

EFFECTIVE STRESS LAW FOR THE PERMEABILITY OF CLAY-RICH SANDSTONES

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INTRODUCTION

Petrophysical properties of porous sedimentary rocks depend on both the confining pressure, P_c , and the pore pressure, P_p . For example, if hysteresis is neglected, a property such as the permeability k can be expressed as some function $k = f(P_c, P_p)$. If the permeability can be expressed as a function of the single parameter $P_c - n_k P_p$, i.e., $k = f(P_c - n_k P_p)$, we then say that it follows an *effective stress law*, with n_k being the effective stress coefficient, and $P_c - n_k P_p$ being the *effective stress*. For a rock whose mineral phase consists of a single mineral, say quartz, the effective stress coefficient is not expected to exceed unity [1]. However, Zoback and Byerlee [2] showed that the effective stress coefficient of some clay-rich sandstones can in fact be as high as 3-4. Walls and Nur [3] found that n_k increased with clay fraction, and reached values as high as 7 for sandstones with volumetric clay fractions of 20%.

To explain this behavior, Zoback and Byerlee proposed a model in which the rock consists of quartz, permeated with cylindrical pores that are lined with a shell-like layer of clay. As the inner clay layer is more compliant than the outer quartz layer, such a rock should be more sensitive to changes in pore pressure than to changes in confining pressure. Although this model has frequently been invoked qualitatively, a quantitative discussion of this model does not seem to have been given. A related model is one in which the clay is in the form of particles that are only tangentially attached to the pore walls. In this paper, we utilize newly-developed solutions of the equations of elasticity for these models, along with previously available solutions for viscous fluid flow in these geometries, to find the effective stress coefficients. Using recently collected data on the elastic deformation of clays [4], which show clays to be about twenty times more compliant than quartz, we find that both models do indeed yield effective stress coefficients that increase with increasing clay content. The second model gives higher values of n_k , which are in somewhat closer agreement with those found in the literature.

REVIEW OF RESULTS FOR CLAY-FREE ROCKS

Before describing the two models used in our study, it is worth reviewing the results for a rock consisting of a single mineral. If the pores are assumed to be cylinders of radius a , flow through each pore will be governed by Poiseuille's law [5], which states that the hydraulic conductance of the tube is proportional to a^4 . Other factors will influence the

overall permeability, such as the interconnectedness of the pores, but these are assumed not to vary with stress. Hence, to find the dependence of permeability on stress, we need only find the variation of the pore radius a with stress, as shown more rigorously below.

It follows from the expression $k = f(P_c - n_k P_p)$ that the effective stress coefficient for permeability, n_k , can be defined as the ratio of the sensitivity of permeability to changes in pore pressure, to the sensitivity of the permeability to changes in confining pressure:

$$n_k = \frac{-(dk / dP_p)_{dP_c=0}}{(dk / dP_c)_{dP_p=0}}. \quad (1)$$

If k depends on the stresses only through the pore radius a , use of the chain rule gives

$$n_k = \frac{-(dk / dP_p)_{dP_c=0}}{(dk / dP_c)_{dP_p=0}} = \frac{-(dk / da)(da / dP_p)_{dP_c=0}}{(dk / da)(da / dP_c)_{dP_p=0}} = \frac{-(da / dP_p)_{dP_c=0}}{(da / dP_c)_{dP_p=0}}. \quad (2)$$

Hence, according to this model, the effective stress coefficient for k is essentially the same as that for a (and, for the pore volume).

If the elasticity equations are solved for a cylindrical pore in an elastic body, it is found that the effective stress coefficient depends on the porosity ϕ and the Poisson ratio ν of the medium, but never exceeds unity [5,6]. For simplicity, and with little loss of generality, it is convenient to assume a typical value of ν , such as 0.25, in which case

$$n_k = (2 + \phi) / 3. \quad (3)$$

If the pores were assumed to be elliptical rather than cylindrical, the effective stress coefficient would increase, approaching unity in the limit of thin crack-like pores, but never exceeding it [5,6]. Hence, it seems that values of $n_k > 1$ should only be expected to occur in a sandstone if it contains clay. Indeed, the data collected from various sources by Kwon *et al.* [7] shows that n_k increases almost linearly with clay content, reaching values as high as 7 when the clay content is 20%.

DESCRIPTION OF MODELS FOR CLAY-RICH SANDSTONES

To explain the results mentioned above in a quantitative way, two different pore-clay models are examined here. The first model is the Zoback and Byerlee shell model, in which the rock consists mainly of a single mineral, say quartz, permeated with cylindrical pores that are lined with shell-like layer of clay (Fig. 1a). In the second model, the rock again consists mainly of quartz permeated with cylindrical pores, but with the clay situated as particles that are touching, but only weakly coupled to, the rock matrix (Fig. 1b). These two somewhat idealized models may be expected to represent extreme cases with regards to the extent of coupling between the clay and rock.

