SCA2003-30: CHARACTERIZATION OF THE INTRINSIC POROSITY EXPONENT THROUGH DUAL-SALINITY MEASUREMENTS OF ELECTRICAL CONDUCTIVITY

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ABSTRACT
Dual-salinity measurements of electrical conductivity allow the identification of a petrofacies-specific intrinsic porosity exponent m* without having to determine any shaly-sand parameters. The method requires measurements of the conductivity of a reservoir rock when it is fully saturated with two different electrolytes of known conductivity. The method has been tested and benchmarked for sands ranging from clean to very shaly. It is not significantly impacted by low-salinity effects in most oilfield situations. A key element of the approach is that the pairs of electrolyte conductivities do not have to be the same for each sample. This is especially useful where a database covers several generations of core analysis. Field examples illustrate how the dual-salinity method can be used to quantify m* and identify petrofacies units simultaneously. The method requires fewer data than traditional multiple-salinity conductivity studies and it is more definitive than approaches that use an electrochemical measurement of the non-Archie conductivity. In these respects the dual-salinity approach offers a balance between the cost of data acquisition and the need to contain uncertainty.

INTRODUCTION
Archie's first law is an empirical relationship between porosity ϕ and formation resistivity factor, or simply formation factor, F [1]. The quantity F is the ratio of the resistivity of a fully water-saturated reservoir rock R_o to the resistivity of the saturating water R_w, or

$$F = C_w/C_o \qquad (1)$$

where C_o and C_w are the conductivities of the fully water-saturated rock and the saturating water, respectively. Archie proposed that F did not change as C_w was varied. The relationship between F and ϕ was written:

$$F = 1/\phi^m \qquad (2)$$

where m is the Archie porosity exponent. Equation (1) can be re-arranged to give the following expression for the conductivity of a fully water-saturated rock:

$$C_o = C_w/F \tag{3}$$

Equation (3) relates to Archie rocks. It does not describe the conductivity of non-Archie rocks, which is greater than that predicted by equation (3). To accommodate this extra conductivity, equation (3) is expanded to include a non-Archie conductivity term X:

$$C_o = C_w/F^* + X \tag{4}$$

where the formation factor is written F^* to denote that it is intrinsic, i.e. it has been corrected for non-Archie effects so that it is independent of C_w. Equation (4) can be seen as the sum of an Archie term and a non-Archie term. Where C_w is large and X is small, the Archie term is large and the non-Archie term is small, so that such a rock behaves as an Archie rock and equation (4) effectively reduces to equation (3).

Equation (4) has been rewritten for mathematical convenience with X broken down into a non-Archie conductivity x that is scaled by the same intrinsic geometric factor as the Archie term, following Waxman & Smits [2]. Thus:

$$C_o = C_w/F^* + x/F \tag{5}$$

Equation (5) describes a linear relationship between C_o and C_w if C_w is sufficiently large (Figure 1) [2]. We will consider only those reservoirs for which $C_w \geq 2.0$ S/m referred to a temperature of 25 °C: this conductivity corresponds to a water salinity of about 12 000 ppm NaCl. Above this salinity, any non-linear departure from equation (5) does not need to be considered for present purposes. With this caveat, equation (5) has traditionally been characterized by using multiple-salinity conductivity data to solve for F^* and x.

As noted earlier, the Archie definition of formation factor has to be modified in non-Archie rocks, because the ratio C_w/C_o is no longer independent of C_w. The quantity F^* is an intrinsic formation factor that has been corrected for non-Archie effects. Where these corrections have been made for non-Archie rocks, Archie's first law is rewritten:

$$F^* = 1/\phi^{m^*} \tag{6}$$

where m^* is an intrinsic porosity exponent and ϕ is total porosity. Equations (5) and (6) form the basis of the dual-salinity method.

DUAL-SALINITY METHOD

In essence, the dual-salinity method allows equation (5) to be solved simultaneously for F^* provided that we have two pairs of measurements of C_o and C_w. Values of F^* can then be correlated with porosity to obtain the intrinsic exponent m^* as per equation (6). At first sight the concept might seem straightforward. After all, a dual-salinity solution has already been proposed for

determining values of the intrinsic saturation exponent n*, a problem that seems far more complex [3]. However, the difference with the porosity exponent is that it is usually determined using data from several different core samples, whereas the partially saturated case uses desaturation data from the same sample, for which other physical properties such as porosity necessarily remain constant. This means that if we are to draw upon the benefits of a dual-salinity approach to characterizing Archie's first law, we have to be sure that we are dealing with data from the same rock type. To do this, we would need to work with a population of core samples for each of which the determined value of m* is the same to within the limits of uncertainty. This exposes a problem, because the whole purpose of equation (5) is to obtain F* for correlation with porosity in order to characterize m* through equation (6). Therefore, we do not know at the time we use equations (5) and (6) what the range of values of m* will turn out to be. The problem has been approached by using equations (5) and (6) differently.

