AN ALTERNATIVE APPROACH TO THE DETERMINATION OF THE INERTIA FLOW COEFFICIENT

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ABSTRACT

A new approach to the estimation of the inertia flow coefficient, also called the turbulence factor, is proposed. This parameter, often named β, is the coefficient in the non-Darcy (second order) term of the general flow equation, for which the value normally needs to be determined through special core measurements, or derived from isochronal or multi-rate production tests.

The proposed formulation takes into account permeability, as well as porosity and formation resistivity through the Archie formation resistivity factor. The significance of the resistivity parameter is explained. The formulation utilizes a concept of actual fluid velocity, rather than volumetric flux in the derivation of the formulae for the β-factor.

The formulation has been tested against a wide range of core measurements – in addition to well test results. A regression analysis including a total of 83 cores, geographically scattered, with permeabilities varying from 0.01 mD to 3.6 Darcy and porosities in the range of 2.2 to 27 percent, yielded a regression coefficient (R^2) of 0.96. Dolomite samples fitted the correlation to the same degree as sandstones.

The proposed expression also leads to an alternative formulation of the general flow equation, indicating that the flow of a single-phase fluid through a porous medium to a great extent resembles the flow through pipes. The alternative formulation includes the definition of a porous media friction factor, equivalent to the friction factor used in the Fanning equation describing pressure gradients in pipes as a function of fluid velocity.

INTRODUCTION

More than 100 years ago, Philippe Forchheimer [1] published his paper: “Wasserbewegung durch Boden” where he demonstrated that permeability, as defined by Darcy’s law, appeared to decrease under high fluid velocity conditions.

Based on this observation, he formulated an extension to Darcy’s law, resulting in the well-known Forchheimer’s equation:

$$\nabla P = \frac{\mu u}{k_d} + \beta \rho u^2 \qquad (1)$$

Where:

∇P = pressure gradient
u = superficial velocity (volumetric flux)
β = inertial flow coefficient.
k_d = darcy permeability

For other parameters, see nomenclatures.

The essential parameter in this equation is the inertial flow coefficient, also called the turbulence factor, non-Darcy flow coefficient and various other names dependent on the author's view of what it represents. In this paper we will call it the beta-factor or simply β.

Fluid velocity conditions for which the second order term of the Forchheimer's equation will become important may frequently occur near the well bore of high flow rate gas wells and in the proppants pack of hydraulic fractures. In the reservoir itself, or in normally operated oil or water wells, non-Darcy flow is rarely a challenge.

Even though β is frequently called the turbulence factor, several researchers, including J. Geertsma [2] and Katz and Firoozabadi [3] rejects that the phenomena has anything to do with turbulence, mainly because the fluid velocities, even in well-bore proximities, are generally far too low for turbulence to occur.

Other authors, such as Jones [4] and Barre [5] have observed that β is not a constant, but will tend to decrease with increasing rates. The Forchheimer equation will thus over-predict the pressure loss at very high rates if this decrease in β is not accounted for. Barre suggests the use of an apparent permeability model in the form of a Log-Dose equation, with the definition of a Minimum Permeability Plateau (k_{min}), with the ability to cover the whole range of rates from normal Darcy flow to beyond the normal applicability of the Forchheimer equation.

Nevertheless, even though we do not contest the formulation proposed by Barre and others, we still believe that the Forchheimer equation, assuming β to be constant, is applicable and sufficiently accurate for most practical purposes. It also serves as a simple basis for the determination of non-darcy flow, in the lab as well as in the field.

For laboratory measurements, equation (1) is integrated and rearranged into the following form:

$$\frac{M(P_1^2 - P_2^2)}{2zRT\mu Lu} = \frac{u}{\mu} + \frac{1}{k} \qquad (2)$$

Where:

M = Molecular weight of the gas
P_l *= Inlet pressure*

P_2 = Outlet pressure
T = Absolute temperature
z = Real gas deviation factor at average pressure
μ = Gas viscosity at average pressure
L = Length of core
R = Real gas constant
u = Mass flux

If the temperature is constant and the pressure differences are small relative to the absolute pressure in the core, z and μ can be regarded as constants, and a plot of the left hand side of the equation vs u/μ will give a straight line. The slope of the line gives the beta-factor, and the intercept gives the reciprocal of the permeability.