Model 1

In this model, the clay is equally distributed over all of the pore walls, forming a thin layer (Fig. 1a). As in the clay-free case, the permeability will depend only on the radius of the pore tube, a , and the effective stress coefficient will be given by the ratio shown in equation (2). The dependence of a on the two applied stresses will of course be different in this case.

If a composite elastic medium such as this is subjected to a confining pressure P_c along its outer boundary $r = b$, and a pore pressure P_p along its inner boundary $r = a$, the radial displacement u will be a function only of the radius, r . The displacement will have the form $u = Ar + B/r$, where A and B are constants [8]. Different values of A and B will apply in the clay region, $a < r < c$, and in the rock region, $c < r < b$. The values of these four constants are found by applying the following boundary conditions:

$$\tau_{rr}(r = a) = -P_p, \quad \tau_{rr}(r = b) = -P_c, \quad (4)$$

$$u \text{ and } \tau_{rr} \text{ continuous at } r = c, \quad (5)$$

where τ_{rr} is the radial normal stress. This stress is related to the displacement through

$$\tau_{rr} = (K + \frac{4}{3}G) \frac{du}{dr} + (K - \frac{2}{3}G) \frac{u}{r}, \quad (6)$$

where K and G are the bulk and shear moduli. These four conditions allow the four constants, $\{A_{rock}, B_{rock}, A_{clay}, B_{clay}\}$, to be found in terms of the four elastic moduli, the porosity, the clay fraction, and the two applied pressures. The resulting expressions are lengthy, and the algebraic details can be found in [9].

Model 2

In this model the clay exists in the form of particles that are tangentially connected to the pore walls. In this configuration, the clays will have essentially no influence on the effect that the confining pressure has on the pore geometry. An increase in confining stress will cause the pore channel to deform in the same manner as if the clay were not present, and furthermore will have essentially no influence on the geometry of the clay particles. The pore pressure will cause the pore wall at $r = a$ to expand radially, exactly as in the clay-free case. But the pore pressure, which acts over essentially the entire outer boundary of the clay particle, will also cause a uniform hydrostatic compression of the clay particle. Hence, this model does not require the solution to any new elasticity problems.

In this model, the permeability will depend on the geometry of the region of the pore that is not occupied by the clay particles. Hence, we need a solution for the viscous flow problem within the open pore space. In order to yield a tractable, two-dimensional flow problem, we assume that the clay exists as a solid cylinder of radius c , touching the pore wall. (Alternative models could include, for example, spherical clay particles attached to the pore walls at random locations.) The region available for fluid flow is then the region

between two eccentric cylinders, of radii a and c , in the limiting case in which the inner cylinder is touching the outer one. Formally, the derivatives appearing in equation (1) must then be calculated as

$$\frac{dk}{dP_p} = \frac{dk}{da} \frac{da}{dP_p} + \frac{dk}{dc} \frac{dc}{dP_p}, \quad (7)$$

and similarly for the confining pressure. The partial derivatives of a and c with respect to pressure are already known, as explained in the previous paragraph. The derivatives of k with respect to a and c are found by differentiating the solution to the viscous flow problem, which is given in [10] in the form of a complicated infinite series. After making several simplifications and approximations to this solution, the details of which can be found in [9], we eventually arrive at the following expression for the effective stress coefficient for model 2:

$$n_k = \frac{\phi + 2}{3} + \frac{c(1 - \phi)\gamma}{10(0.56 - c)}, \quad c = \sqrt{(1 - \phi)F_c}, \quad (8)$$

where ϕ is the porosity of the rock, F_c is the clay fraction (defined here as the volume of clay divided by the total volume of solids), and $\gamma = G_{rock} / G_{clay}$ is the stiffness ratio.

RESULTS AND DISCUSSION

Model 1

The effective stress coefficient predicted by model 1 is plotted in Figure 2a as a function of clay fraction. The porosity is taken to be 20%, and, the Poisson's ratio of both the rock and the clay are taken to be 0.25. The different curves represent different values of the stiffness ratio. At zero clay content, all curves begin at the value 0.733, given by equation (3). In the limiting case on which the stiffness ratio is 1, *i.e.*, the clay and rock have the same elastic properties, then the system is equivalent to a uniform rock without clay, and the effective stress coefficient is consequently insensitive to clay fraction. For higher stiffness ratios, the effective stress coefficient increases with clay content, at a rate that increases with increasing stiffness ratio. However, no combination of parameters seem to be capable of yielding values of n_k that are greater than about, say, 3 or 4.

Model 2

Again, a rock with 20% porosity is assumed, and the effective stress coefficient n_k is plotted as a function of clay fraction, F_c , at different stiffness ratios, γ (Figure 2b). The results are qualitatively similar as those of model 1, in that the coefficient n_k increases as clay fraction increases, with the effect more enhanced for higher values of the stiffness ratio. But the numerical values of n_k are larger for model 2 than for model 1. For example, for a stiffness ratio of 20 and a clay fraction of 0.2, model 1 yields an effective stress coefficient of 1.93, whereas model 2 predicts a value of 4.60.

