

# **SCA2003-67: ACCURACY ANALYSIS OF WATER SATURATION MODELS IN CLEAN AND SHALY LAYERS**

**Hamada, G.M.**

**Petroleum Engineering Department, Faculty of Engineering, Cairo University, Egypt.**

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## **ABSTRACT**

An accurate determination of initial oil in place in the early life of reservoirs or evaluation of developed reservoir is required to well estimate the hydrocarbon volumes either in place or left in the reservoir. Modified Archie formula ( $S_w = (a R_w / \phi^m R_t)^{1/n}$ ) is the basic equation to compute water saturation in clean formation or suitable shaly water saturation models in shaly formation. The exactness of water saturation value for given reservoir conditions depends on the accuracy of Archie parameters  $a$ ,  $m$  and  $n$ .

This paper presents a new technique to determine Archie parameters  $a$ ,  $m$  and  $n$ . The developed technique is based on the concept of three dimensional- regression (3-D) plot of water saturation, formation resistivity and porosity. 3D technique provides simultaneous values of Archie parameters. Also, 3D technique overcomes the uncertainty problems due to the separate use of formation resistivity factor- porosity and water saturation equations to get  $a$ ,  $m$  and  $n$  parameters.

A field example is given to show the applicability of 3-D technique and three other techniques: 1) common values of Archie parameters, 2) conventional technique and 3) core Archie parameter estimation (CAPE) technique. The comparison between the four techniques has shown that 3-D technique provides an accurate and physically meaningful way to get Archie parameters  $a$ ,  $m$  and  $n$  for given core samples. Water saturation profiles, using Archie parameters deduced from the four techniques, have been produced for the studied section in the well. These profiles have shown a significant difference in water saturation values. This difference could be mainly attributed to the uncertainty level for Archie parameters from each technique.

## **INTRODUCTION**

Petroleum literature presents an exhaustive review of the results determining Archie's parameters and also water saturation computation processes. In quantitative log interpretation, an accurate water saturation requires good values of Archie's parameters for being used either in Archie saturation equation in clean formation or in a shaly sand water saturation model in shaly formation (e.g., Ref. 1-5).

In this paper, the author proposed a new technique to determine Archie's parameters, three dimensional regression (3-D) technique which is based on the analytical expression of 3-D

plot of  $R_t/R_w$  versus  $S_w$  and  $\phi$ . Water saturation profiles were calculated using common values ( $a = 0.65$ ,  $m = 2.15$  and  $n = 2$ ), conventional; CAPE and 3-D techniques for selected well. In the case of clean sands Archie formula was used to compute water saturation. In the case of shaly sand sections three shaly sand models were applied and the final water saturation has been the average value of the three models derived water saturations.

## CALCULATION OF ARCHIE'S PARAMETERS

### Conventional Determination of a,m and n

In 1942 Archie proposed an empirical relationship between rock resistivity,  $R_t$ , with its porosity,  $\phi$ , and water saturation  $S_w$ ,

$$S_w^n = a R_w / \phi^m R_t = R_o / R_t = 1 / I_r \quad (1)$$

**Conventional Determination of a and m** The conventional determination of a and m is based on Eq. 2 and is rewritten as:

$$\log F = \log a - m \log \phi \quad (2)$$

Plot of  $\log F$  vs.  $\log \phi$  is used to determine a and m for the core samples. Cementation factor m, is determined from the slope of the least square fit straight line of the plotted points, while tortuosity factor is given from the intercept of the line where  $\phi = 1$ . Note that in this plot only points of  $S_w = 1.0$  are used. Archie parameters a and m were determined as 1.36 and 2.03 for well A.

**Conventional Determination of n** The classical process to determine saturation exponent, n, is based on Eq. 1. This equation is rewritten as:

$$\log I_r = -n \log S_w \quad (3)$$

Log - log plot of  $I_r$  versus  $S_w$  gives a straight line with negative slope n. The retained saturation exponent is the average values of the 6 core samples measurements. Saturation exponent equals to 2.1 for well A. It is obvious that the conventional technique treats the determination of n as a separate problem from a and m. This separation is not physically correct, thereby, it induces an error in the value of water saturation using Eq. 1.

