CHARACTERIZATION OF HETEROGENEITIES AT THE CORE-SCALE USING THE EQUIVALENT STRATIFIED POROUS MEDIUM APPROACH

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ABSTRACT

Tracer tests are often used to characterize heterogeneous porous media. Generally, breakthrough curves are interpreted using the advection-dispersion equation (ADE), with the dispersion coefficient as key parameter. However, the integration of the ADE at the core-scale assumes a constant dispersion coefficient along the core. This assumption does not hold for porous media with significant heterogeneities. This paper presents a new approach for characterizing heterogeneities at the core scale. The basic idea is to represent a heterogeneous porous medium by using an equivalent stratified medium. The theoretical model for tracer transport through stratified porous media assumes that the displacement of the tracer in each layer is piston-like, with no dispersion, no molecular diffusion, and no mass transfer across layers. It is extended to heterogeneous media by defining a new parameter, the stratification factor (F_s), which is related to the spatial properties of the media. Values of F_s are determined from tracer tests presented at the 2005 Society of Core Analysts meeting. The heterogeneous porous media consisted of various carbonate cores. For each core, tracer concentration was measured as a function of time and distance using X-ray CT. Tracer flux was then calculated using the mass balance equation. The results show that the stratification factor is a decreasing power function of the distance from the inlet. Empirical relationships for F_s are proposed. Using the results as input to the model leads to good predictions of tracer flux as functions of distance and time.

INTRODUCTION

Modeling tracer displacement in heterogeneous porous media is essential for enhanced gas and oil recovery, chemical engineering, nuclear waste disposal, and geothermal energy production. Other areas of interest are salt water intrusion in coastal aquifers, water management, and subsurface pollution.

In the conventional approach the tracer transport is modeled by advection-dispersion equation (Bear, 1972). This equation can be solved to determine the tracer concentration at distance x and time t provided that the flow rate, porosity and dispersion coefficient are known. Several studies have demonstrated that the dispersion coefficient determined experimentally for any given macroscopic uniform flow conditions in a homogeneous porous medium is constant (Strivastava et al., 1992; Stenberg 2004). However, it is well known that this coefficient has spatial dependency for heterogeneous media (Domenico and Robbins, 1984; Gelhar et al., 1992; Rajaram and Gelhar, 1993) even at laboratory scale (Fourar et al., 2005). It has also been demonstrated that the dispersion coefficient depends on the travel time. All these effects have been attributed to the medium heterogeneity. Nevertheless, most commonly used analytical transport models are based on the convection-diffusion equation with constant dispersion coefficient. This assumption does not hold for porous media with significant heterogeneities.

The aim of this paper is to present a new approach to modeling tracer convection in heterogeneous porous media. The basic idea is to represent a heterogeneous porous medium by using an equivalent stratified medium, for which the tracer transport can be modeled in a consistent manner. Our approach involves the definition of a new coefficient, the stratification factor, which is related to the spatial distribution of the formation properties. This approach is validated by using experimental results of tracer tests presented at the 2005 Society of Core Analysts meeting (Fourar et al., 2005).

LABORATORY EXPERIMENTS

Several carbonate samples of 38 mm diameter and 80 mm length were selected. They present heterogeneous structures at scales much larger than pore scale which might affect flow and tracer characterization of core samples.

Tracer test experiments were performed with a standard experimental setup. The system includes a Hassler core-holder, two peristaltic pumps, and a conductimeter. The core was first saturated with sodium chloride (NaCl) brine of known concentration (10 g/l) using the first pump. Tracer test was then performed by displacing the resident brine with a solution of a different (150 g/l) concentration using the second pump. The in-situ tracer concentration variation at several positions along the core was measured by using a Hispeed FX/i medical scanner (General Electric). For each sample, one-millimetre width slices were recorded every 16 millimetres. Each image was reconstructed in a 512×512 mm^2 matrix, thus providing a voxel of $0.12 \times 0.12 \times 1$ mm^3. The effluent tracer concentration was measured using the conductimeter placed at the outlet of the core-holder. It was calibrated over the range of used tracer concentrations. The experiment was stopped when the conductimeter indicated the same concentration as the injected brine. All experiments were conducted at a constant room temperature of 21°C. The injection flow rates were chosen high enough (200 cc/h) to make molecular diffusion negligible in comparison with convection.