Suppose we have two measurements of C_o for different electrolytes, simulated formation water (electrolyte 1) and a water of significantly higher salinity (electrolyte 2). Let the rock conductivities be designated C_{o1} and C_{o2} and the corresponding electrolyte conductivities be C_{w1} and C_{w2}, respectively. Then equation (5) can be written twice as follows:

$$C_{o1} = C_{w1}/F^* + x/F^* \tag{7}$$

$$C_{o2} = C_{w2}/F^* + x/F^* \tag{8}$$

Subtracting one equation from the other and substituting for F* from equation (6):

$$C_{o2} - C_{o1} = (C_{w2} - C_{w1}) \, \phi^{m^*} \tag{9}$$

Rearranging and taking logarithms:

$$\log_{10} ((C_{o2} - C_{o1})/(C_{w2} - C_{w1})) = m^* \log_{10} \phi \tag{10}$$

For convenience the composite quantity $(C_{o2} - C_{o1})/(C_{w2} - C_{w1})$, which is a (dimensionless) ratio of differences in conductivity, is here termed the *conductivity difference ratio* (C_{DR}). The quantity C_{DR} is the reciprocal of intrinsic formation factor F* where the C_o vs. C_w data distribution is perfectly linear (Figure 1). Otherwise it is an approximation to 1/F*. Equation (10) indicates that a bilogarithmic plot of ϕ vs. C_{DR} can be interpreted in terms of m*. A single linear data trend indicates that there is one petrofacies unit within the dataset for purposes of determining m* and thence characterizing equation (6) (Figure 2a). The recognition of a single linear trend is based on the following criterion. A data point with a value of m* that is displaced from a fitted linear trend value by more than ±0.1 is not part of the petrofacies unit to which that trend relates and it must be either assigned to another petrofacies or designated as an outlier (Figure 2a). This criterion relates to a single plug. It is based on the distribution of probable errors in single-sample values of m* obtained by applying partial-differential equations derived from equation (10) to the

test database that is described in the following section. A scattered data field is indicative of more than one petrofacies unit within a dataset (Figure 2b). The same criterion has been adopted for sorting the data into petrofacies units. Once the petrofacies units have been identified and separated, each subset can then be data-fitted to obtain petrofacies-specific values of m* (Figure 2b). Note that by using equation (10) in the above manner, it is possible to accommodate directly those datasets of ϕ and C_{DR} within which the two values of C_w are not the same throughout, provided that the conductivity behaviour of each plug can be described by the quasi-linear portion of Figure 1.

TESTING THE METHOD

The method has been tested using the core data listed in Table 3 of Waxman and Smits [2]. This database is described in Table 1. It allows a benchmarked value of m* to be established through a comprehensive suite of multiple-salinity conductivity data. It is also pre-sorted according to the degree of shaliness, into test dataset 1 (clean sandstone), test datasets 2 and 3 (shaly sandstone) and test dataset 4 (very shaly sandstone). The initial input data related to the following two values of water conductivity: $C_{w1} = 8.19$ S/m; $C_{w2} = 22.8$ S/m. Measurement temperature is 25 °C. For present purposes, C_{w1} will be taken as the conductivity of simulated formation water and C_{w2} the conductivity of the higher salinity water used specifically for the dual-salinity method. Figure 3 shows a composite bilogarithmic plot of ϕ vs. C_{DR} for the entire database. There is evidently more than one petrofacies unit represented here. The above criterion for distinguishing between petrofacies units can be used to sort the data for the purposes of the specific objective of characterizing m*. The starting point is the pre-sorting within the source data.

Test Dataset 1 - Clean Sandstone

Figure 4 shows that most of the data points lie on a contiguous trend, and they clearly belong to the same petrofacies unit (Petrofacies A) for the purposes of this particular petrophysical objective, i.e. characterizing m*. However, there are two other points that do not belong to this group and they seem to lie on a second possible trend (Petrofacies B). A value of m* has been determined for each petrofacies unit (Table 2).

Test Datasets 2 & 3 - Shaly Sandstone

Both these datasets furnish the same value of m* to within one standard error (Table 2). Therefore they have been combined into a single unit (Petrofacies C). The data fit for the composite Petrofacies C is shown in Figure 5: there are no outliers. Petrofacies C is a striking example of a heterogeneous petrophysical facies that can nevertheless be described by a single form of Archie's first law within the adopted limits of m* ± 0.1.

Test Dataset 4 - Very Shaly Sandstone
Figure 6 shows the data distribution for very shaly sandstone. There is more than one petrofacies unit. The contiguous data envelope has been treated as one unit (Petrofacies D). Figure 7 shows the data fit for this unit. The remaining points are assigned to Petrofacies E. The resulting values of m* are listed in Table 2.