### Core Archie-Parameter Estimation (CAPE)

Maute et al [7] has presented a data analysis approach to determine Archie's parameters m and n and optionally a from standard resistivity measurements on core samples. The analysis method, Core Archie- Parameters Estimation (CAPE) determines m and n and optionally a by

minimizing the error between computed water and measured water saturations. The mean square saturation error  $\varepsilon$ , is given by

$$\varepsilon = \sum_j \sum_i [S_{wij} - (aR_{wij}/\phi_i^m R_{t\ ij})^{1/n}]^2 \quad (4)$$

where  $j$  = core index,  $i$  = index for each of the core  $j$  measurements,  $S_{wij}$  =  $i$ th laboratory measured water saturation for core  $j$  (fraction),  $R_{t\ ij}$  =  $i$ th laboratory measured resistivity for core  $j$ ,  $\Omega.m$ , and  $\phi_j$  = core  $j$  porosity (fraction). Eq.4 calculates the minimum error between measured core water saturation and computed water saturation by Archie's formula, this is by adjusting  $m$ ,  $n$  and optionally  $a$  in the equation.

### Three Dimensional Regression (3-D)

We contend that, so far as Archie's parameters are concerned, the error in the water saturation value should be kept minimum. This is because water saturation quantity is desired and physically meaningful quantity. Here, we have developed a method to determine Archie's parameters  $a$ ,  $m$  and  $n$  using standard resistivity measurements on core samples.

### Methodology

The basis of the 3-D technique is to view  $S_w$  in Archie's formula Eq. 1 as a variable in three dimensional regression plot of  $S_w$ ,  $R_w/R_t$  and  $\phi$ . The 3-D technique determines Archie's parameters  $a$ ,  $m$  and  $n$  by solving three simultaneous equations of  $S_w$ ,  $R_w/R_t$  and Eq. 1 is rearranged after taking the logarithm of both sides.

$$\log R_w / R_t = - \log a + m \log \phi + n \log S_w \quad (5)$$

The left hand side of Eq. 5 is a dependent variable of the two independent variables  $S_w$  and  $\phi$ . Eq. 5 is an equation of a plane in three dimensional (3-D) space of coordinates  $x$ ,  $y$  and  $z$  ( $x = \log \phi$ ,  $y = \log S_w$  and  $z = \log R_w/R_t$ ). The intersection of this plane with the plane ( $x = 0.0$ ) gives a straight line of slope ( $m$ ), with the plane ( $y = 0.0$ ) giving a straight line with slope ( $n$ ) and with the plane ( $z = 0.0$ ) provides the value of ( $a$ ) parameter.

For a given set of data for a core sample, we can obtain an equivalent set of variables  $x$ ,  $y$  and  $z$ . Eq. 5 will take the following form for  $i$  measurement points:

$$Z_i = - A + m X_i + n Y_i \quad (6)$$

After normalizing Eq. 6 for  $N$  readings, we obtain the following three simultaneous equations

$$\sum Z_i = - N A + m \sum X_i + n \sum Y_i \quad (7)$$

$$\sum X_i Z_i = - A \sum X_i + m \sum X_i^2 + n \sum X_i Y_i \quad (8)$$

$$\sum Y_i Z_i = - A \sum Y_i + m \sum X_i Y_i + n \sum Y_i^2 \quad (9)$$

The solution of Eqs. 7-9 provides the values of Archie's parameters  $a$ ,  $m$  and  $n$  for one core sample. For  $j$  core samples, running the same analysis for  $j$  core samples produces average values of Archie's parameters. **Table 1** shows Archie's parameters calculated by 3-D technique in addition to, CAPE, conventional and common values.

