THE MISSING LINK BETWEEN PORE-SCALE ANCHORING AND PORE-SCALE PREDICTION

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ABSTRACT

An earlier paper by the authors (SCA 2001-15) discussed the predictive capability of pore-scale network models by using real experimental data as lithological "anchors". The ultimate goal of this approach was to produce an anchored model capable of relative permeability prediction. An additional advantage of having such a calibrated model would be that a wide range of rock/fluid sensitivities could be examined without recourse to additional experiments.

Paper SCA 2001-15 presented a preliminary methodology — utilising mercury injection capillary pressure (MICP) data — that could permit the *matching* of existing experimental gas/oil relative permeability curves. Results demonstrated that a basic four-parameter model was sufficient to reproduce the vast majority of experimental drainage relative permeability curves that were examined. The constrained set of adjustable parameters in the macropore network model comprised: coordination number (z), pore size distribution exponent (n), pore volume exponent (ν) and pore conductivity exponent (λ). Each of these quantities has a clear physical interpretation.

However, we also showed that anchoring network models to mercury intrusion data alone was insufficient for predicting relative permeabilities *a prior*. There was an interdependence of parameters and, consequently, an infinite set of parameter combinations could be chosen that matched the MICP data but gave very different relative permeability predictions. It was concluded that future analysis of MICP data should be performed in conjunction with the analysis of some other independent experiment — an experiment that would give the additional data that could form the "missing link" between anchoring and prediction.

We have developed two approaches to evaluating this missing link. In the present paper, we present one potential approach and show how a *unique* parameter combination can be derived using only MICP data and routinely-measured residual saturations (S_{rw} and S_{rnw}). The methodology incorporates some simple ideas from percolation theory and is first verified using synthetic MICP data and residual saturations obtained from anonymous network simulations. The anonymous data is analysed using our new approach and we find that we can successfully retrieve the unique parameter combination used to generate the synthetic data. Having obtained the relevant parameter combination, we then go on to predict the corresponding relative permeability curves. Predictions are shown to be in excellent agreement with the anonymous data and we then go on to examine laboratory core data using the same approach.

INTRODUCTION

Large-scale simulation of immiscible multiphase fluid flow in porous media necessarily requires a number of petrophysical parameters to be specified. In the context of two-phase flow, the requisite parameters include: (i) absolute permeability, (ii) capillary pressure (the relationship that links wetting and non-wetting phase pressures as a function of saturation), and (iii) two-phase relative permeabilities (the relative conductance of each phase as a function of saturation). Unfortunately, laboratory determination of this data — especially (iii) — is time-consuming and costly, and so a number of different approaches have been developed in an attempt to predict missing measurements using data that are more easily acquired. For example, the empirical relationships published by Burdine [1], Brooks and Corey [2], and van Genuchten [3] are often used to derive relative permeability functions from capillary pressure curves when no core flood data are available.

Pore-scale network models offer an alternative approach towards the calculation of single and multiphase flow properties by explicitly incorporating interconnected pore elements into a three-dimensional framework (see [4] for a fuller review). The key term here is *interconnected* — although great progress can be made using capillary bundle models (indeed some of the empirical models mentioned above are based upon such a simplification), it is the interconnected nature of the underlying pore structure that is largely responsible for the richness of behavior observed in the laboratory. Here, the predictive capability of pore-scale network models is extended by using real experimental data as lithological "anchors". The aim of the study is to anchor pore-scale network models using cheap, routinely-measured data — such as mercury injection capillary pressure (MICP) data — and then go on to *a priori* prediction of gas-oil relative permeability data sets in the presence of S_{wi}.

Results from an earlier phase of this work (SCA2001-15) were extremely encouraging and considerable progress had already been made towards the ultimate goal of producing a fully predictive model. However, it was found that an infinite set of network parameter combinations were capable of producing almost indistinguishable capillary pressure curves. Hence, in order to increase the predictive capability of network models, some additional research is required — perhaps by considering an *independent* measure of one of the associated anchoring parameters or by the development of a new analytical approach. The current paper describes an example of the latter and endeavours to find the "Missing Link" between pore-scale *anchoring* and pore-scale *prediction*.

The approach taken is both simple and extremely powerful, ultimately leading to a unique solution to the anchoring problem. A brief description of the method is presented first and this is followed by its application to interconnected networks and some ideas are presented concerning how necessary measures could be obtained from routine SCAL

data. Finally, the new methodology is applied to some experimental data and relative permeability predictions are presented.