Petrofacies Units
It can be seen from Table 2 that each of the five petrofacies units A - E has an exclusive value of m* at least at the 95% level (plus/minus two standard errors). It is not claimed that each of these units has been sufficiently sampled to be adequately characterized in terms of m*. That is a separate issue, which has to be based on statistical considerations of the data inputs [4]. However, a fivefold petrofacies classification is supported by the existing database but solely within the context of the single characterizing parameter m*. There is an element of judgement in identifying petrofacies units on the basis of just one characterizing parameter. In a field study, the identification process would involve other parameters such as the intrinsic saturation exponent n*.

Comparisons with Benchmarking Data
Porosity has been crossplotted with F* determined using the full set of multiple-salinity conductivity measurements [2]. The resulting m* is taken as the benchmark. This has been done separately for all petrofacies units. The dual-salinity results are compared with the benchmarks in Table 3. The agreement is excellent, even though there are limited data for petrofacies B and E. Thus, the dual-salinity determination of m* is validated.

Use of Different Pairs of Values of C_w
Different input data have been used in order to examine the impact of different pairs of values of C_w on the determined value of m*. All datasets relate to Petrofacies C. All stated conductivities are at 25 °C. The first case was that used above, i.e. $C_{w1} = 8.19$ S/m and $C_{w2} = 22.8$ S/m. The second case used the following two values of water conductivity: $C_{w1} = 3.73$ S/m and $C_{w2} = 8.19$ S/m. The third case used $C_{w1} = 3.73$ S/m and $C_{w2} = 22.8$ S/m. The final case used the mixture of values indicated in Table 4. The results of these four analyses are listed in Table 4. It can be seen that all determined values of m* are the same to within the limits of uncertainty at the 68% level (plus/minus one standard error). This confirms that the dual-salinity method is independent of the reference water conductivities provided that these do not extend below the linear portion of Figure 1.

FIELD EXAMPLES
These examples cover three distinct cases. The first dataset is from a slightly shaly sandstone reservoir that shows only minor departures from Archie conditions. The second describes limited data from a single petrofacies unit in a moderately shaly sand saturated with water of salinity less than that of sea water: this combination of factors makes the reservoir distinctly non-Archie. The

third and final dataset relates to shaly sandstones. The resulting values of m* and the associated standard errors are listed in Table 5.

Reservoir 1 - Slightly Shaly Sandstone

The dataset relates to 21 core plugs. The formation water conductivity is nominally $C_{w1} = 7.69$ S/m at 25 °C, and this corresponds to a salinity of about 50 000 ppm NaCl. The higher water conductivity is $C_{w2} = 13.49$ S/m at 25 °C: this corresponds to a salinity of 100 000 ppm NaCl. A bilogarithmic plot of ϕ vs. C_{DR} shows two petrofacies units F and G (Figure 8). The composite value of m* = 1.99. The data are plotted separately for Petrofacies F and G in Figures 9 and 10, and the resulting values of m* are 1.89 and 2.14, respectively. There are no outliers. It can be seen from Table 5 that in terms of m* alone, Petrofacies Units F and G are distinct even at the 99% level (plus/minus three standard errors).

Reservoir 2 - Moderately Shaly Sandstone

This is a limited dataset of seven plugs. The formation-water conductivity is $C_{w1} = 3.72$ S/m at 25 °C and this corresponds to a salinity of about 23 000 ppm NaCl. The higher water conductivity is $C_{w2} = 12.35$ S/m at 25 °C: this corresponds to a salinity of about 90 000 ppm NaCl. The dual-salinity data are plotted in Figure 11. There is a well-defined trend, designated Petrofacies H for which m* = 1.87 (Table 5). There are no outliers.

Reservoir 3 - Shaly Sandstone

These data relate to 26 samples of Palaeogene sandstone from Bulgaria [5]. The lower water conductivity is nominally 7.94 S/m at 20 °C and this corresponds to a salinity of about 55 000 ppm NaCl. The higher water conductivity is 22.57 S/m at 20 °C; this corresponds to a salinity of 250 000 ppm NaCl. A bilogarithmic plot of ϕ vs. C_{DR} shows more than one petrofacies unit (Figure 12). The composite value of m* = 1.73. The data are plotted separately for two units, designated Petrofacies J and K, in Figures 13 and 14, and the resulting values of m* are 1.70 and 1.89, respectively. There is an outlier for Petrofacies J. There are no outliers for Petrofacies K. Again, it can be seen from Table 5 that in terms of m* alone, Petrofacies Units J and K are distinct at the 99% level, although the sample populations are unequal.

DISCUSSION

These field examples demonstrate that the dual-salinity method allows m* to be characterized for groups of core samples from different petrofacies units within diverse reservoirs of extreme lithological character. The limiting value of the most probable error in m*, i.e. ± 0.1, is an important part of this process. This tolerance has been substantiated for the test database of Waxman & Smits (1968). It could readily be re-established for any other benchmarking database. In particular, the dual-salinity method has been established using data measured at 25 °C. For higher temperatures the range of C_w over which the method is applicable will need to be re-examined.

