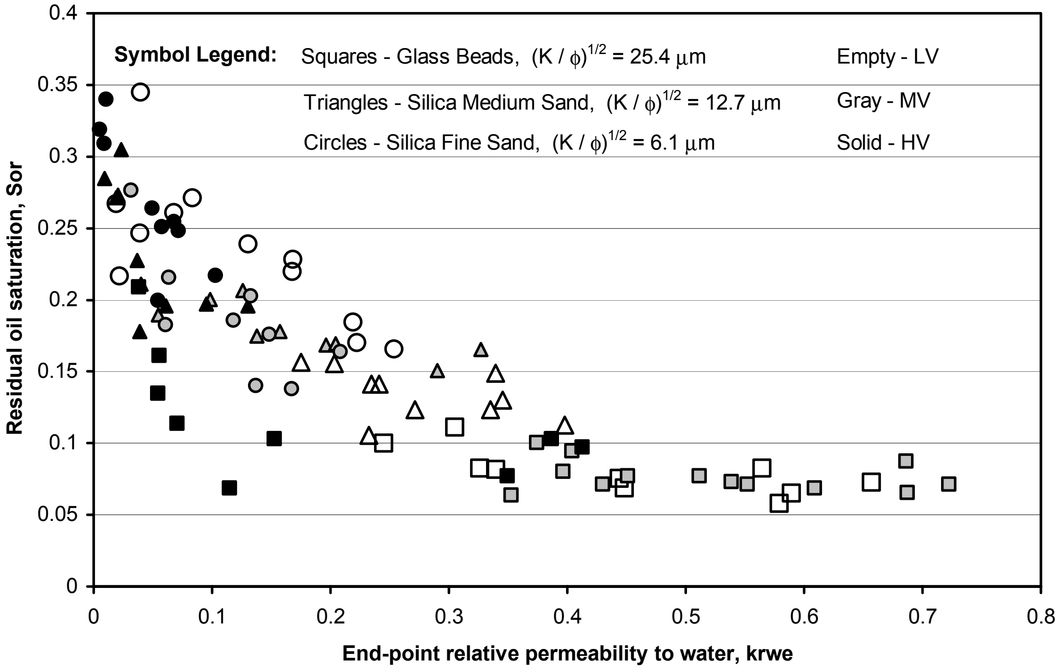

Figure 2. Residual saturation of oil versus the end-point relative permeability to water.

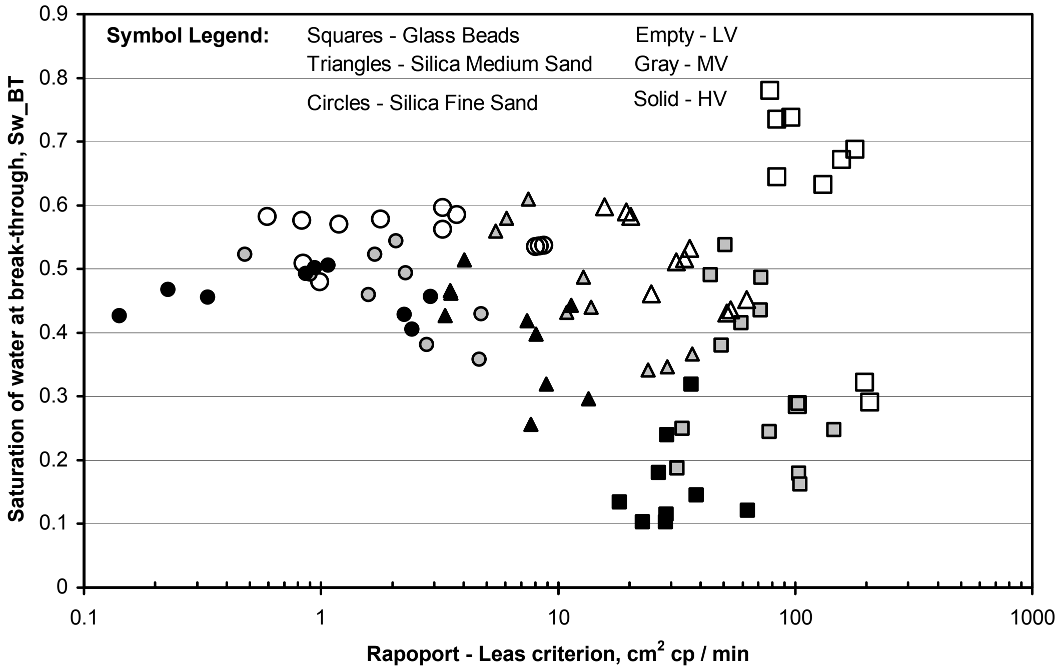

Figure 3. Saturation of water at break-through versus the Rapoport and Leas criterion.