Using the tracer concentrations measured at the voxel scale, we determined the mean tracer concentration for different cross sections as a function of time t and distance x. The tracer flux was then calculated from the concentration by using the mass balance equation. All the results presented below are dimensionless. They were calculated by dividing the flux f by the total flux QC_0, the distance x by the length of the medium L, and the time by the characteristic time defined as $AL\phi/Q$. The results are interpreted by using the equivalent stratified porous medium approach.

EQUIVALENT STRATIFIED POROUS MEDIUM APPROACH

We first recall the theoretical model for the displacement of a tracer injected continuously through a perfectly stratified porous medium. The model is then applied to heterogeneous porous media by introducing the stratification factor. Details of calculations are developed in Fourar (2006).

Theoretical Model for Tracer Transport in Stratified Porous Media

Let us consider a continuous tracer injection through a perfectly horizontal stratified porous medium. The stratified model assumes that: i) the porosity of the medium is uniform; ii) the permeability of the layers is a random function; iii) the effect of pore-scale dispersion and molecular diffusion are negligible; and iv) the flow is parallel to the layers. With these assumptions, the tracer flux calculation leads to:

$$f(x,t) = \int_{K^*}^{K_{\max}} C_0 \, dq = C_0 Q \frac{1}{<K>} \int_{K^*}^{K_{\max}} K \, G(K) dK \tag{1}$$

where $f(x,t)$ is the tracer flux at position x and time t, C_0 the injected brine concentration, $G(K)$ the probability distribution function of the permeability K, $K_{\max}$ the maximum value of K, K^* the permeability of the layer where the tracer front reaches position x at time t, $<K>$ the mean permeability, and Q the tracer injection flow rate.

In the case of the lognormal distribution, the probability distribution function is written:

$$G(K) = \frac{1}{\sigma_{\ln K}\sqrt{2\pi}K} \exp\left(-\frac{\left(\ln K - <\ln K>\right)^2}{2\sigma_{\ln K}^2} \right) \tag{2}$$

where $<\ln K>$ and $\sigma_{\ln K}^2$ are the mean and variance of $\ln K$, respectively.

It should be recalled that $<\ln K>$ and $\sigma_{\ln K}^2$ are related to the mean and variance of K as follows:

$$<K> = \exp\left(<\ln K> + \frac{\sigma_{\ln K}^2}{2} \right) \tag{3}$$

$$\sigma_K^2 = \exp\left(2 <\ln K> + \sigma_{\ln K}^2\right)\left(-1 + \exp\left(\sigma_{\ln K}^2\right)\right) \tag{4}$$

The stratified model defines the heterogeneity factor H as the ratio of the standard deviation to the mean permeability:

$$H = \frac{\sigma_K}{<K>} = \sqrt{-1 + \exp\left(\sigma_{\ln K}^2\right)} \tag{5}$$

This factor, which is constant along the medium, characterizes the degree of the stratified formation heterogeneity.

Substituting the expression of $G(K)$ in Equation (1) and performing the integrations yields:

$$f(x,t) = \frac{1}{2} C_0 Q \, erfc\left(\frac{\ln\left(\frac{x}{Vt} \frac{1}{\sqrt{1+H^2}} \right)}{\sqrt{2\ln\left(1+H^2\right)}} \right) \tag{6}$$

where $erfc$ is the complementary error function, $V = \frac{Q}{A\phi}$ the tracer front mean velocity, with A the cross-sectional area, and ϕ the porosity.

Equivalent Stratification Factor for Heterogeneous Porous Media

Assuming that each domain between the inlet and the cross section x can be represented by an equivalent stratified porous medium, the stratified model can be used to describe the tracer transport through this domain. In the Equivalent Stratified Porous Medium approach, the heterogeneity factor is replaced by the stratification factor F_s, which depends on the distance from the inlet. Therefore, tracer transport through a heterogeneous porous medium can be predicted by the following equation:

$$f(x,t) = \frac{1}{2} C_0 Q \, erfc\left(\frac{\ln\left(\frac{x}{Vt} \frac{1}{\sqrt{1+F_s^2}} \right)}{\sqrt{2\ln\left(1+F_s^2\right)}} \right) \tag{7}$$

The tracer flux curve for each cross section x and for each sample was fitted to Equation (7). Figure 1 shows an example of experimental results of sample 2 fitted to Equation (7) by optimizing the stratification factor. We can observe a very good agreement between the fitted analytical model and experimental data. Figure 2 shows the stratification factor as a function of the distance from the inlet of the medium for different samples. As we can observe, F_s is a decreasing function of the distance. This finding is consistent with the fact that at the inlet, the medium can be considered as stratified. As the tracer advances through the medium, the flow becomes three-dimensional and the stratification effect decreases. Note that Figure 2 shows that the stratification factor is a decreasing power function of the distance:

$$F_s = a\,x^{-b} \tag{8}$$

Values of a and b for different samples are presented in Table 1.