BACKGROUND TO THE MODELLING APPROACH

One of the simplest network modelling approaches for simulating multiphase flow in porous media is the "3Rs" approach, where the medium is approximated using a network of "pore elements". These pore elements are of arbitrary internal geometry, which is essentially parameterized by the associated capillary entry lengthscale. Each element is assigned a capillary entry radius (r), a volume ($V(r)$) and a conductance ($g(r)$) — the volume and conductance are considered to be proportional to the capillary entry radius raised to some power. In addition, the pore size distribution function (PSD), $f(r)$, is also assumed to follow a power law scaling, although the methodology developed here can be used with *any* single parameter distribution (including bell-shaped distributions).

In the discussion below, the various parameters in our pore element model and their physical interpretation are as follows (see [4] for additional information):

- n —pore-size distribution parameter, $f(r) = \dfrac{(n+1)}{(R_{\max}^{n+1} - R_{\min}^{n+1})} r^n$ (R_{max}, R_{min} known);
- z —coordination number of the network; typically $2.5 < z < 6$ in 3D;
- ν —volume exponent. Pore volume $V(r) \sim r^{\nu}$ — typically $1 < \nu < 3$;
- λ —conductivity exponent. Pore conductance $g(r) \sim r^{\lambda}$ — typically $2 < \lambda < 4$;
- the wettability of the pore as defined by $\cos\theta_{ow}$ (for an oil/water system). At present, we will consider the system to be of uniform wettability and it will be taken as water wet.

Hence, the network model has four fairly tightly constrained parameters and these are to be found uniquely. In the approach described below, we must still determine the conductivity exponent, λ, although this appears to be very constrained ($2 < \lambda < 4$). However, some scaling arguments presented elsewhere [5] suggest a way of obtaining the ratio (λ/ν) which offers an additional constraint on the parameter set above. This particular aspect of the problem will not be dealt with directly here, however, and a fixed λ-value of 4 will be used throughout the remainder of the paper.

PERCOLATION THEORY BACKGROUND

Percolation theory was first used by Broadbent and Hammersley [6] to investigate the flow of gas through carbon granules (for the design of gas masks to be used in coal mines). In the present context of fluid flow, percolation theory emphasises the topological aspects of the problem, dealing with the connectivity of a very large number of elemental pores and describing the size and behaviour of connected phase clusters in a well-defined manner.

The primary focus of this section is to discuss how ideas from percolation theory can be applied more specifically to the anchoring problem. As a simple, instructive analogue, consider first a two-dimensional square lattice of pore elements (we use the term "pore

elements" here to emphasize the fact that no precise pore geometry has been assumed. However, the elements can be thought of as simple capillary tubes to aid understanding). The most pertinent concepts from percolation theory will now be discussed using this geometry. Consider the critical behavior of the network. Assume that, initially, all of the tubes are blocked and that they are then opened *at random*. For any given geometry there is a unique fraction of tubes that must be open before flow across the network can commence; this critical fraction is called the *percolation threshold* (P_{th}) and for a simple (infinite) square lattice has the value P_{th}=0.5 exactly. One of the most important aspects of this result is that it is *independent of the radius distribution*; it only depends upon the topological structure of the network (actually, the co-ordination number (z) and the Euclidean dimension (d)). In fact, the percolation threshold and system topology are linked by the equation:

$$z.P_{th} = \frac{d}{(d-1)} \qquad (1)$$

([7], [8]). Table 1 shows percolation thresholds and calculated values of zP_{th} for a variety of two- and three-dimensional geometries. Cubic (3D) geometries of 30x30x30 nodes have been used for calculations here and, not surprisingly, calculations using larger networks give even greater accuracy.

Now, if instead of opening pores of random sizes, the pores are opened systematically beginning with those of largest radius (although these are still *located* at random positions in the network), it is clear that flow will commence once a cluster of large open pores spans the system (we refer to this model as a pure top-down fill and it is used purely for didactic purposes). The radius at which flow occurs is known as the *percolation radius*, R_p, and is defined implicitly by the equation:

$$P_{th} = \int_{R_P}^{R_{\max}} f(r)dr \qquad (2)$$

where f(r) is the normalized tube radius distribution function and P_{th} the percolation threshold. This is illustrated in Figure 1, where it is clear that the process yields a spanning cluster, together with a number of smaller isolated clusters. However, the simulation of low-rate drainage processes is carried out using a top-down *invasion percolation* model with hydraulic trapping of the wetting phase. In this case, the injected nonwetting phase first fills the largest pores *connected to the inlet face of the network*, and then proceeds sequentially occupying smaller and smaller pores. Although this process is very different from the pure top-down pore filling illustrated in Figure 1, the resulting flowing clusters are, in fact, identical. This is shown in Figure 2, where a physically realistic invasion percolation process has been implemented. Hence, the invasion percolation spanning cluster also appears at $R=R_p$ (although $p(R_p)$ and $S_{Hg}(R_p)$ will both tend to zero for very large networks, as the percolating cluster itself is highly fractal in nature).