BETA-FACTOR CORRELATIONS

Several correlations of the beta-factor have been published. All correlations found in literature use permeability or combinations of permeability and porosity, as correlation parameters.

The general form of these equations is:

$$\log(\beta) = a\log(\varphi^m k^n) + b \quad (3)$$

In which a, b, m and n are given different values.

The beta-factor must have the unit of reciprocal length [ft^{-1}] in order to be consistent with the general flow equation. Only correlations where a = 1 and n = -0.5 will give the correct unit *as long as $\log^{-1}(b)$ is a dimensionless correlating constant*. The only correlation found that has this quality is the one proposed by Geertsma[2]:

$$\log(\beta) = \log(\varphi^{-5.5} k^{-0.5}) - 2.3 \quad (4)$$

or on a more useful form (using standard oilfield units):

$$\beta = 0.005\varphi^{-5.5} k^{-0.5} \left[ft^{-1}\right] \quad (5)$$

Compared to other correlations found in literature this is about average with regard to correlating quality.

A new attempt was made to develop a more accurate correlation. Included in the correlation basis (see Figure 1) were data from analysis of sandstone cores from North Sea reservoirs (well 1 through 3), data published in the papers from Geertsma [2] and Firoozabadi [4] and data collected from the Handbook of Natural Gas Engineering [6]. A total of 82 cores were included, geographically scattered, with permeabilities ranging from 0.01 mD to 3.6 Darcy and porosities ranging from 2.2 to 26.7 percent. Limestone (dolomite) samples collected from [6] were also included.

The result of this correlation exercise, using all the available core measurements, was the following equation for the beta-factor:

$$\log(\beta) = \log\left(\varphi^{-1.7} k^{-1}\right) - 5.65 \qquad (6)$$

Or in linear form:

$$\beta = \lambda \varphi^{-m} / k \qquad (7)$$

In which $\lambda = 2.24\text{x}10^{-6}$ ($=10^{-5.65}$) and the exponent m equals 1.70.

The correlation is shown in Figure 1. The regression analysis gives a correlating coefficient (R^2) of 0.96. The standard error of the log(β) estimates is 0.28. Inclination ratio of the regression line (a) is 1.00.

With n = -1, the correlating parameter λ must have the unit of length [m] in order for equation (6) *to be unit consistent.*

As may be observed from Figure 1 the discrepancies between calculated and measured data are most severe in the low permeability/high beta-factor region. This should be expected due to the greater uncertainty, both in permeability and beta-factor measurements in this range.

Analysis of distribution of errors of the log(β) estimates indicates typical normal distribution, even though some extremes (as mentioned above) tend to disturb. The normal distribution, which is very typical for experimental errors, is also confirmed by the fact that 68 % of the estimates lie within ± one standard deviation, in this case 0.28.

Geertsma's correlation (4) applied to the same data gives a correlating coefficient of 0.85 with a standards error of log(β) estimate of 0.56.

THE ACTUAL FLUID VELOCITY CONCEPT

The equations generally used as basis for experimental determination of the beta-factor, (1) and (2) have volume or mass rate per unit area i.e. volume or mass flux as the independent variable. The actual velocity of the fluid in the core is thus not accounted for in these types of experiments.

The obvious reason for this is that the actual velocity is difficult to determine as long as the actual length and cross section of the flow paths are unknown. Moreover when considering only the linear portion of the flow equation, i.e. Darcy's law, the use of flux is acceptable since a fixed ratio will always exist between the flux, actual velocity and pressure gradient independent of the flux value.

However, it is reasonable to assume that the phenomena determining the pressure gradient in a fluid flowing through a porous medium are functions of the actual velocity of the fluid, rather than the volumetric flux.