CONCLUSIONS

The dual-salinity method of characterizing the intrinsic porosity exponent m* for a core sample population avoids the measurement of electrochemical parameters such as cation exchange capacity. It is also data-economical relative to the benchmarking multiple-salinity conductivity method. The method has been tested using a heterogeneous core database for sands that range from clean to very shaly and that were measured at 25 °C. At this temperature, the method is applicable where formation water conductivity $C_{w1} \geq 2$ S/m. Over this conductivity range and for the test database, the dual-salinity method can furnish a single-sample value of m* with a most probable error that is no greater than ±0.1. This affords a basis for assigning samples to petrofacies units on the basis of m* alone. For multiple-sample applications, the dual-salinity method has furnished values of m* that agree with reference multiple-salinity benchmarks to within plus/minus one standard error. Tests of the dual-salinity method using electrolytes of different salinity for the same core sample population have delivered values of m* that are similarly consistent. Field examples have resulted in the characterization of petrofacies units with determined values of m* that are distinct at the 99% level. The approach can be merged with other areas of petrophysical characterization to generate a petrofacies classification based on all the key characterizing core-derived parameters that are needed for well log analysis. All this points to a method of characterizing m* that is flexible, robust and cost-effective.

REFERENCES

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Table 1   Database of multiple-salinity conductivity measurements

Suite	Description	No. of samples	Sample Nos.	
			Waxman-Smits [2]	**This study**
1	Clean sandstone	11	1 - 11	1 - 11
2	Shaly sandstone	15	1 - 15	12 - 26
3	Shaly sandstone	6	1 - 6	27 - 32
4	Very shaly sandstone	23	1 - 23	33 - 55

Table 2   Dual-salinity values of m* for the test database

Pre-sorting	Description	Petrofacies	No. of samples	m*	Standard error
Dataset 1	Clean sandstone		11	1.87	0.03
		A	9	1.82	0.01
		B	2	2.02	0.01
Dataset 2	Shaly sandstone		15	1.93	0.02
Dataset 3	Shaly sandstone		6	1.94	0.04
		C	21	1.93	0.02
Dataset 4	Very shaly sandstone		23	2.35	0.03
		D	17	2.43	0.02
		E	4	2.21	0.04

Table 3   Comparison of dual-salinity and benchmark values of m*

Petrofacies	Dual-salinity		Benchmark	
	m*	standard error	m*	standard error
A	1.82	0.01	1.81	0.01
B	2.02	0.01	2.01	0.01
C	1.93	0.02	1.92	0.02
D	2.43	0.02	2.42	0.03
E	2.21	0.04	2.23	0.05

Table 4   Dual-salinity values of m* for Petrofacies C using different pairs of values of C_w

Case No.	Sample Nos.	C_{w1} (S/m)	C_{w2} (S/m)	m*	Standard error
1	12 - 32	8.19	22.8	1.93	0.02
2	12 - 32	3.73	8.19	1.90	0.02
3	12 - 32	3.73	22.8	1.93	0.02
4	12 - 18	2.13	22.8	1.93	0.02
	19 - 25	3.73	22.8		
	26 - 32	8.19	22.8		

Table 5   Dual-salinity values of m* for field examples

Reservoir	Description	Petrofacies	No. of samples	m*	Standard error
1	Clean sand		21	1.99	0.03
		F	11	1.89	0.01
		G	10	2.14	0.02
2	Moderately shaly sand	H	7	1.87	0.03
3	Shaly sand		26	1.73	0.02
		J	21	1.70	0.02
		K	5	1.89	0.03
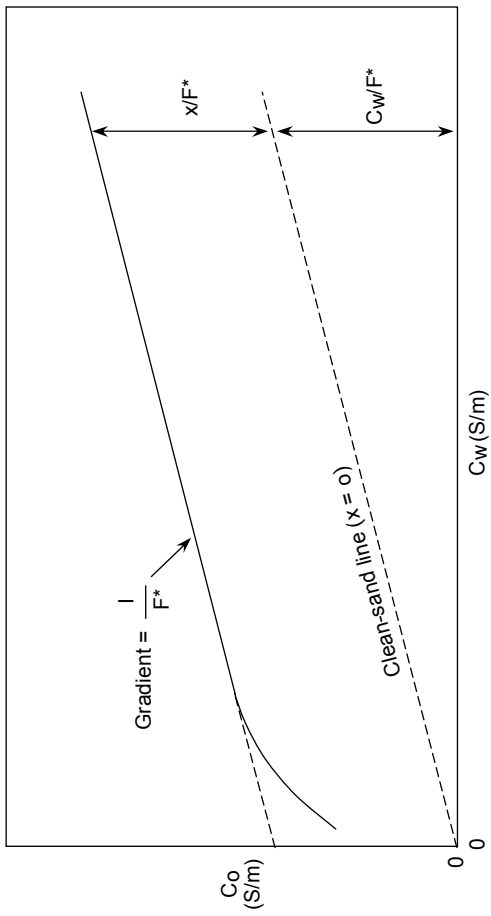
Fig.1 Schematic plot of C_o vs. C_w

(a)