The foregoing discussion means that (1) and (2) can also be utilised to study the invasion percolation process characterizing a drainage process. Rearranging (1) and (2) gives, for a 3D network (i.e. d=3):

$$\int_{R_p}^{R_{\max}} f(r)dr \approx \frac{3}{2z} \tag{3}$$

If a pore-size distribution (PSD) function of the form f(r) ~ r^n is assumed, then this becomes:

$$\int_{R_p}^{R_{\max}} \frac{(n+1)r^n}{(R_{\max}^{n+1} - R_{\min}^{n+1})} dr = \frac{(R_{\max}^{n+1} - R_p^{n+1})}{(R_{\max}^{n+1} - R_{\min}^{n+1})} \approx \frac{3}{2z} \tag{4}$$

Therefore, assuming that R_{min}, R_{max}, and R_p can be measured, equation (4) constitutes a relationship between coordination number (z) and PSD exponent (n) — i.e. one equation for 2 unknowns. How can a second equation be found? To answer this, consider trapping of a wetting phase during drainage (say, at the end of an oil-water or gas-water capillary pressure measurement). Symmetry arguments can be used to show that the number fraction of trapped pores at the end of drainage is given by:

$$\int_{R_{\min}}^{R_t} f(r)dr \approx \frac{3}{2z} \tag{5}$$

where R_t refers to the radius at which hydraulic continuity of the wetting phase is lost — that is to say, wetting phase trapping occurs at a capillary pressure $P_{ct} \sim 1/R_t$. Once again, assuming a power-law PSD, this can be rewritten as:

$$\int_{R_{\min}}^{R_t} \frac{(n+1)r^n}{(R_{\max}^{n+1} - R_{\min}^{n+1})} dr = \frac{(R_t^{n+1} - R_{\min}^{n+1})}{(R_{\max}^{n+1} - R_{\min}^{n+1})} \approx \frac{3}{2z} \tag{6}$$

(Note —it may be simpler to understand the symmetry argument if one considers the initial stages of a pure "bottom-up" imbibition process, where the smallest oil-filled pores in a network are displaced by water via film flow).

If R_t can be determined, then (6) is the missing second relationship linking coordination number and PSD exponent. The former can now be eliminated from equations (4) and (6) to give:

$$R_{\max}^{n+1} - R_p^{n+1} = R_t^{n+1} - R_{\min}^{n+1} \tag{7}$$

which can be solved uniquely for n, given R_{min}, R_{max}, R_p and R_t. Once n has been found, then z can be determined from (4) or (6).

The main assumptions in this method are the following: (i) that the porous medium can be approximated by a homogeneous network model in the first place, and (ii) that the network model is of a sufficient size to adequately make use of the results from percolation theory. The distribution of pore size should also either be random in space or any local spatial correlation structure - of length l, say - should be small compared with the system size, L (i.e. $l \ll L$). Whilst point (i) above is a matter for debate, point (ii) is simply a matter of required accuracy. This will be discussed later.

Having solved for coordination number and PSD exponent, attention next focuses upon determining the volume exponent (ν) — where the volume of a pore is given by $V(r) \sim r^{\nu}$. Now, the nonwetting phase saturation can be calculated at any capillary pressure (~1/R) for a fully accessible network from the integral:

$$S_{nw}(R) = \frac{\int_{R}^{R_{\max}} r^{(n+\nu)} dr}{\int_{R_{\min}}^{R_{\max}} r^{(n+\nu)} dr} = \frac{(R_{\max}^{n+\nu+1} - R^{n+\nu+1})}{(R_{\max}^{n+\nu+1} - R_{\min}^{n+\nu+1})} \qquad (8)$$

For invasion percolation simulations, however, an accessibility effect exists at low nonwetting-phase saturations. To circumvent this difficulty, equation (8) must be used at high nonwetting-phase saturations — ($1-S_{wr}$) for example, where S_{wr} refers to the residual wetting phase saturation. Hence, the volume exponent can be obtained by solving:

$$\frac{(R_{\max}^{n+\nu+1} - R_t^{n+\nu+1})}{(R_{\max}^{n+\nu+1} - R_{\min}^{n+\nu+1})} = 1 - S_{wr} \qquad (9)$$

where ν is the only unknown remaining.

Having described the background to the new methodology, the analysis will next be applied to a series of network simulations of mercury injection, where the data is no longer analytic in nature.