## APPLICATION

**Fig.2** shows flow chart for a computer program used to compute water saturation in clean and shaly sections with the use of certain Archie's parameters from each technique Conventional, Cape and 3-D and also common Archie's parameters for sands ( $a= 0.65$ ,  $m= 2.15$  and  $n = 2$ ).

Water saturations have been computed for both clean sections and shaly sand sections using Archie parameters calculated by conventional , CAPE , 3-D techniques and common values. This has been done by repeating the computer programm four times, each time with certain Archie parameters  $a$ ,  $m$  and  $n$  values, **Fig. 1**. Considering the fact that water saturation has been computed in high shale volume sections as well as in low shale volume sections, we used three shale models and final water saturation value is taken as the average value of the three shaly sand water saturation models. It is recommended in these field examples to compute water saturation by three models Poupon et al, Doll and Indonesia and then water saturation will the average value of the calculated water saturations derived from the three models.

**Fig. 2** shows water saturation profiles for well A using Archies parameters  $a,m$  and  $n$  caculated by four techniques. Water saturation was computed in clean sand section with Archie's formula while in shaly sand section it was computed using shaly sand water saturation models. Water saturation has been measured in the laboratory on 10 core samples from the studied section in the well, the average water saturation of all core samples was 30.5 %. **Fig. 2a** illustrates water saturation profile with depth using common values (  $a = 0.65$ ,  $m = 2.15$  and  $n = 2$ ). Examination of figure 5a shows that water saturation is generally high and the average water saturation for 85 points was 34.4 % with standard of deviation in water saturation( $\sigma_{sw}$ ) values equals to 0.34. **Fig. 2b** is the water saturation profile versus depth with use of Archie's parameters caculated by conventional technique (  $a = 1.36$ ,  $m = 2.03$  and  $n = 2.1$ ). The average water saturation of the studied section of 85 points was 47% with standard deviation ( $\sigma_{sw}$ ) = 0.17. It is still high, but it is better than in the case of coomon values. **Fig. 2c** shows water saturation profile derived from water saturation equations using Archie's parameters calculated by CAPE technqiue (  $a = 3.21$ ,  $m = 1.7$  and  $n = 2.21$ ). The average water saturation of the retained section was about 28 % with standard deviation ( $\sigma_{sw}$ ) = 0.067. CAPE technique provided water saturation value much closer to the average value of core-measured water saturation (  $S_{wcore} = 30.5$  %) than conventional and common techniques. **Fig. 2d** depicts the water saturation profile deduced from the use of Archie's parameters caculated by 3-D technique (  $a = 2.94$ ,  $m = 1.56$  and  $n = 2.15$ ). Average value of 85 water saturation points was found 31.2 %. With standard of deviation( $\sigma_{sw}$ ) equals to 0.069. **Table 2** summarizes average water saturation by the four techniques and errors for well A.

## CONCLUSIONS

1. Conventional technique optimizes the two functions  $F$  vs  $\phi$  and  $R_t$  vs.  $S_w$  rather than water saturation values. While CAPE technique confirms that the quantity one should optimize is not the two functions but rather the water saturation
2. 3-D technique provides simultaneously the values of Archie's parameters from standard resistivity measurements on core samples. Unlike the conventional method, which ignored the values of  $S_w < 1.0$  in the determination of  $a$  and  $m$ , the 3-D method uses all data of  $S_w$  points.
3. 3-D technique answers the controversial question of whether tortuosity factor  $a$  should be fixed at unity or not. It gives directly  $a$ ,  $m$  and  $n$ , and thereby, it is recommended to consider the case of the three variables  $a, m$  and  $n$ .

