A DETAILED ANALYSIS OF PERMEABILITY AND KLINKENBERG COEFFICIENT ESTIMATION FROM UNSTEADY-STATE PULSE-DECAY OR DRAW-DOWN EXPERIMENTS

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ABSTRACT

Estimation of the intrinsic permeability, k_l, and Klinkenberg coefficient, b, of tight rock plugs is routinely performed using unsteady state gas-flow experiments. These experiments popularized by Jones (1972) and referred to as pulse-decay or draw-down methods consist in recording the differential pressure, $\Delta P(t)$, at the edge of a core plug when the inlet of this plug is connected to a gas tank initially put at a given pressure. Using adequate flow models and an inverse technique, k_l and b are estimated from the pressure decay. Our purpose in this work is to determine optimum conditions under which precise estimations of both k_l and b can be performed. A complete 1D isothermal gas flow model including Klinkenberg effect was developed and direct numerical simulations were used to determine reduced sensitivity of the pressure decay to k_l and b. Conditions under which these two parameters can be estimated independently from a single pressure decay signal were analyzed. Optimal parameters of the experiment including volumes of the upstream and downstream tanks, length and diameter of the plug, initial pressure in the upstream tank and pressure decay recording period are deduced from precision criteria on the estimation of k_l and b.

INTRODUCTION

Unsteady-state gas permeability measurement of reservoir core samples has been routinely used to circumvent some difficulties associated with a steady state method. In fact, this latter technique, when applied on very low permeability samples (less than 10^{-15} m^2), can become time consuming due to the period required to reach steady state flow and is made difficult by the very small flow rates to be measured. Typically, for a 1D experiment, time required to reach steady state roughly varies with the square of the sample length and is inversely proportional to the intrinsic permeability when constant pressures applied at the upstream and downstream faces of the sample are considered. Moreover, with this method, the identification of the intrinsic permeability and Klinkenberg coefficients, respectively denoted k_l and b in this work, requires several

measurements each performed at different mean pressure levels (Rushing *et al.*, 2004, Blanchard *et al.*, 2006). On the contrary, unsteady-state experiments are usually fast since, as indicated further in this work, a single experiment is sufficient to estimate the two coefficients, k_l and b.

The use of unsteady-state experiments to determine core permeability as currently employed was early suggested by the work of Bruce *et al.* (1952). In this work, direct numerical solutions of the governing equations for 1D unsteady-state gas flow were compared to experimental pressure profiles measured along a 1D sand pack. Shortly after, Klinkenberg effects were introduced in the physical model and some recommendations were put forth to carry out porosity, apparent and intrinsic permeability measurements (Aronofsky, 1954). During the same period, experimental set-ups were proposed to perform such measurements often referred to as the pulse-decay technique (Wallick and Aronofsky, 1954). Typically, the experiment consists in recording the evolution of the differential pressure ΔP(t) at the core plug edges, each face being connected to a tank, the upstream one being submitted to a pressure increment (Aronofsky *et al.*, 1959). During the following decade, interest of the technique was renewed with applications in tight gas reservoir characterization and geological nuclear waste storage issues (Brace *et al.*, 1968; Jones, 1972). It was later extended to the measurement of liquid permeability (Trimmer, 1982; Amaefule *et al.*, 1986). Variants of this experiment, both in the experimental apparatus and procedure, were proposed either to i) extend the method to partially water saturated samples (Newberg and Harastoopour, 1986; Homand *et al.*, 2004), ii) shorten the experimental time (Jones, 1997), iii) provide an unsteady-state version of a minipermeameter (Jones 1994), iv) simplify the pulse-decay experiment by removing the downstream reservoir. This latter experiment (referred to as the draw down experiment in the rest of this paper) is a special case of the pulse-decay technique and can be treated with the same set of equations with the downstream reservoir volume taken as infinite featuring a constant pressure boundary condition at the plug outlet.