(b)
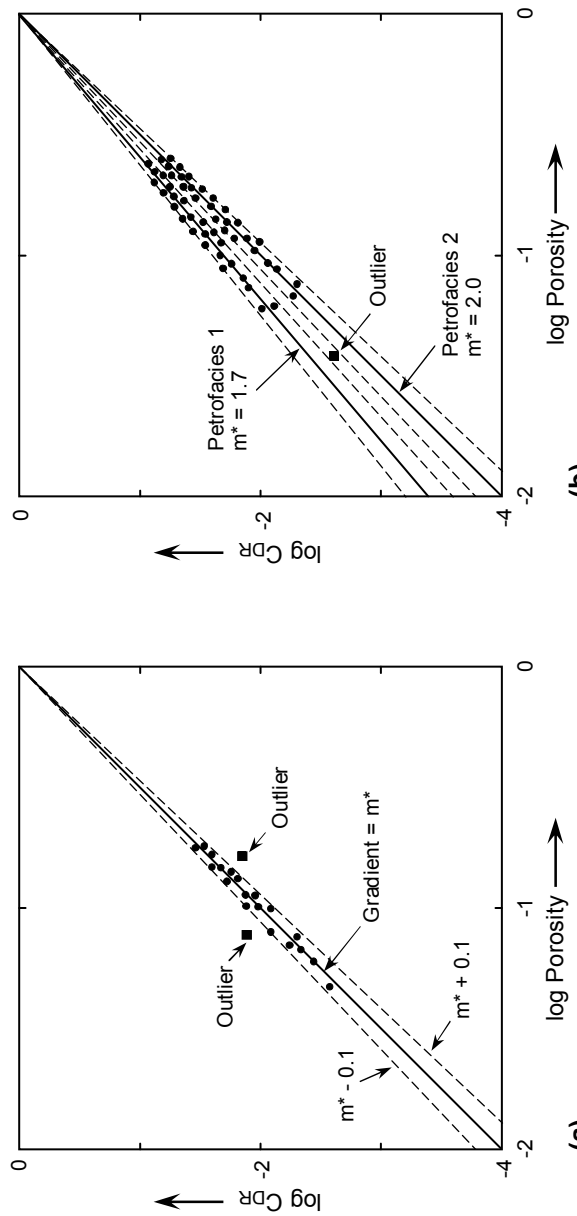
Fig.2 Schematic bilogarithmic plots of porosity vs. C_{DR} for: (a) single petrofacies unit with outliers; (b) two petrofacies units
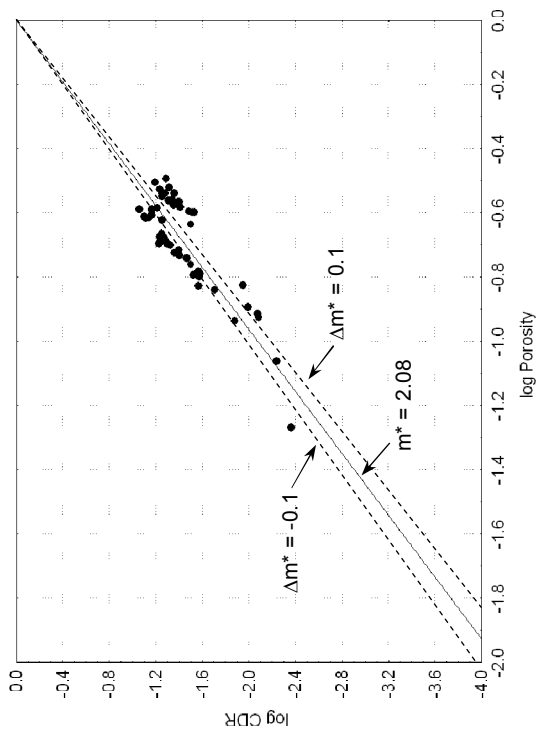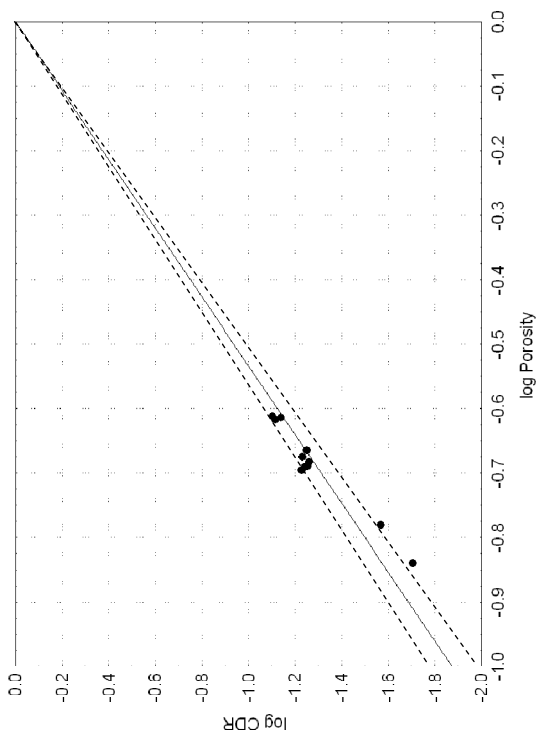
Fig.3   Porosity vs. C$_{DR}$ plot for the entire test database
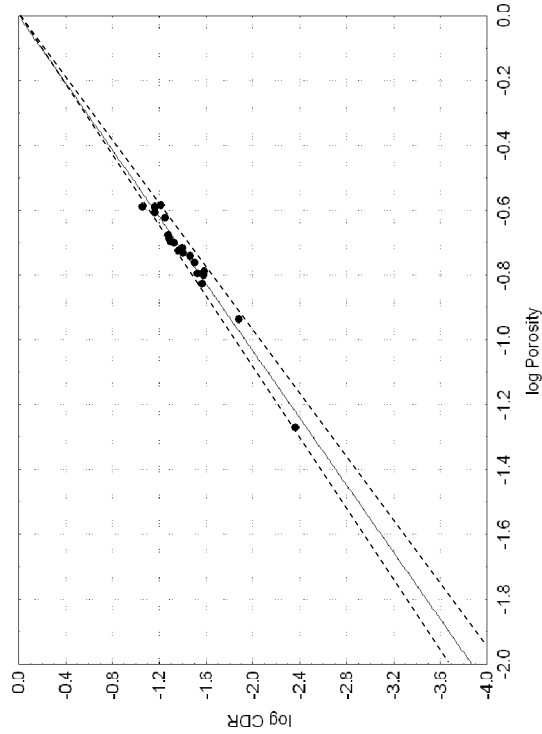