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Table 1 Archie's parameters values from different techniques

Technique	A	m	N
Common values	0.65	2.15	2
Conventional	1.36	2.03	2.1
CAPE	3.21	1.7	2.21
3-D	2.94	1.56	2.15

Table 2 Average water saturation , error degrees and Archie's parameters techniques

Technique	$S_w$	$\sigma_w$
Common values	34.4	0.34
Cnventional	47	0.17
CAPE	28	0.067
3-D	31.2	0.069

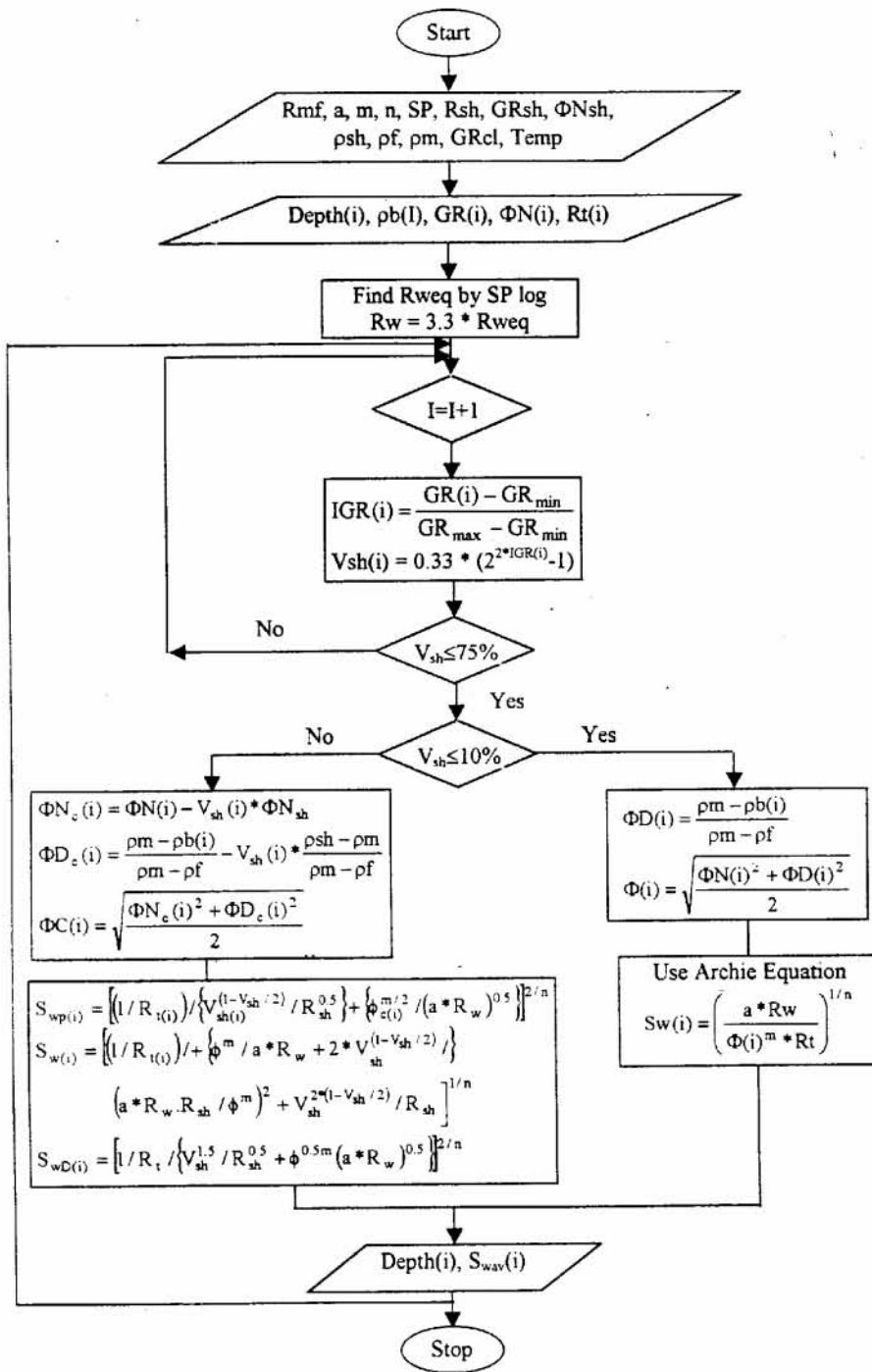


Fig. 1 Flow chart to compute water saturation.