APPLICATION TO INTERCONNECTED NETWORKS

To investigate this issue, a number of different network simulations were undertaken using a wide range of (*z, n, ν*) parameter combinations. These are listed in Table 2. Sensitivities to network size, random number generator seed, and capillary pressure step-size were also studied. Values for R_p and R_t were found directly from each simulation by noting; (i) the capillary pressure at which the nonwetting phase relative permeability first became non-zero (~$1/R_p$), and (ii) the capillary pressure at which the wetting phase relative permeability dropped to zero (~$1/R_t$). Methods for determining these radii from the capillary pressure data alone — without recourse to flow data — are discussed later. Unless otherwise stated, all simulations were run on a 30x30x30 network and 96 capillary pressure steps were taken. R_{min}=2μm and R_{max}=50μm in all cases, although the method is equally applicable to any parameter values.

Table 3 shows the relative % errors in R_p, R_t, and S_{wr} (and S_{nwt} after imbibition, see later) between the measured (network) values and the expected values from percolation theory. For R_p, R_t, and S_{nwt}, the agreement between the actual data and theoretical predictions is excellent. The average relative % error between the actual and predicted values of R_t was only 2.52%, whilst the corresponding average error for the percolation radius (R_p) was only 1.49%. Note that the relative % errors for S_{wr} are not particularly accurate — these residual wetting phase saturations tend to be relatively small and so any statistical effects

are magnified by the relative % error measure. Predicted values of S_{nwt} were far more accurate, with an average relative error of 4.8% — it will be shown later that a more accurate prediction of volume exponent can be achieved by taking into account both S_{wr} and S_{nwt}. In Summary, these results clearly demonstrate that the analytical formulae based on percolation theory are directly applicable here. In general, those predictions that were most in error came from analysis using small (10^3) networks and a scan of the tabulated results shows that more accurate predictions of R_p and R_t could be expected as network size is increased.

Having measured R_p and R_t from the "mock experiment" (i.e. blind simulation), attention next turned to the prediction of the underlying network parameter combination (z, n, ν) using the percolation-based analysis of Section 3. R_p, R_t, and S_{wr} (and S_{nwt} if available) are now considered to be "experimental" inputs (indeed, when the method is applied to real rocks, these three or four values would be measured or inferred from experiment). The methodology is applied as follows:

(i) a GoalSeek-type macro tool is used in a spreadsheet to solve equation (7) for the pore-size distribution exponent n,
(ii) equation (4) is solved for the coordination number, z.
(iii) the GoalSeek macro is then used to solve equation (9) for the volume exponent, ν.

The results of applying the procedure to all cases tabulated in Table 2 are shown in Table 4 and Figure 3. The figure shows plots of the relative % error in the predicted values of z, n, and ν, compared to the actual values used in the simulations. It is clear that values for the predicted volume exponent, derived from S_{wr} measurements (ν_{pred}^{Swr}), are not particularly accurate — the reasons for this have been discussed earlier. In an attempt to improve upon this, imbibition simulations were performed for all cases and values of trapped *nonwetting* phase saturation, S_{nwt}, were used to predict (ν_{pred}^{Snwt}). These results are also shown in Table 4, together with the average ($\nu_{pred}^{Swr} + \nu_{pred}^{Snwt}$)/2.

The graphs presented in Figure 3 present direct visual evidence for the success of the percolation approach in *predicting* network parameters *a priori*. Remember that the simulation data was treated as "experimental", in that the input parameters used to generate the resulting capillary pressure curves were assumed to be unknown — i.e. the predictive procedure was applied blind to the raw output data. Predicted values for coordination number (z) and PSD exponent (n) were found to be in excellent agreement with the input parameters: an average relative error of only 2.14% was found between the predicted and actual values for z, whilst a corresponding average relative error of only 2.21% was found for n. This, in effect, means that the raw capillary pressure data arising from *any* network model drainage simulation can be interrogated using the procedure outlined here to *uniquely* determine the underlying coordination number and PSD exponent of that network. Note also that this approach is equally valid for site and site-bond models, where the connectivity and size distribution of pore throats could be similarly inferred.

One question remains, however — can the volume exponent (ν) be uniquely determined? The relative % error in the predicted volume exponent using S_{wr} (ν_{pred}^{Swr}) was 19.0%, with $\nu_{pred}^{Swr} < \nu$ — once again, small residual wetting phase saturations mean that any statistical effects are magnified by the relative % error measure and so it is not particularly surprising that ν_{pred}^{Swr} is a poor predictor. Rather surprisingly, the relative % error in ν_{pred}^{Snwt} — the predicted volume exponent using the trapped nonwetting phase saturation after imbibition — was equally poor (18.98%, with $\nu_{pred}^{Snwt} > \nu$). As this prediction comes from analysis involving S_{nwt}, which is itself predicted relatively accurately by the analysis (only 4.8% relative error), then the poor predictions of ν using ν_{pred}^{Snwt} must simply be a consequence of the power-law formulation expressed in the S_{nwt} equation analogous to (9) (this issue is currently under closer examination). All is not lost, however: the arithmetic average of the two volume exponent predictors (ν_{pred}^{Swr} and ν_{pred}^{Snwt}) gives an extremely accurate prediction of the actual simulation value. Indeed, the average relative error is only 3.32% (and, in some cases, the relative error can be as low as 0.5%).