Fig.4   Porosity vs. C$_{DR}$ plot for Test Dataset 1

Fig.5   Porosity vs. C$_{DR}$ plot for Test Datasets 2 & 3
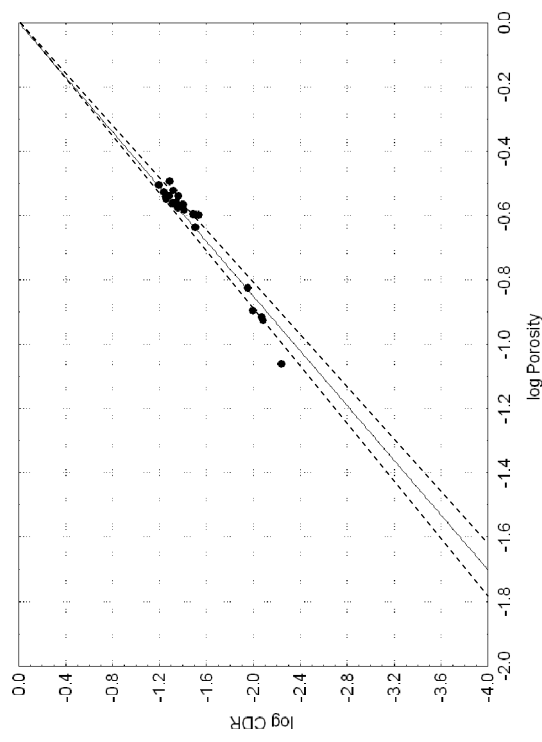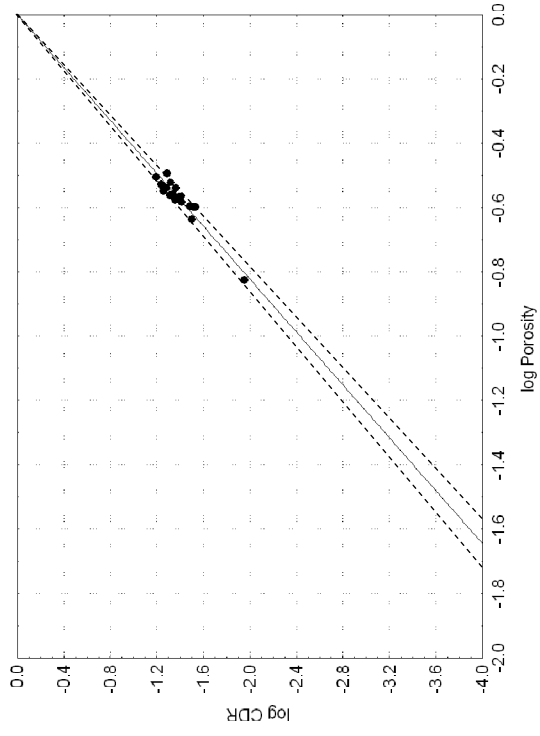