In most of references where data interpretations of gas-pressure pulse decay are proposed, simplifying assumptions are made allowing analytical solution for ΔP(t) to identify plug permeability. Among others, the main hypotheses lie in negligible Klinkenberg effects and constant gas density along the plug leading to a solution under the form of series (Brace *et al.*, 1968; Hsieh *et al.*, 1981; Neuzil *et al.*, 1981; Chen and Stagg, 1984; Haskett *et al.*, 1988; Dicker and Smits, 1988, Wang and Hart, 1993), error functions (Bourbie, 1982) or exponential decay (Dana and Skoczylas, 1999). A similar type of approach was adopted for the radial configuration (Gillicz, 1991). On the basis of this hypothesis, it was found that estimation accuracy is improved when upstream and downstream tank volumes are taken equal (Dicker and Smits, 1988, Jones, 1997) and close to the pore volume of the sample when porosity is to be determined (Wang and Hart, 1993). Approximated analytical interpretations for the draw-down experiment including Klinkenberg (and Forchheimer effects) were also proposed (Jones, 1972).

On the analysis of the pressure signal, one should note the work of Ruth and Kenny (1989) who examined the conditions under which Klinkenberg (and Forchheimer) effects on pressure decay can be discriminated from the error on the measurement of the upstream pressure in a draw-down experiment. Despite a short sensitivity analysis reported by Wang and Hart (1993) but restricted to the case without Klinkenberg effects recently completed by Escoffier *et al.* (2005) and a more complete work performed by Finsterle and Persoff (1997) where a formalized inverse technique with a complete physical model was proposed, the impact of experimental parameters on estimation accuracy has not been analyzed carefully so far. It is hence the objective of this work to perform a sensitivity analysis of the pressure signal these parameters on the estimations of k_l and b in order to guide the optimal design of an unsteady state experiment. With this in mind, we first recall the configuration under study and the associated physical model. A sensitivity analysis is performed in a second step finally leading to concluding recommendations.

CONFIGURATION AND PHYSICAL MODELLING

The configuration under study is that of classical gas pressure pulse decay as described above. For this experiment, we assume a 1D linear homogeneous non-deformable sample and an isothermal gas flow at very low Reynolds number (this is usually the case in practise) so that no significant inertial (or Forchheimer) effects are present. In addition, gas is considered as ideal, which is a valid approximation for gases like N_2 or He at experimental operating pressures. Combining the mass, momentum and constitutive equations yields the following initial boundary value problem

$$\frac{\partial}{\partial x}\left[(P+b)\frac{\partial P}{\partial x}\right] = \frac{\varepsilon\,\mu}{k_l}\,\frac{\partial P}{\partial t} \qquad 0 < x < e \qquad t > 0 \tag{1}$$

Initial conditions

$$P(0,0) = P_{0i} \tag{2}$$
$$P(x,0) = P_{1i}, \quad x > 0 \tag{3}$$

Boundary conditions

$$\frac{k_l\,S}{\mu\,V_0}\left[P(0,t)+b\right]\frac{\partial P}{\partial x}(0,t) = \frac{\partial P}{\partial t}(0,t) \tag{4}$$

$$\frac{k_l\,S}{\mu\,V_1}\left[P(e,t)+b\right]\frac{\partial P}{\partial x}(e,t) = -\frac{\partial P}{\partial t}(e,t) \tag{5}$$

When Klinkenberg effects are neglected (b=0) and gas density is taken as a constant along the plug axis, x, one finds the classical equation extensively used in the literature (Brace *et al.*, 1968; Hsieh *et al.*, 1981; Bourbie, 1982; Chen and Stagg, 1984; Haskett *et al.*, 1988; Dicker and Smits, 1988)

$$\frac{\partial^2 P}{\partial x^2} = \frac{\varepsilon\,\mu}{k\,\overline{P}}\,\frac{\partial P}{\partial t} \tag{6}$$

where $\overline{P}$ is the mean pressure over the sample length. This last equation admits analytical solutions (see above references) whereas, when $b \neq 0$, the original one (1) does not (see some iterative procedures proposed by Jones (1972) when $V_1 \rightarrow \infty$). For this reason a

direct numerical resolution was adopted in this work. To do so, it is convenient to reformulate the original problem using the new variable $\phi = (P + b)^2$ yielding