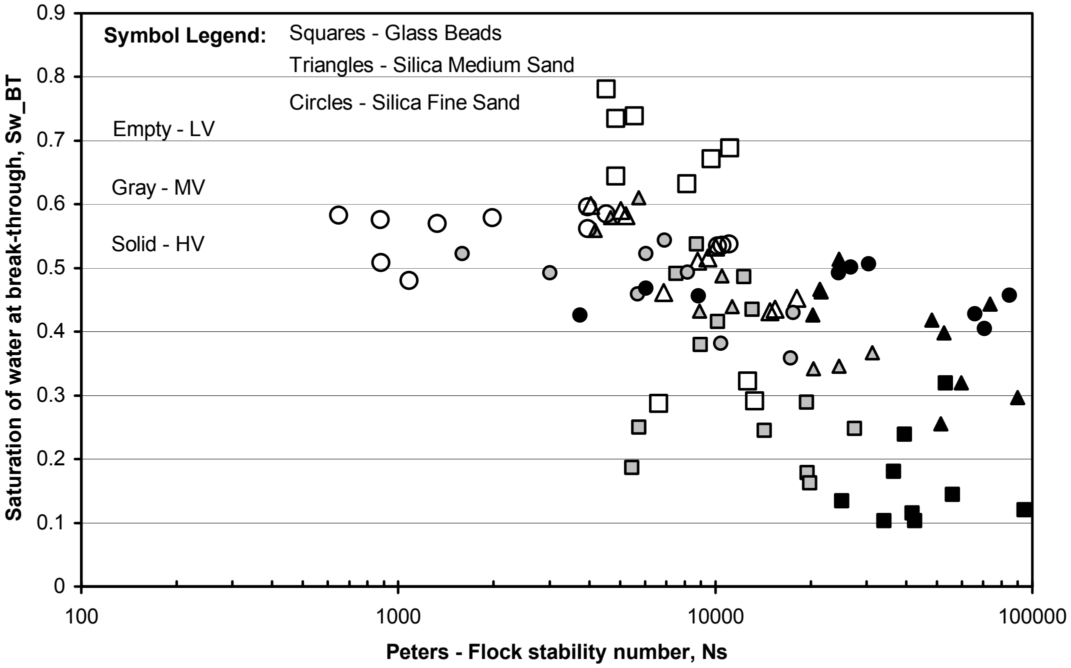

Figure 4. Saturation of water at BT versus the Peters and Flock stability number, *N*s.

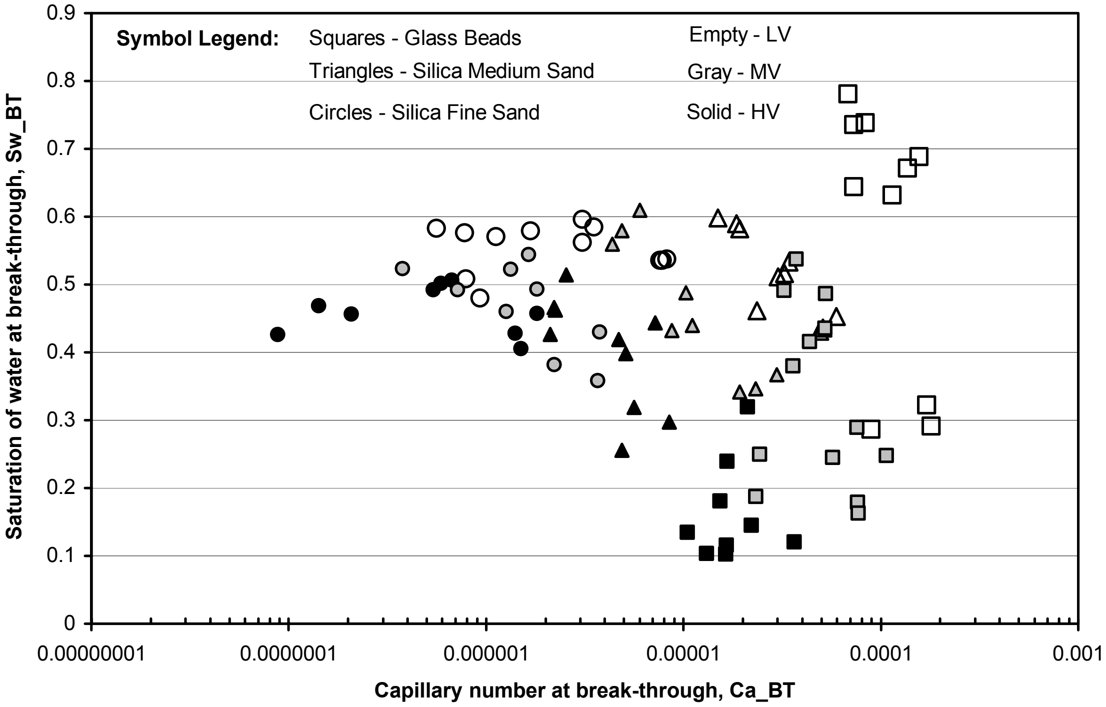

Figure 5. Saturation of water at break-through versus capillary number.