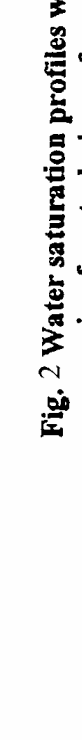
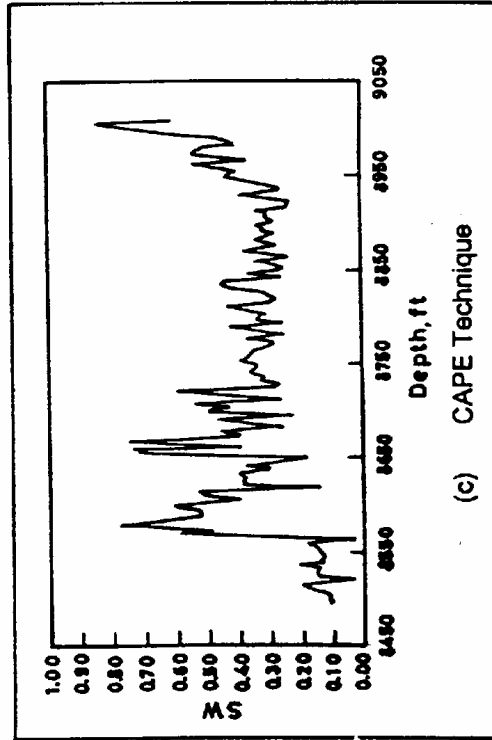
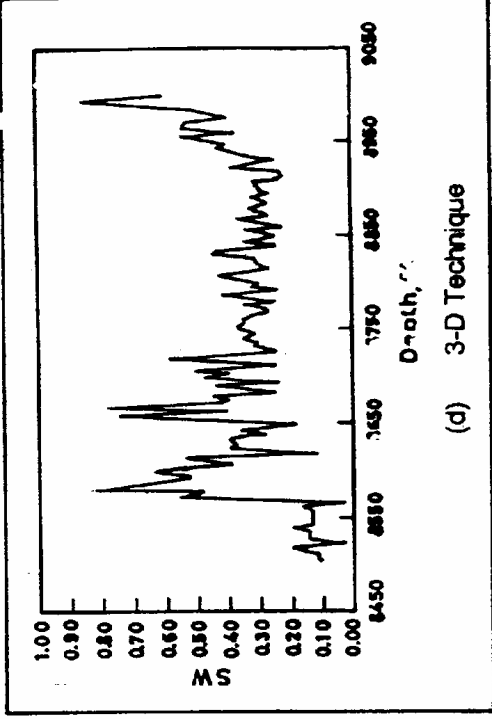
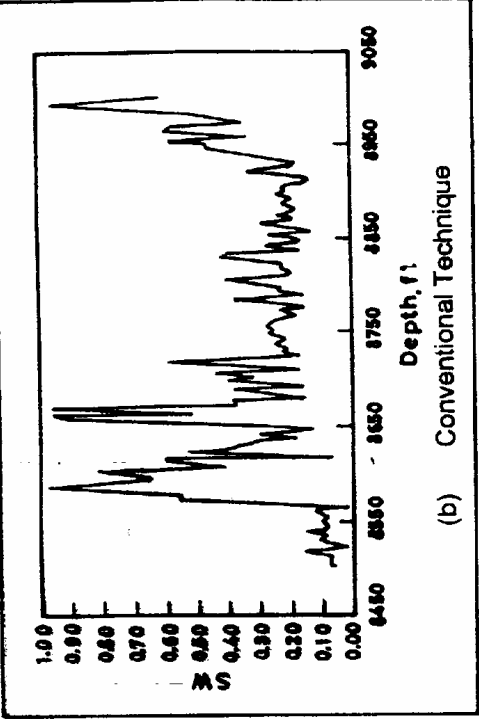


Fig. 2 Water saturation profiles with different Archie's parameters using four techniques for well A