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{\alpha}{\sqrt{\phi}} \frac{\partial \phi}{\partial t}, \qquad \alpha = \frac{\varepsilon \mu}{k_1} \tag{7}$$

Initial conditions

$$\phi(0,0) = (P_{0i} + b)^2 \tag{8}$$

$$\phi(x,0) = (P_{1i} + b)^2, \quad x > 0 \tag{9}$$

Boundary conditions

$$\frac{\partial \phi}{\partial t}(0, t) = \frac{k_1 S}{\mu V_0} \left(\sqrt{\phi} \frac{\partial \phi}{\partial x} \right)_{(0,t)} \tag{10}$$

$$\frac{\partial \phi}{\partial t}(e, t) = -\frac{k_1 S}{\mu V_1} \left(\sqrt{\phi} \frac{\partial \phi}{\partial x} \right)_{(e,t)} \tag{11}$$

The numerical solution is sought on the basis of an explicit finite difference scheme which is second order in space and first order in time. Using the notation $\phi_i^n = \phi\big((i-1)\Delta x, (n-1)\Delta t\big)$, and m space discretization nodes, the scheme is given by

$$\phi_1^{n+1} = \phi_1^n + \frac{\phi_2^n - \phi_1^n}{\Delta x} \frac{\Delta t \sqrt{\phi_1^n}}{\beta + \alpha \Delta x/2} \tag{12}$$
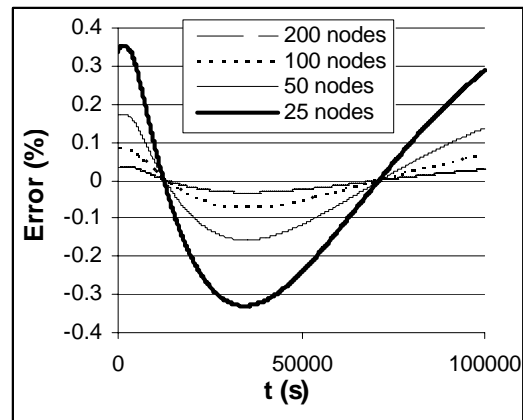
$$\phi_i^{n+1} = \frac{\Delta t}{\alpha \Delta x^2} \sqrt{\phi_i^n} \left(\phi_{i+1}^n - 2\phi_i^n + \phi_{i-1}^n \right) + \phi_i^n, \qquad 2 < i < m-1 \tag{13}$$

$$\phi_m^{n+1} = \phi_m^n - \frac{\phi_m^n - \phi_{m-1}^n}{\Delta x} \frac{\Delta t \sqrt{\phi_m^n}}{\delta + \alpha \Delta x/2} \tag{14}$$

where $\beta = \mu V_0/k_1 S$ and $\delta = \mu V_1/k_1 S$. Stability of the overall algorithm is subjected to a criterion on the time step due to the explicit character of the time scheme. This criterion is a classical one for a diffusion-like equation and is such that $\Delta t < \varepsilon \mu \Delta x^2/2 k_1 (P_{0i} + b)$.
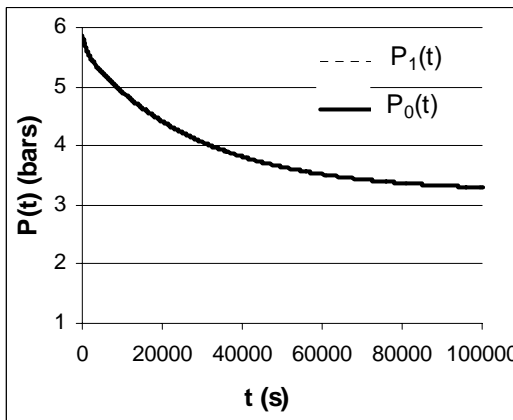
a)   *b)*

Figure 1. Direct simulation performed with $k_l = 10^{-19}$ m^2, $\varepsilon = 0.05$, $D = 0.05$ m, $e = 0.05$ m, $V_0 = V_1 \approx 1.96\ 10^5$ m^2, $P_{0i} = 6\ 10^5$ Pa, $P_{1i} = 10^5$ Pa. a) Evolution of P_0 and P_1. b) Evolution of the relative error on $P_0(t)$ as a function of the number of nodes for space discretization.