Figure 2 show very schematic drawings of two hypothetical flow paths through a core. In *core a*, the flow path follows a straight line through the core, while in *core b* (representing a real porous media) the length of the flow path is longer than the length of the core. L is the length of the cores while the sum of L_1 through L_3 makes up the total length, L_e of the flow path in *core b*. The section L_2 in *core b* gives the flow path additional length in excess of the core length. The pore section L_2 also contributes to an increase in porosity which does not result in an increase of the fraction of the core cross-section open to flow. *A* and *a* denominates the x-sectional area of the cores and the pores respectively.

If both cores are exposed to the same flow rate *q*, the average velocity of a fluid particle traveling through core *a*, (v_1) will be:
$v_1 = q/a$

while for core *b* where the path is longer, the average velocity (v) will be:

$v = (q/a)*(L_e/L) = v_1*(L_e/L)$

The porosity of the cores is:

Core a: $\varphi_1 = a/A$

Core b: $\varphi = (a/A) *(L_e/L) \Rightarrow a = \varphi A*(L/L_e)$

Substituting this equation for *a* into the equation for *v* above, we get:

$$v = \frac{q\left(L_e/L\right)}{\varphi\ A\left(L/L_e\right)} = \frac{q}{A}\frac{\left(L_e/L\right)^2}{\varphi} = u\frac{\left(L_e/L\right)^2}{\varphi} \qquad (8)$$

Where *u* is the volumetric flux through the core.

The expression $(Le/L)^2/\varphi$ found in (8) is the same as the general expression for the formation resistivity factor (also called formation factor) i.e. the ratio between the resistivity of a completely brine saturated rock to the resistivity of saturating brine i.e.:

$$F = R_o / R_w = (L_e / L)^2 / \varphi \qquad (9)$$

The ratio $(Le/L)^2$ is commonly termed 'tortuosity'. Particularly for clean sandstones this ratio can be successfully correlated with porosity resulting in the following formula for the formation resistivity factor (Archies formula):

$$F = \varphi^{-m} \qquad (10)$$

The exponent m in (10) is frequently called the cementation factor. For clean sandstones it normally exhibits a value between 1.5 and 2.0 (somewhat higher for limestones). The cementation factor generally increases with the amount of overburden, thereby increasing the formation factor.

The development of equation (9) is based on the same logic as is the basis for (8), using electric current rather than actual fluid flow. It should therefore be reasonable to assume that the measured formation factor is also an adequate expression for the ratio of the average fluid velocity in the core to the volumetric flux through the core particularly for a single-phase flow.

Combining equations (8), (9) and (10) gives the following expression for actual fluid velocity in the core:

$$v = uF = u\varphi^{-m} \qquad (11)$$

By replacing u with v/F and substituting β with the expression given in (7) into the general flow equation (1), we obtain:

$$-\nabla P = \mu v/(kF) + \lambda\varphi^{-m}\rho v^2/(kF^2)$$

And assuming that the factor m in the expression for the beta-factor is the same m which is found in the formation factor equation (10) the general flow equation will get the following form in terms of actual velocity:

$$-\nabla P = \mu v/(kF) + \lambda F\rho v^2/(kF^2)$$

or

$$-\nabla P = \mu v/(kF) + \lambda\rho v^2/(kF) \qquad (12)$$

and

$$\beta = \lambda F / k \qquad (13)$$

Approximately 30 of the cores used in the beta-factor correlating exercise described above did also have formation factor reported. Regression analysis of the beta-factors measured on these cores, using actual formation factor rather than the average m value of 1.7, verifies equation (13) (see Figure 3). The inclination of the regression line is again very close to unity (0.99). The correlating coefficient (R^2) is 0.95 and the standard error of log(β) estimate is 0.18. λ is equal to 2.7 μm.