This method has been used to produce the sample relative permeability predictions shown in Figure 4. These correspond to cases (1), (5), and (7) from Table 2. The results are excellent in all cases.

APPLICATION TO A CLASTIC SAMPLE

Having verified the predictive methodology using mock data, attention next turned to its application to gas-oil displacements in real rock samples. This work is still in progress but the first example is presented here. Experimental mercury injection capillary pressure data and residual phase saturations were examined in light of the theory presented earlier. Values for R_{min}, R_{max}, R_p, and R_t were identified, and equation (7) was solved for the PSD exponent (n). Equation (4) was then solved for coordination number (z), and the residual wetting phase saturation was used to derive a volume exponent (ν) using (9). A comparison between the measured and predicted relative permeabilities is shown in Figure 5 and the agreement is excellent. An optimised conductivity exponent of 3.8 was used in the simulation (although varying this parameter between 3.5 and 4 gave curves that were very closely grouped).

CONCLUSIONS AND FUTURE WORK

A powerful new approach to pore-scale ***prediction*** of two-phase relative permeability has been presented in this paper. Some percolation-based analysis has been used to produce a set of 2 simultaneous equations involving the system coordination number (z) and the pore-size distribution exponent (n). We have shown that these equations can be solved uniquely for the two network parameters concerned and, subsequently, that a simple residual saturation can be used to derive a unique volume exponent (ν).

The method is extremely simple and utilises 4 capillary entry radii that could be inferred from routine measurements (maximum and minimum capillary entry radii, R_{max} and R_{min},

together with 2 percolation radii, R_p and R_t). Values for R_p and R_t are easily found from "mock experimental" (i.e. blind simulation) data by noting; (i) the capillary pressure at which the nonwetting phase relative permeability first becomes non-zero ($\sim 1/R_p$), and (ii) the capillary pressure at which the wetting phase relative permeability no longer exists ($\sim 1/R_t$). S_{wr} and S_{nwt} are found similarly. However, this requires knowledge of the associated relative permeabilities, and methods for determining these radii from the capillary pressure data alone — without recourse to flow data — would be highly desirable when applying the method to experimental data.

Future work will focus on precisely this issue, although some thought has already gone into this. Firstly, a value for R_p could be determined directly from the slope of a capillary pressure curve (it corresponds to the peak in the so-called "pore-size distribution" function produced by commercial porosimeters). Secondly, it should be possible to estimate R_t and S_{rw} from a 2-phase drainage capillary pressure curve or even from MICP data (Cf. the R-plot analysis and "kink saturation" discussed in paper SCA 2001-15 — this, rather surprisingly, appears to correlate well with S_{rw}). S_{nwt} could be found from a trapped gas saturation measurement, 2-phase imbibition capillary pressure curve, or possibly mercury extrusion data. Finally, although R_{max} and R_{min} are rather well constrained, additional checks are required to ascertain that the chosen values are consistent with other macroscopic measures (for example, network simulations should reproduce the ratio of K_{abs} to K_{nw} @ S_{wr} observed experimentally)

In fact, even if no 2-phase data or direct endpoint saturations are available, it may still be possible to derive a unique network parameter combination from a mercury intrusion-extrusion cycle alone. This could then be used to construct an approximate network analogue of the particular experimental sample and a suite of relative permeability predictions could be made. Although this cost-effective approach based upon mercury intrusion data may have its limitations (i.e. correlations between 2-phase residual saturations and mercury residuals would have to be either measured or assumed), it is, nevertheless, based upon sound physical principles and should be a useful addition to the list of existing methods for relative permeability prediction.

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Table 1. Different bond networks satisfy the equation $Z.P_{th}=d/(d-1)$. In the tables below, P_{cb} is equivalent to P_{th}.

Network	Z	P_{th}	Z. P_{th}
Honeycomb (2D)	3	0.6527	1.96
Square (2D)	4	0.5	2
Kagome (2D)	4	0.522	2.088
Triangular (2D)	6	0.3473	2.084
Diamond (3D)	4	0.3886	1.55
Cubic (3D)	6	0.2488	1.49
BCC (3D)	8	0.1795	1.44
FCC (3D)	12	0.198	1.43