On figure 1a, we have represented an example of a simulation result showing the evolution of $P_0(t)$ and $P_1(t)$ respectively while, in figure 1b, we have reported the relative error on $P_0(t)$ obtained with, 25, 50, 100 and 200 nodes in space using a reference result obtained with 1000 nodes in space. This last figure clearly indicates that the result on $P_0(t)$ is not affected significantly by the number of nodes used to perform the simulation. In the rest of the analysis, 50 nodes were employed for space discretization.

SENSITIVITY ANALYSIS

The aim of the sensitivity analysis is to define the best experimental conditions of a gas pressure pulse-decay leading to an optimal estimation of the intrinsic (or liquid) permeability k_l and of the Klinkenberg coefficient b. The numerical procedure detailed above to solve the initial boundary value problem (7) to (11) was used to study the influence of the different adjustable experimental parameters on the pressure difference $\Delta P(t) = P_0(t) - P_1(t)$, namely

- volumes V_0 and V_1,
- diameter D and length e of the sample,
- initial pressure P_{0i} in the high pressure volume V_0,
- duration t_f of $\Delta P(t)$ recording.

The sensitivity analysis was carried out for three typical porous materials with low permeability and having the following characteristics

1. $k_l = 10^{-17}$ m^2 ; $b = 2.49.10^5$ Pa ; $\varepsilon = 0.02$
2. $k_l = 10^{-17}$ m^2 ; $b = 2.49.10^5$ Pa ; $\varepsilon = 0.1$
3. $k_l = 10^{-19}$ m^2 ; $b = 13.08.10^5$ Pa ; $\varepsilon = 0.02$

In each case, the Klinkenberg coefficient was estimated from the correlation proposed by Jones (Jones, 1972), *i.e.*

$$b = 1.89\ 10^{-1}\ k_l^{-0,36} \tag{15}$$

where b and k_l are in *SI* units. It shall be noted that the porosity ε is considered as a known datum is this study.

If $\Delta P(t) = f(t, k_l, b)$ were a linear relationship, optimal values of k_l and b, that could be estimated from n pressure drop experimental data sets $\Delta \hat{P}(t_i)$, would be calculated using the following matrix relation (Beck and Arnold, 1977)

$$[B] = \left[X^t\ X \right]^{-1} [X]^t \left[\Delta \hat{P} \right] \tag{16}$$

where

$$[B] = \begin{bmatrix} k_l \\ b \end{bmatrix} \text{ and } [X] = \begin{bmatrix} \dfrac{\partial f}{\partial k_l}(t_1) & \dfrac{\partial f}{\partial b}(t_1) \\ \vdots & \vdots \\ \dfrac{\partial f}{\partial k_l}(t_n) & \dfrac{\partial f}{\partial b}(t_n) \end{bmatrix} \tag{17}$$

The standard deviation of the estimation error on the parameters k_l and b, *i.e.* the covariance matrix of the error on [B], $[cov(eB)]$, would be then calculated according to

$$[cov(eB)] = [\sigma(\Delta P)]^2 [X^t X]^{-1} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \tag{18}$$

where $\sigma(\Delta P)$ represents the standard deviation of the measurement error on the pressure drop. The parameters a_{11} and a_{22} enable an estimation of the standard deviations of the estimation error on k_l and b given by:

$$\sigma_{k_l} = \sqrt{a_{11}} \tag{19}$$

$$\sigma_b = \sqrt{a_{22}} \tag{20}$$

In the case of a non-linear function f, the relation (16) enabling an estimation of the optimal values of k_l and b is no longer valid. Nevertheless, relations (19) and (20) may be used to estimate the standard deviation of the estimation error on k_l and b with the hypothesis that the function f may be considered as locally linear for weak variations of k_l and b in the neighbourhood of their optimal values (Beck et Arnold, 1977).