SIMILARITY TO PIPELINE FLOW

The Fanning equation describes pressure loss in a single-phase pipeline flow as:

$$-\nabla P = 2 f \rho v^2 / d_h \qquad (14)$$

Where:

∇P = pressure gradient
f = Fanning friction factor
ρ = fluid density
v = fluid velocity
d_h = hydraulic diameter

For fully laminar flow, the friction factor can be expressed as:

$$f = 16 / NR_e = 16\mu / (\rho v d_h) \qquad (15)$$

(NR_e is the Reynolds number) and the Funning equation (14) turns into the Hagen-Poiseuille equation for laminar flow:

$$-\nabla P = 32 \mu v / d_h^{\,2} \qquad (16)$$

By comparing (14) and (16) to the general flow equation in terms of average velocity (12) we see that the linear part of this equation (Darcy's law) is similar to the Hagen-Poiseuille equation for laminar flow, provided that:

$$kF = d_h^2/32$$

which brings about that the average hydraulic diameter of the pores for which laminar flow exist will be equal to:

$$d_h = 4\sqrt{2kF} \qquad (17)$$

Substituting the expression for d_h given in (17) into (15) will give the following equation for the Fanning friction factor for laminar (low velocity) flow through porous media:

$$f_l = 2\sqrt{2}\mu / \rho v \sqrt{kF} \qquad (18)$$

By defining a high velocity or "turbulent flow" friction factor for porous media (dimensionless) as:

$$f_h = 2\sqrt{2}\lambda / \sqrt{kF} \qquad (19)$$

we may substitute this friction factor together with (17) for d_h into the Fanning equation (14). This makes the Fanning equation become equal to the non-Darcy part of the flow equation (12):

$$-\nabla P = 2 f_h \rho v^2 / d_h = 2\sqrt{2}\lambda / \sqrt{kF}\, \rho v^2 / (4\sqrt{2kF}) = \lambda \rho v^2 / (kF) \qquad (20)$$

Adding the two together, we get a total Fanning friction factor, accounting for both high and low-velocity flow:

$$f_t = f_l + f_h = (2\sqrt{2}/\sqrt{kF})(\mu/\rho v + \lambda) \quad (21)$$

Substituting this friction factor into the Fanning equation and replacing d_h with (17) gives the following general equation for single phase flow through porous media:

$$-\nabla P = (2\sqrt{2}/\sqrt{kF})(\mu/\rho v + \lambda)\rho v^2/(4\sqrt{2}\sqrt{kF}) \quad (22)$$

which implies that

$$-\nabla P = (\mu v + \lambda\rho v^2)/(2\mathrm{kF})$$

A porous medium friction factor may be defined as:

$$f_r = f_t/2\sqrt{2} = (\mu/\rho v + \lambda)\sqrt{kF} \quad (23)$$

And the general flow equation will take the following form:

$$-\nabla P = f_r \rho v^2/\sqrt{kF} \quad (24)$$

DISCUSSION

Equation (24) does not represent a new definition of the general flow equation. If the expression for the porous media friction factor given in (23) is substituted into (24), and v and λ are defined as in (7) and (11), Forchheimers equation will reappear. The formulation of the flow equation given in (24) does however indicate a strong similarity between flow through pipes and flow through porous media.

This similarity, and the existence of a parameter in the friction factor formula (23) which is apparently independent of the fluid flow is pointing in the direction that turbulent, or turbulence-equivalent, flow may exist in the core along with laminar flow. To claim this now, after most researchers have definitely abandoned this theory, may be inadequate. However, some reasons exists - as discussed below - for re-inventing this concept.

For pipeline flow, the Reynolds number is defined as:

$$NR_e = \rho v d_h/(16\mu) \quad (25)$$

Substituting (17) for d_h and otherwise using gas and flow data reported from the beta-factor experiments give average Reynolds numbers ranging from 1 through 5 for most experiments. According to the definition of semi-turbulent flow in pipes with normal relative roughness, i.e. with Reynolds number less than approximately 2300, this supports the theory that turbulent flow is not the reason for the observed departure from Darcy's law.

However, it should be easy to imagine that a flow path through a core will vary significantly in cross-sectional area. Assuming that the same gas volume has to pass through all sections of the flow path in a given time interval, the expression above gives that the NRe at any point in the path will be inversely proportional to the hydraulic diameter (d_h) of the path at that point. The actual Reynolds number may therefore, at several places in the core, be *significantly greater* than the average.