Experimental Data from Literature

Figure 3 shows the predictions of our two models, compared with some experimental data from the literature. Data for different clay-bearing sandstones collected in [7] from various sources [2,3,11,12,13] are shown in the figure, along with our model results. Note that each experimental data point corresponds to a rock having a different porosity; in order to compare the data to the model predictions, we choose a porosity of 20% for the model calculations. Similarly, we use the same stiffness ratio of 20:1 to generate our theoretical curves, although the actual values for the rocks probably varied from this value. Nevertheless, we see that the both models give the same trend as is observed in the data. In particular, model 2 gives a reasonable fit. This fit could be greatly improved by assuming a stiffness ratio of 30, which is not unreasonable.

CONCLUSIONS

We have discussed the implications of two conceptual models for clay-rich sandstones. The first model is the shell model proposed by Zoback and Byerlee [2], in which the clays line all of the pore walls, and the second is a model in which the clay exists as particles tangentially attached to the pore walls. Both models yield effective stress coefficients that increase with increasing clay content. The rate of increase depends on the stiffness ratio, $\gamma = G_{rock} / G_{clay}$. Using realistic values of the stiffness ratio, model 1 cannot yield values of n_k larger than about 3 or 4. Model 2, on the other hand, can give much higher values, and can roughly fit the data set collected by [7].

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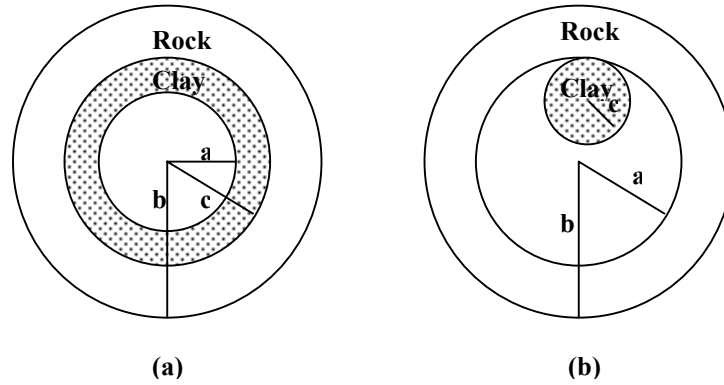


Figure 1. Cross sections of cylindrical pore clay models: (a) pore lined with shell-like layer of clay, (b) clay forming a cylindrical rod sitting along side wall of the pore.

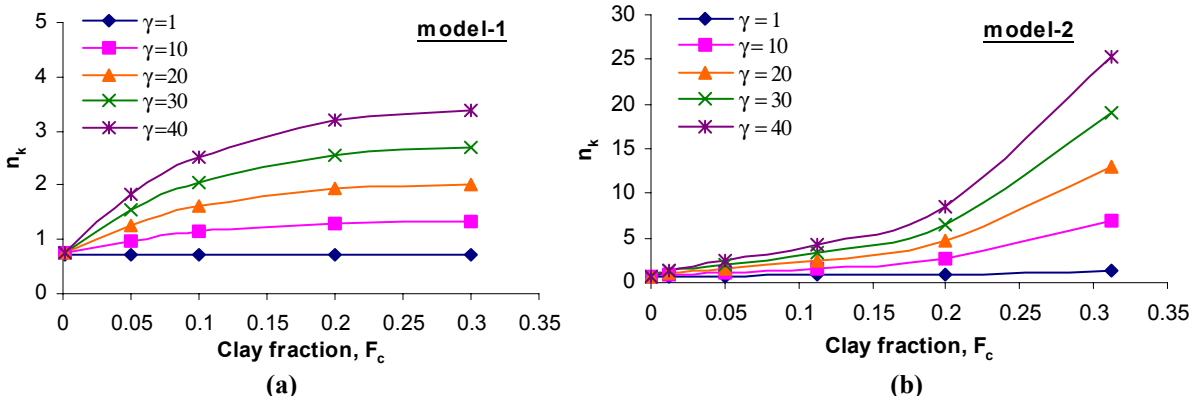


Figure 2. Effective stress coefficient n_k , as a function of clay fraction F_c for different stiffness ratios γ , for the two models.

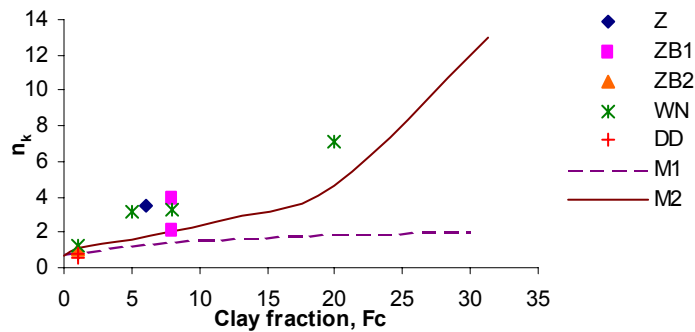


Figure 3. Effective stress coefficient n_k , as a function of clay fraction F_c . The points refer to data from literature and the lines refer to results from the models.