In this study, the standard deviations σ_{k_l} and σ_b were computed using relations (19) and (20) with a constant standard deviation $\sigma(\Delta P) = 5.10^{-3} (P_{0i} - P_{1i})$. To do so, the matrix [X] was computed with the numerical procedure described in the previous paragraph.

Sensitivity To V_1/V_0

The influence of the volume ratio V_1/V_0 was first investigated. To illustrate our conclusion that remains independent of the sample characteristics k_l and b, results obtained on the material of type 3 are discussed. For this analysis, the following values of the parameters were considered: $P_{0i} = 6.10^5$ Pa, $P_{1i} = 10^5$ Pa, D = 0.05 m, e = 0.05 m. In figure 2 we have represented the values of the standard deviations σ_{k_l} and σ_b as a function of V_1/V_0. Clearly, errors on the estimations of k_l and b are maximum when $V_0 = V_1$ and decrease quickly when V_1/V_0 increases.
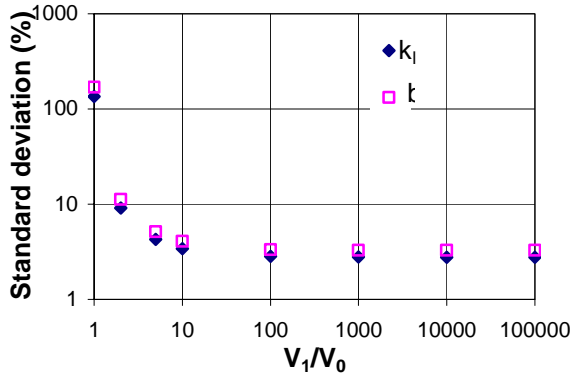
Figure 2. Standard deviations of the estimated values of k_l and b as a function of V_1/V_0.
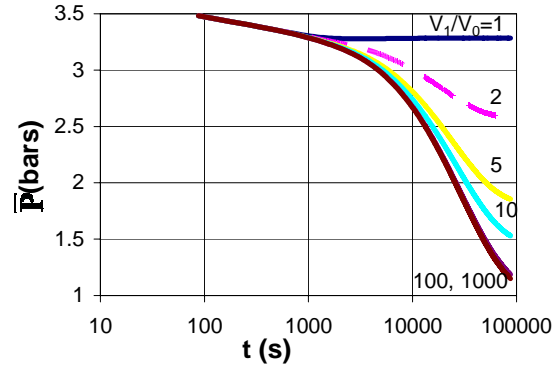
Figure 3. Evolution of $\overline{P}(t)$ as function of the ratio V_1/V_0.

A close attention to the evolution of the mean pressure $\overline{P}$ over the sample length can better explain this behaviour. In fact, when $V_0 = V_1$, the mean pressure in the sample remains almost constant during the whole experiment as indicated in figure 3 where we have represented $\overline{P}$ versus time. Because of that, the apparent permeability, k_g, remains also rather constant since $k_g = k_l\left(1 + b/\overline{P}\right)$. As a result, a single experiment does not provide enough mean pressure contrast to allow a separate estimation of k_l and b when V_0/V_1 is close to unity. When this is the case, several experiments with different values of P_{0i} and P_{1i} are necessary to obtain several values of $k_g(\overline{P})$ as previously reported (Finsterle and Pershoff, 1997).

The study of sensitivity analysis of $\Delta P(t)$ to the ratio V_1/V_0 demonstrates that the best configuration to estimate separately k_l and b from a single measurement is the draw down experiment (corresponding to an extremely high value of V_1). This last configuration will be the only one considered in the rest of this paper.

Sensitivity To V_0

The volume V_1 being fixed to a very high value ($1m^3$), the sensitivity of $\Delta P(t)$ to the volume V_0 was further studied for the following conditions: $P_{0i} = 6.10^5$ Pa, $P_{1i} = P_{atm} = 10^5$ Pa, D = 0.05 m, e = 0.05 m. Standard deviations of the estimated values of k_l and b were computed over an interval of time $[0, t_f]$, t_f corresponding to $P_0(t_f) = 2.10^5$ Pa. Computation was performed for the three types of porous materials mentioned above and results of the analysis is reported in table 1.