It should also be possible to accept that the relative roughness of a flow path in a core could be enormous compared to relative roughness found in pipes. From pipeline flow theory, it is known that the relative roughness of the pipe determines the limit for Reynolds number for which the flow becomes fully turbulent and the friction factor becomes a function of relative roughness only. This limit function can be expressed as

$$NR_e > 2000d/(3\varepsilon) \qquad (26)$$

where ε is the absolute roughness of the pipe wall. This limit function indicates that a laminar flow is more easily converted to completely turbulent flow in a high roughness environment.

For Reynolds numbers greater than this limit, the flow is fully turbulent, and the friction factor can be estimated by the von Karman equation:

$$1/\sqrt{f} = -4\log(\varepsilon/3.7d) \qquad (27)$$

Equation (27) is obviously empirical. For very high values of relative roughness, the friction factor calculated by this equation becomes roughly proportional to the relative roughness of the pipe. Or if the absolute roughness is constant, it becomes inversely proportional to the hydraulic diameter. This is equivalent to the high velocity porous medium friction factor (f_h) being inversely proportional to the square root of kF (19). By analogy, the λ will then be a function of the absolute roughness in the flow paths.

The theory described above has been developed and tested for pipeline flow, where all parameters are well defined, and the friction factor and relative roughness stay within reasonable limits. Applying this theory to porous media may seem unreasonable, but the fact that a friction factor for a porous media can be defined, obviously equivalent to the friction factor defined for turbulent pipe flow, and apparently independent of other variables than the average diameter of the pores, is indicating that the theory described above may apply to porous media as well as to pipes. The formulation of the porous medium friction factor (23) indicates that single phase flow through a porous medium is either fully laminar or fully turbulent, no intermediate -semi-turbulent - regime exists. However, the two flow regimes may coexist within the same porous media.

The fact that the departure from Darcy's law increases gradually with increased velocity may be explained by the great variations in flow path diameter mentioned earlier. As the velocity of the fluid increase, the fraction of the flow path where the flow is turbulent will also gradually increase, giving an apparent smooth departure from Darcy's law for the total pressure drop.

CONCLUSIONS

An equation has been developed (13) which defines the beta-factor (β) in terms of permeability, formation resistivity and a third factor, λ, with a unit of length. A further determination of the factor λ has not been successful, but there are reasons to believe that it may be a function of the absolute roughness of the flow conduits inside the porous media.

The variations in λ are fairly narrow. For the majority of the cores used in this study, and for which formation factor data are available, the variations might in fact be explained by experimental errors in the measurements of permeability, formation resistivity factor and/or beta-factor.

Considering experimental errors in routine core analysis and uncertainties introduced by applying laboratory determined core data to subsurface reservoir environment and conditions, it is likely that determination of the beta-factor by (13), using an average value of 2.5 μm for λ, would yield sufficient accuracy for most practical purposes. Particularly for sandstone reservoirs, from which most of the data used in the regression analysis were obtained, using the mentioned value for λ should be acceptable.

The formation resistivity factor required in this beta-factor correlation will normally also be available from well log analysis. It should thus be possible to obtain reliable beta-factor data from single rate well tests, using standard Horner analysis to determine permeability, even if core data from the test intervals are unavailable.

A new formulation of the general flow equation has also been developed, expressing pressure gradient in terms of average fluid velocity rather than volumetric flux. The equation strongly resembles the Fanning equation used to calculate pressure drop in pipelines. The formulation includes a porous media friction factor equivalent to the Fanning friction factor used for pipelines. This friction factor is a combination of a fluid and flow dependent part and a rock or pore geometry dependent part. This indicates, by pipeline flow analogies, that laminar and turbulent flow may coexist in a porous media and be the main reason for the observed departure from Darcy's law at high velocities.