Table 2. A list of network parameter combinations used in the present study

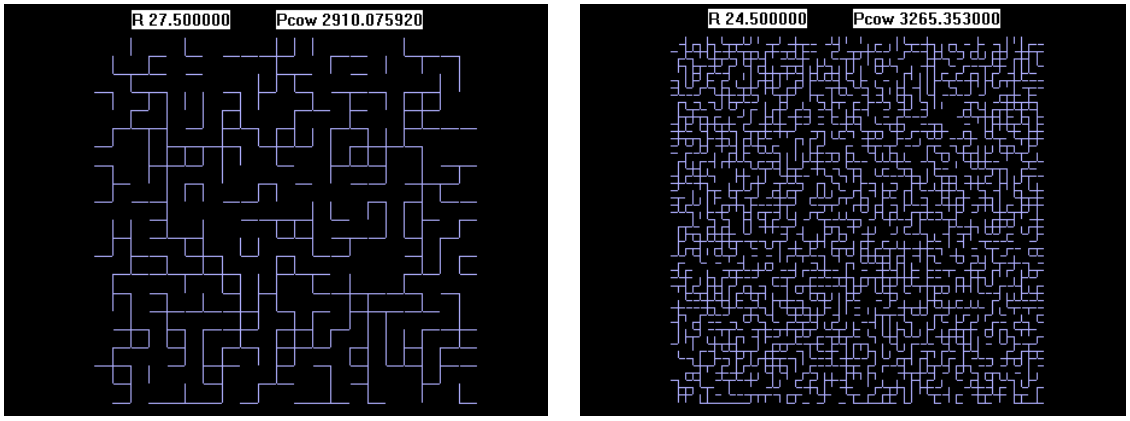
RUN	Z	n	ν	Net. Size	Seed	Pc Steps
1	6	0	2	30^3	21	96
2	4	0	2	30^3	21	96
3	4	0	1	30^3	21	96
4	6	-0.99	1	30^3	21	96
5	4	-0.99	2	30^3	21	96
6	6	3	2	30^3	21	96
7	4	3	1	30^3	21	96
8	4	0	2	10^3	21	96
9	4	0	2	15^3	21	96
10	4	0	2	20^3	21	96
11	4	0	2	40^3	21	96
12	4	0	2	10^3	21	192
13	4	0	2	15^3	21	192
14	4	0	2	20^3	21	192
15	4	0	2	30^3	21	192
16	6	0	2	30^3	12	192
17	6	0	2	30^3	23	192
18	6	-0.99	1	30^3	12	192
19	6	-0.99	1	30^3	23	192
20	4	3	1	30^3	12	192
21	4	3	1	30^3	23	192

Table 3 % Relative Errors in predicted percolation radii and residual saturations

RUN	Rp % Rel Error	Rt % Rel Error	Swr % Rel Error	Snwt % Rel Error
1	0.00	0.00	22.73	5.70
2	0.00	2.50	35.94	3.25
3	0.00	2.50	16.35	5.08
4	0.35	0.44	9.80	5.23
5	0.92	3.99	87.50	0.77
6	0.06	0.40	12.80	6.27
7	0.09	0.33	8.84	5.63
8	6.25	10.00	15.63	1.90
9	1.56	0.00	45.31	5.15
10	3.13	7.50	31.25	5.28
11	1.56	0.00	37.50	4.07
12	7.19	10.00	17.19	1.63
13	0.62	1.00	43.75	4.74
14	3.13	7.50	32.81	5.42
15	0.00	2.50	35.94	3.52
16	0.79	0.00	27.27	5.35
17	0.53	2.14	31.82	6.60
18	1.86	0.44	13.73	4.18
19	2.57	0.44	13.73	6.10
20	0.36	0.33	10.20	7.66
21	0.36	0.84	9.18	7.21
Ave.	**1.49**	**2.52**	**26.63**	**4.80**

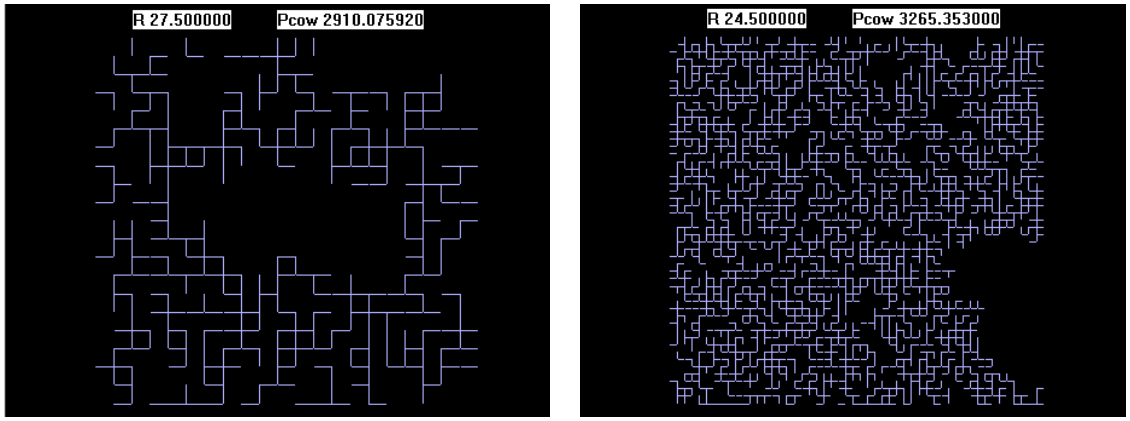
Table 4 % Relative Errors in predicted values of underlying network parameters