Table 1. Influence of V_0 on the duration of the experiment and on standard deviations of the estimation error on k_l and b.

V_0	*Material 1*			*Material 2*			*Material 3*		
	t_f	σ_{k_l}	σ_b	t_f	σ_{k_l}	σ_b	t_f	σ_{k_l}	σ_b
m^3	s	%	%	s	%	%	s	%	%
5.10^{-6}	810	1.5	2.9	909	3.6	6.7	24371	5.3	6.3
10^{-5}	1561	1.4	2.8	1730	3.4	6.5	47605	5.2	6.2
5.10^{-5}	7614	1.4	2.8	7863	3.2	6.3	231901	5.1	6.1

These results show that, for given values of both the initial and final pressures in the upstream reservoir, the value of V_0 has no significant influence on the precision expected on the estimation of k_l and b. However, this parameter has a great influence on the duration of the experiment. This conclusion was also validated for e = 0.025 m and e = 0.075 m as well as for $P_{0i} = 10^6$ Pa and $P_{0i} = 2.10^6$ Pa.

Sensitivity To D
The boundary condition on the upstream end face of the sample (see equation (4) or (10)) is the only relationship where the cross sectional area, S, of the sample appears. As a consequence, the influences of S and V_0 on $P_0(t)$ are proportional. This indicates that the use of a larger sample diameter, D, leads to a shorter experiment but does not influence significantly the precision of the estimation of k_l and b.

Sensitivity To e
The influence of sample length, e, was studied by considering successively the following values of e: 0.025, 0.05 and 0.075 m. Values of the other parameters were taken as: $P_{0i} = 6.10^5$ Pa, $P_{1i} = P_{atm} = 10^5$ Pa, D = 0.05 m, $V_0 = 10^{-5}$ m^3. Standard deviations of the estimated values of k_l and b were calculated between times t = 0 and t_f such that $P_0(t_f) = 2.10^5$ Pa. Calculations were repeated for the three types of materials under consideration in this work and results are reported in table 2.

Table 2. Standard deviations of k_l and b as a function of the sample length, e.

e	*Material 1*			*Material 2*			*Material 3*		
	t_f	σ_{k_l}	σ_b	t_f	σ_{k_l}	σ_b	t_f	σ_{k_l}	σ_b
m	s	%	%	s	%	%	s	%	%
0.025	773	1.4	2.8	821	3.3	6.5	23496	5.1	6.1
0.05	1571	1.4	2.8	1730	3.4	6.5	47605	5.2	6.2
0.10	3240	1.5	2.9	3637	3.5	6.7	97486	5.3	6.2

Results indicate that, for given values of both the initial and final pressures in the upstream reservoir, the sample length has no influence on the precision of the estimation of k_l and b. However, the use of a shorter sample decreases significantly the duration of the experiment.

Sensitivity To P_{0i}

Influence of the initial pressure P_{0i} in the upstream reservoir was studied by varying this parameter in the range 2 bars - 50 bars, the other parameters remaining constant: $P_{1i} = 1$ bar, e = 0.05 m, d = 0.05 m, $V_0 = 10^{-5}$ m^3. Standard deviations on the error expected on k_l and b were computed using 1000 points on the signal $P_0(t)$, t ranging between t = 0 and t = t_f so that $P_0(t_f) = 0.2 \, P_{0i}$. Calculations were repeated for the three materials under consideration and results of this analysis are reported in table 3.

Table3. Standard deviations of k_l and b as a function of the pressure P_{0i}.