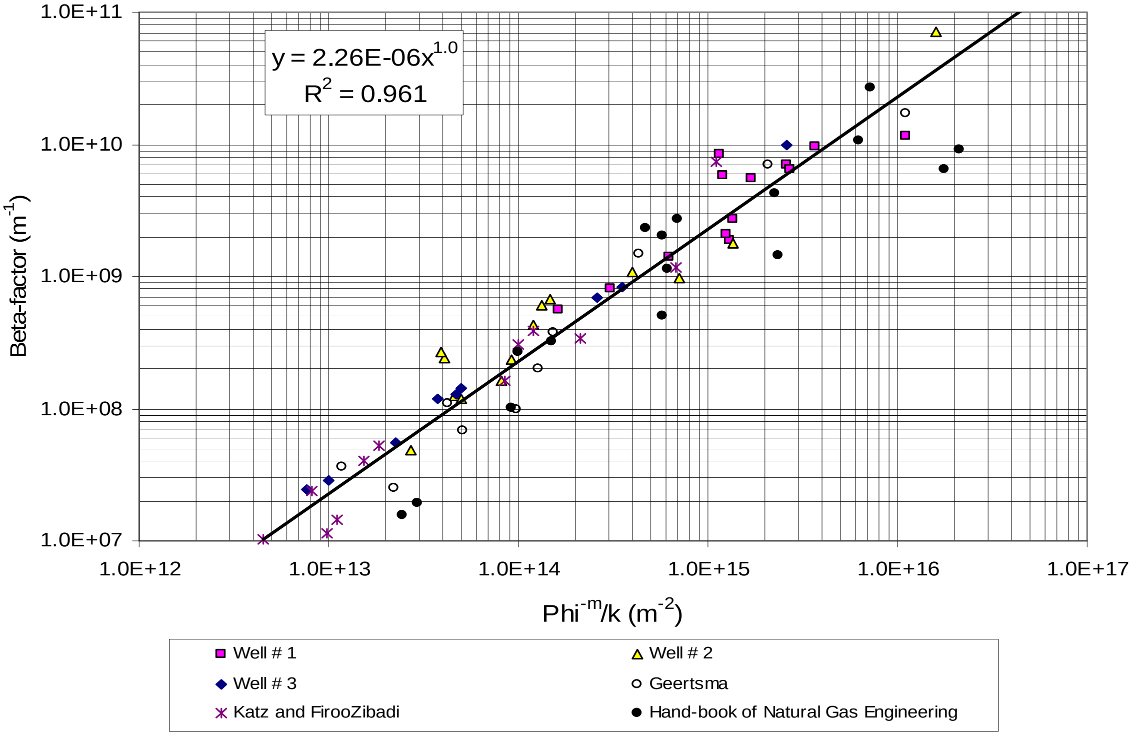

Figure 1: General β-factor correlation

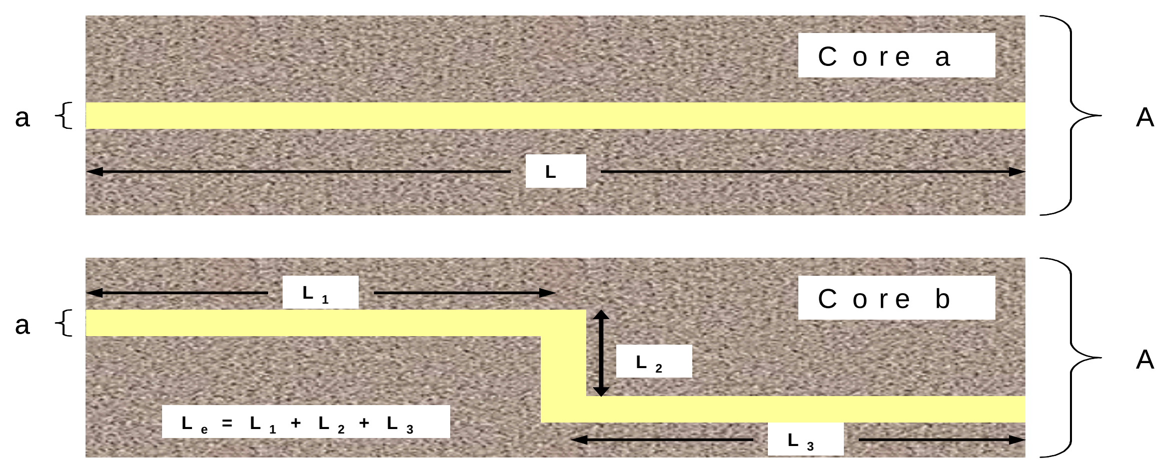

Figure 2: Schematic flow path

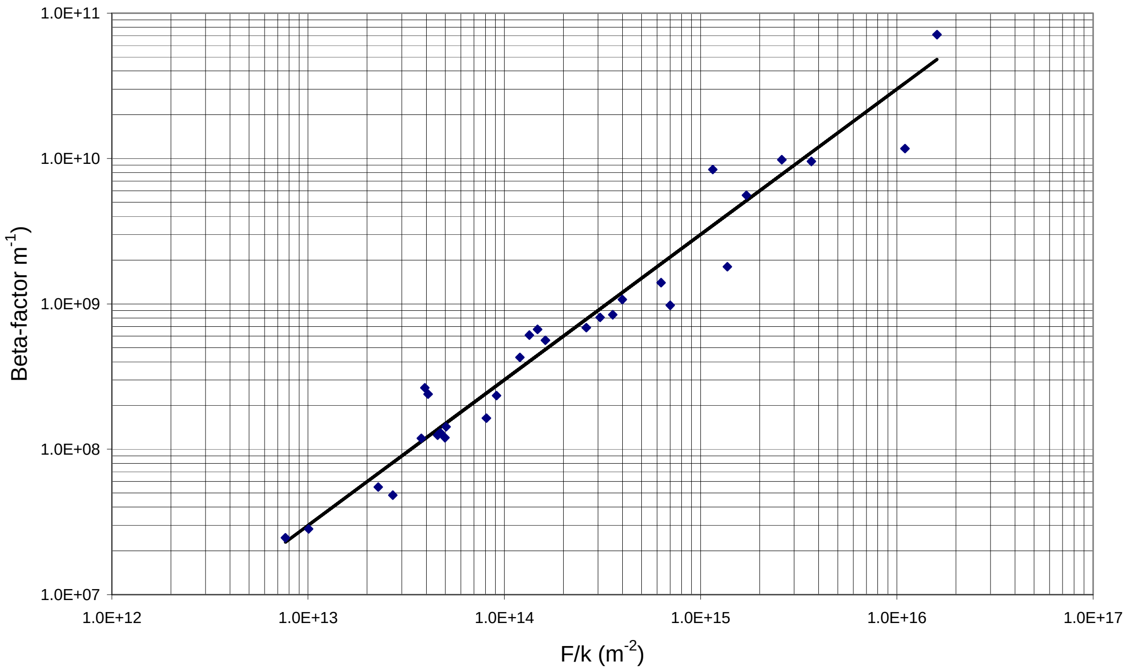

Figure 3: β-factor correlation using actual formation resistivity (F)

NOMENCLATURE

A = cross-sectional area
d_h = hydraulic diameter
F = formation resistivity factor
f = Fanning friction factor for pipeline flow
f_l = friction factor for laminar flow in porous media
f_h = friction factor for turbulent flow in porous media
$_{fr}$ = general porous media friction factor
k = absolute permeability
L = length of core
Le = length of pore
M = molecular weight of gas
m = cementation factor
NRe = Reynolds number
P = pressure
P_1 = inlet pressure
P_2 = outlet pressure
R = real gas constant
Ro = resistivity of brine saturated rock
Rw = resistivity of saturating brine
T = absolute temperature
u = superficial velocity (volumetric flux)
v = velocity
z = real gas diviation factor
β = turbulence factor (beta-factor)
ε = absolute roughness
ρ = density
λ = new turbulent flow correlation factor
μ = fluid viscosity
φ = porosity

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