RUN	Z Rel % Error	$(n_{ave}-n)$ Rel % Error	ν_{Swr} Rel % Error	ν_{Snwt} Rel % Error	ν_{ave} Rel % Error
1	0.00	0.00	8.50	13.50	2.5
2	1.25	2.00	20.00	11.50	4.25
3	1.25	2.00	20.00	19.00	0.5
4	0.00	0.00	4.00	10.00	3
5	1.25	1.01	17.00	3.50	6.75
6	1.00	0.50	14.50	20.00	2.75
7	1.00	1.00	40.00	44.00	2
8	12.50	0.00	22.50	30.50	4
9	1.50	2.00	20.00	13.00	3.5
10	0.75	9.00	21.00	11.50	4.75
11	1.50	2.00	16.50	8.50	4
12	13.75	1.00	24.00	33.00	4.5
13	0.00	2.00	20.50	14.50	3
14	0.75	9.00	21.50	12.00	4.75
15	1.25	2.00	19.50	12.50	3.5
16	1.50	1.00	10.50	17.00	3.25
17	2.00	0.00	9.00	12.50	1.75
18	1.33	0.50	8.00	12.00	2
19	1.33	1.01	5.00	9.00	2
20	0.75	4.00	38.00	45.00	3.5
21	0.25	6.50	39.00	46.00	3.5
Ave	**2.14**	**2.21**	**19.00**	**18.98**	**3.32**



(a) (b)

Figure 1. Cluster structure at percolation threshold during primary drainage — the largest pores are filled first regardless of their position in the lattice (i.e. these figures refer to pure top-down bond percolation simulations). (a) 20 x 20 lattice, (b) 50 x 50 lattice.



(a) (b)

Figure 2. Cluster structure at percolation threshold during primary drainage — in this case, the largest pores are filled first but they must have hydraulic continuity to the inlet (LH) face of the lattice (i.e. these figures refer to invasion percolation simulations). (a) 20 x 20 lattice, (b) 50 x 50 lattice. Notice that the spanning clusters are identical to those in Figure 1.