P_{0i}	Material 1			Material 2			Material 3		
	t_f	σ_{k_l}	σ_b	t_f	σ_{k_l}	σ_b	t_f	σ_{k_l}	σ_b
bar	S	%	%	s	%	%	s	%	%
2	1933	7.7	11.7	2044	8.3	12.5	50935	28.6	31.4
4	1723	2.0	3.6	1929	2.2	3.8	48880	6.7	7.7
6	1573	1.3	2.5	1671	1.4	2.8	47255	3.8	4.6
8	1432	1.0	2.1	1591	1.1	2.4	45939	2.7	3.4
10	1323	0.8	2.0	1470	1.0	2.2	44851	2.1	2.7
15	1107	**0.6**	**1.9**	1241	**0.8**	**2.1**	42215	1.5	2.0
20	957	0.6	1.9	1076	0.7	2.2	39366	1.2	1.7
30	755	0.5	2.1	873	0.6	2.4	35265	0.9	1.5
40	625	0.4	2.4	727	0.5	2.7	32080	0.7	1.4
50	534	0.4	2.7	620	0.5	3.0	29376	**0.6**	**1.4**
60	467	0.4	3.0	548	0.5	3.3	26955	0.6	1.4

Two main observations can be made from this analysis. First, the pressure P_{0i} leading to the lowest standard deviations σ_{k_l} and σ_b does not depend on k_l. Second, there is an optimum value of P_{0i} depending on b, and this optimum increases with b. These results were confirmed with other values of k_l and b. The optimal value is around 15 bars for b = 2.49 bars and around 50 bars for b = 13.09 bars. An initial value around 10 bars for P_{0i} may be used for all materials since increasing P_{0i} beyond this value does not lead to a significant improvement of the precision on the estimation of k_l and b.

Sensitivity To t_f

The effect of the duration, t_f, of the experiment on the expected standard deviations on k_l and b was analyzed for the three materials under concern, using 1000 points for the signal $P_0(t)$, this signal being considered in the interval [0, t_f] such that $P_0(t_f) = \chi \, P_{0i}$ where χ was varied between 0.7 and 0.1. As before, the following parameters were used: $P_{1i} = 1$ bar, e = 0.05 m, d = 0.05 m, $V_0 = 10^{-5}$ m^3. Results on σ_{k_l} and σ_b are reported in table 4.

Table4. Standard deviations of k_l and b as a function of the pressure P_{0i}

χ	Material 1			Material 2			Material 3		
	t_f	σ_{k_l}	σ_b	t_f	σ_{k_l}	σ_b	t_f	σ_{k_l}	σ_b
	S	%	%	s	%	%	s	%	%
0.7	187	3.8	11.6	67	5.7	19.5	7957	8.4	11.6
0.6	307	2.7	7.7	200	3.1	9.6	12298	6.1	8.3
0.5	441	2.0	5.4	334	1.9	5.6	17362	4.5	6.1
0.4	641	1.4	3.7	601	1.4	3.6	23872	3.4	4.5
0.3	909	1.1	2.7	935	1.1	2.8	32553	2.7	3.5
0.2	1323	0.8	2.0	1470	1.0	2.2	44851	2.1	2.7
0.1	2111	0.7	1.6	2472	0.8	1.8	67277	1.8	2.3

It can be observed that standard deviations of the errors on the estimated values of k_l and b decrease almost linearly with χ. This indicates that, as expected, pressure decay recording must be as long as possible to improve estimation of k_l and b. The limiting criterion is an adequate compromise for an acceptable duration of the experiment.