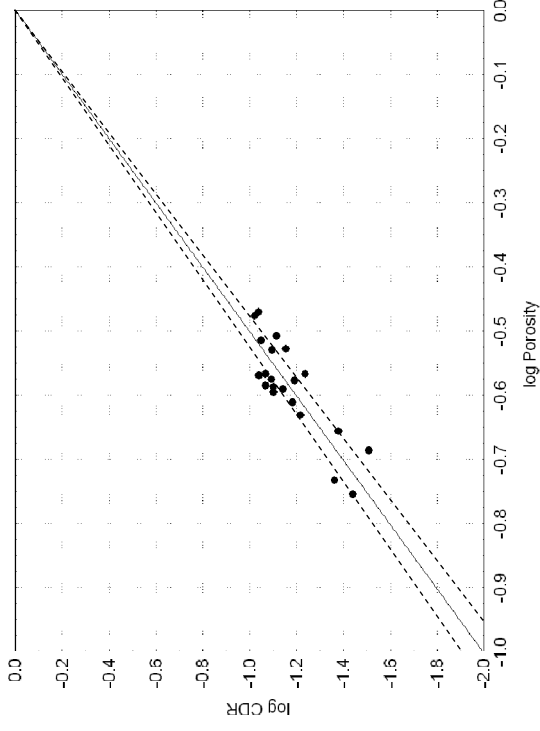Fig.6   Porosity vs. C$_{DR}$ plot for Test Dataset 4

Fig.7　Porosity vs. C$_{DR}$ plot for Test Petrofacies D

Fig.8　Porosity vs. C$_{DR}$ plot for Slightly Shaly Sandstone
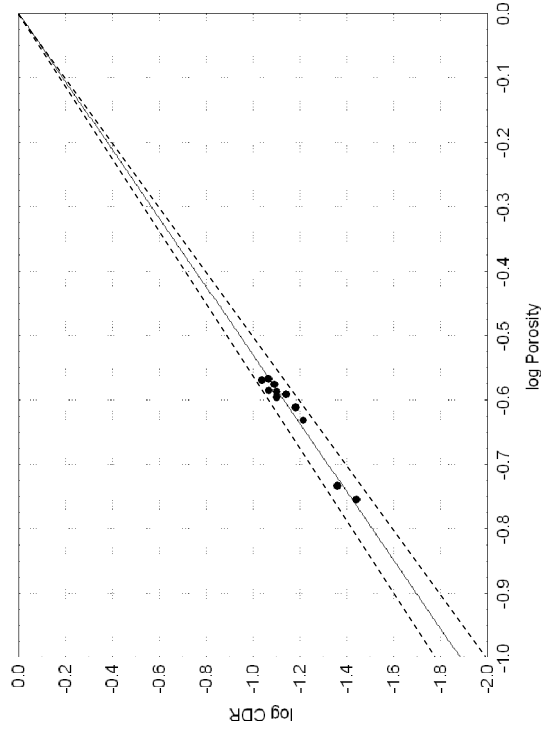
Fig.9　Porosity vs. C$_{DR}$ plot for Petrofacies F
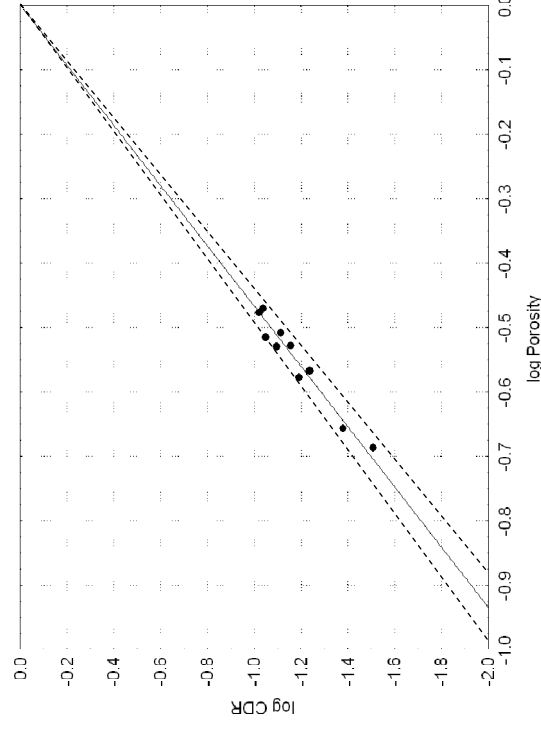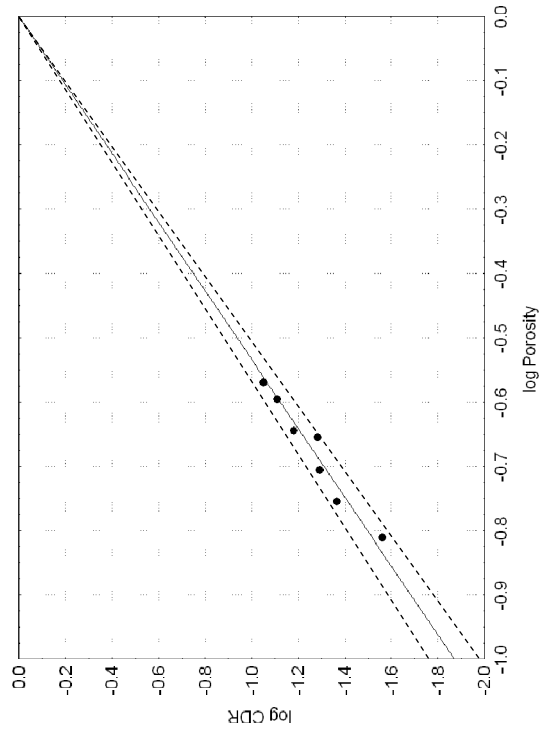