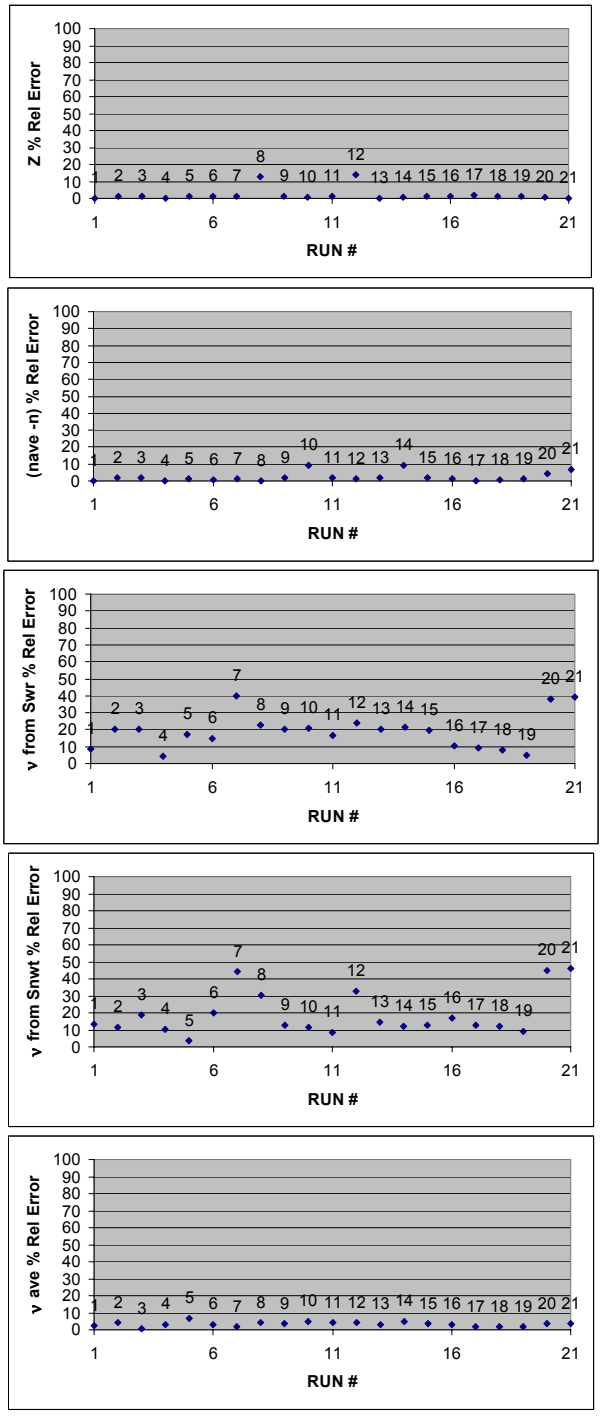
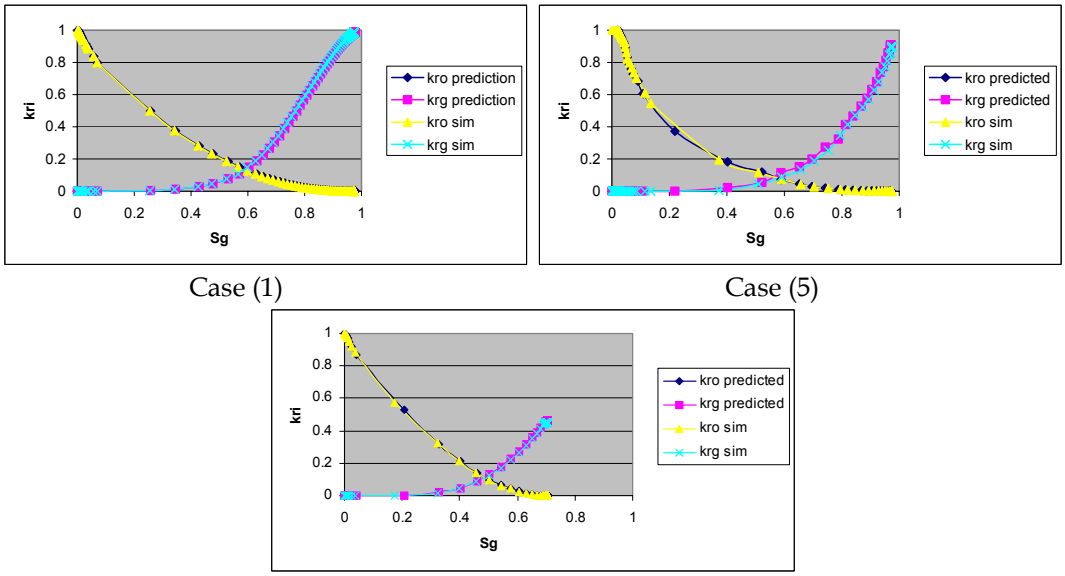


Figure 3. Plots of % Relative Error of *z, n, ν* from 21 network simulations (see Table 2). Errors are those between mock experimental values (actually obtained from numerical simulations) and analytical predictions. Three different predictions of ν were made (ν_{pred}^{Swr}, ν_{pred}^{Snwt} and $\nu_{ave}=(\nu_{pred}^{Swr} + \nu_{pred}^{Snwt})/2$).



Case (1) Case (5)

Case (7)

Figure 4. Predicted vs "mock experimental" (i.e. blind simulation) gas-oil relative permeabilities. These correspond to cases (1), (5), and (7) in Table 3.

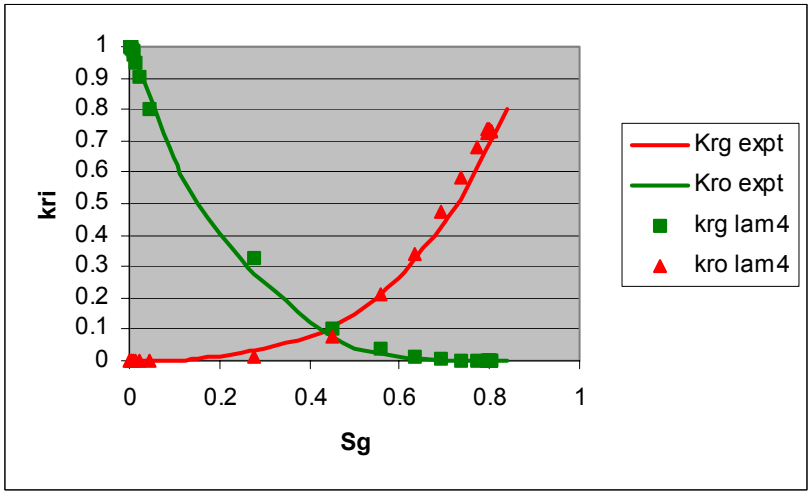


Figure 5. Comparison between experimental and predicted gas-oil relative permeabilities.