Reduced Sensitivities

We finally analyzed the reduced sensibilities $k_1 \, \partial P_0(t)/\partial k_1$ and $b \, \partial P_0(t)/\partial b$ of the pressure signal in the upstream tank to the two characteristics k_l and b that are to be estimated. This was performed for the three materials with $P_0(t)$ with $P_{1i} = 1$ bar, $e = 0.05$ m, $d = 0.05$ m, $V_0 = 10^{-5}$ m^3 and P_{0i} corresponding to the optimal value identified previously, *i.e.* 15 bars for material 1 and 2 and 50 bars for material 3. Reduced sensibilities are reported in figure 4.
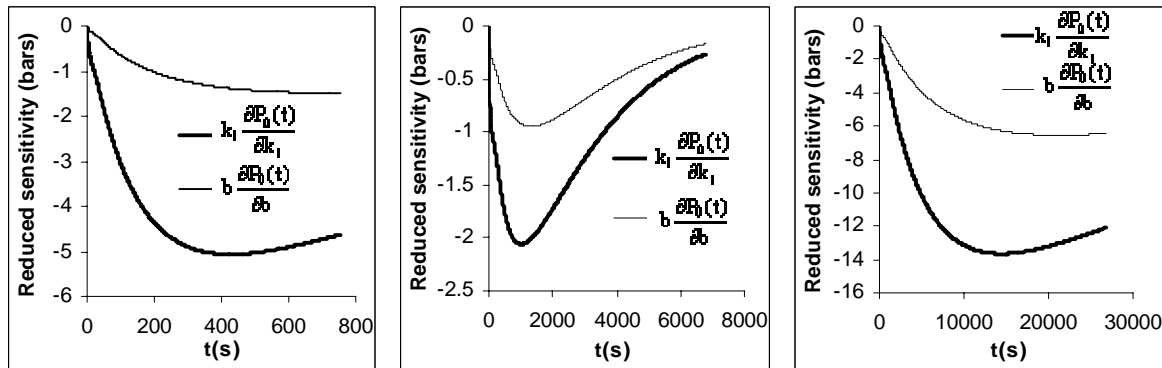
a) Material 1   b) Material 2   c) Material 3
Figure 4. Reduced sensitivity of $P_0(t)$ to k_l and b for the three materials.

This figure clearly indicates that reduced sensitivities of $P_0(t)$ to both k_l and b are large enough -and much larger than the expected sensitivity of a classical pressure sensor- to allow the simultaneous determination of k_l and b from a single recording of $P_0(t)$, although sensitivity to b is always smaller than that to k_l.

CONCLUSION

The sensitivity analysis based on the use of the physical modelling of the pressure difference $\Delta P(t)$, assuming isothermal flow of an ideal gas, leads to the following conclusions:

- The draw-down experiment is the optimal configuration to estimate both k_l and b. This estimation is possible from a single measurement.
- The volume, V_0, of the upstream reservoir has no significant influence on the precision of the estimation but has a strong influence on the duration of the experiment. A small value of V_0 leads to a shorter experiment but may introduce a strong relative uncertainty on its value affecting the precision of the estimation of k_l and b.
- The sample diameter, D, has no significant influence on the precision of the estimation but has a strong influence on the duration of the experiment. A large value of D leads to a shorter experiment and is more representative of the (heterogeneous) material to be characterized.
- The sample length, e, has no significant influence on the precision of the estimation but has a strong influence on the duration of the experiment. A short sample leads to a faster experiment but might be less representative of the (heterogeneous) material.
- While the precision on the estimation of k_l does not depend on the initial pressure in the upstream tank, there is an optimum value of P_{0i} that minimizes the error on the estimation of b. This optimum value increases with b.
- The duration, t_f, of the experiment is an important parameter since the expected precision on the estimation of k_l and b decreases while increasing t_f. This precision increases almost linearly with the amount of decrease of P_{0i}.

Finally, it shall be noted that, in the present work, estimations on the standard deviations σ_{k_l} and σ_b were performed assuming uncertainty on the measurement of $\Delta P(t)$ only, all other parameters being known with perfect exactness. Uncertainties on ε, V_0, e and D should increase σ_{k_l} and σ_b without modifying the above conclusions on the optimal experimental conditions.

NOMENCLATURE

b	Klinkenberg coefficient	Pa		k_l	Intrinsic or liquid permeability	m^2
D	Sample diameter	m		P	Pressure	Pa
e	Sample length	m		t_f	Duration of the experiment	s
k_g	Apparent permeability	m^2		V	Volume	m^3
ε	Porosity			ΔP	Pressure difference	Pa
S	Sample cross sectionnal area	m^2				

Indices

atm	Atmospheric		0	High pressure reservoir
i	Initial values at time t = 0		1	Low pressure reservoir

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