Fig.10　Porosity vs. C$_{DR}$ plot for Petrofacies G
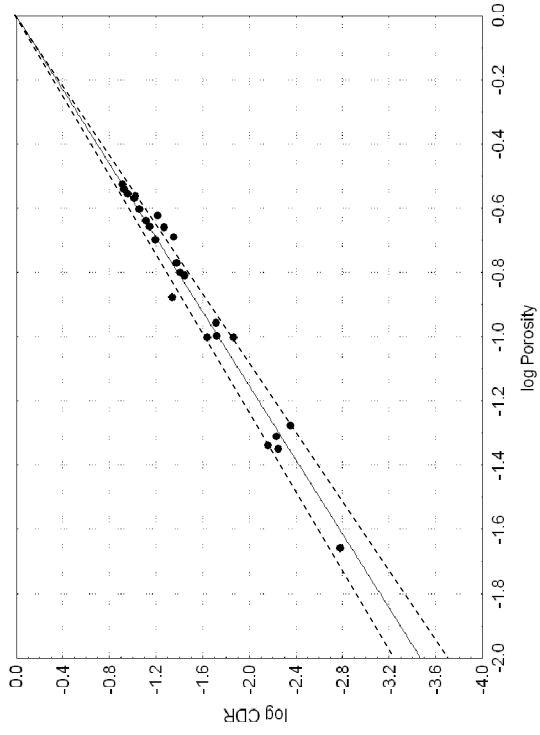
Fig.11   Porosity vs. C$_{DR}$ plot for Moderately Shaly Sandstone
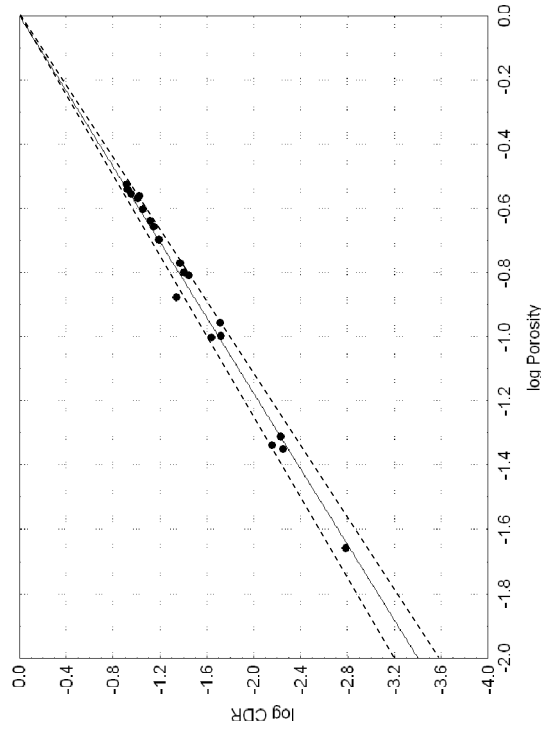

Fig.12   Porosity vs. C$_{DR}$ plot for Shaly Sandstone
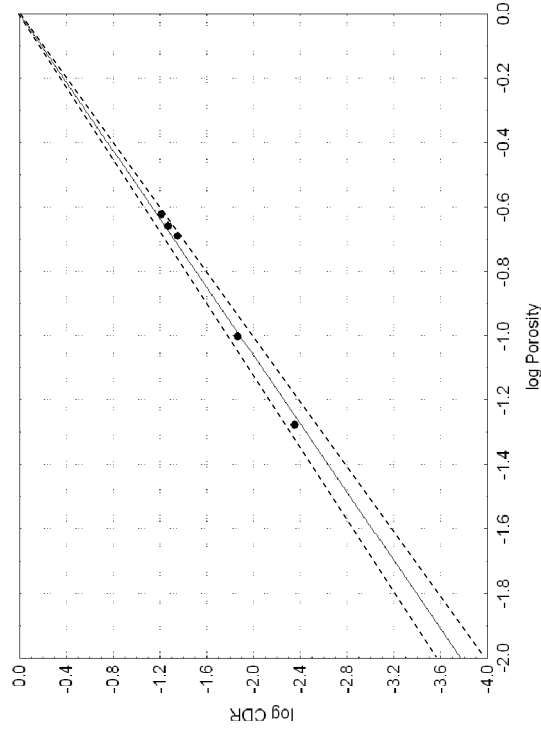

Fig.13   Porosity vs. C$_{DR}$ plot for Petrofacies J

Fig.14   Porosity vs. C$_{DR}$ plot for Petrofacies K