Knowing the stratification factor value at any given position, we can predict the tracer flux. Inserting the value of F_s obtained with Equation (8) into Equation (7) enables us to predict the tracer transport through the medium. Figure 3 shows a good agreement between the experimental results obtained with different samples and the semi-theoretical model using Equations (7) and (8) with values of a and b presented in Table 1.

CONCLUSION

This paper presents a new approach for characterizing heterogeneities at the core scale using the Equivalent Stratified Porous Medium approach. It is based on the representation of a heterogeneous medium by using an equivalent stratified medium.

The stratified model assumes that:

i) the porosity of the medium is uniform;
ii) the permeability of the layers is a random function;
iii) the effects of pore-scale dispersion and molecular diffusion are negligible;
iv) the flow is parallel to the layers.

The model introduces the heterogeneity factor, defined as the ratio of the standard deviation to the permeability mean. Assuming that a heterogeneous medium can be represented by an equivalent stratified medium, the theoretical model was then extended to heterogeneous media by using the stratification factor instead of the heterogeneity factor, which is a function of the distance from the tracer source injection.

Using experimental results of tracer tests through several heterogeneous porous samples, we determined the stratification factor as a function of the distance from the inlet of samples. The results show that stratification factor is a decreasing power function of the distance. We established a simple empirical equation for the stratification factor. This equation, combined with the stratified convective model, serves to predict tracer transport in heterogeneous porous media. Good agreements between experimental and semi-theoretical tracer flux values were obtained.

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Table 1. Values of exponent a and b in Equation (8)

Sample	a	b
1	0.3	0.51
2	0.57	0.32
3	0.66	0.27
4	0.62	0.82
5	0.67	0.75

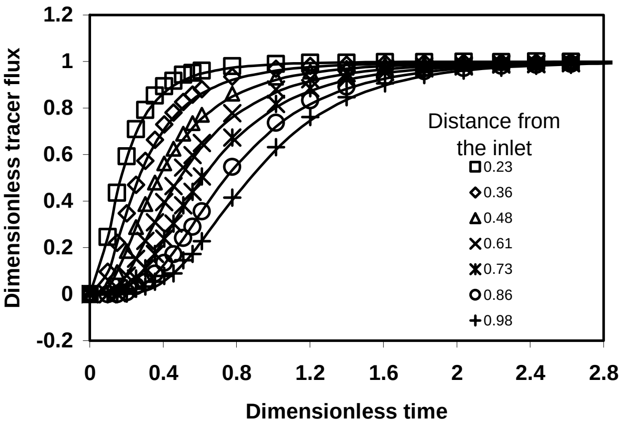

Figure 1. Experimental results of tracer flux versus time fitted to Equation (7).

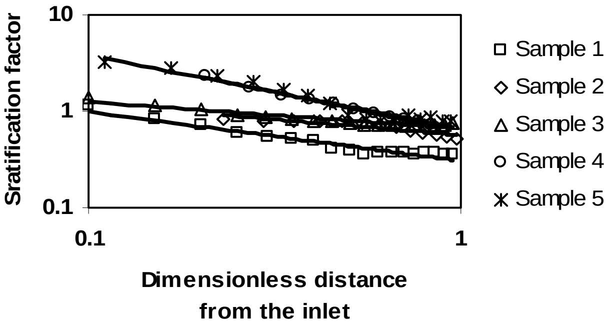

Figure 2. Variation of stratification factor (F_s) with distance.

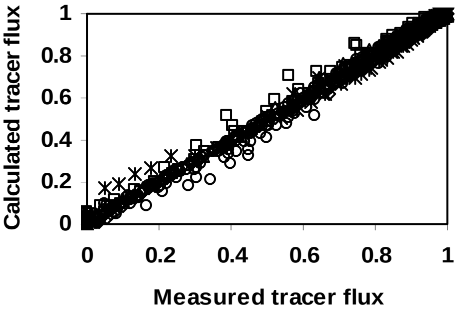

Figure 3. Comparison of measured and calculated tracer flux.