TRAPPING OF FINE PARTICLES IN GAPS IN POROUS MEDIA

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1. ABSTRACT

Pore throats have been typically considered the place where fine particles are trapped. However, the retention apart from filtration of particles smaller than the smallest nominal pore throat has been observed in a number of independent laboratory investigations over the past several decades. Though too small to be trapped in throats, the particles in these experiments were also too large to undergo deep-bed filtration. An explanation for this surprising observation could improve models of well productivity and predictions of reservoir performance. We studied the retention of fines by straining, a purely geometric mechanism, in gaps in a random packing of mono-dispersed spheres. Here gaps are defined as the void space between the centers of two neighboring spheres. The characterization of gaps in this model sediment has confirmed that their occurrence is large enough to trap a considerable number of particles. The geometric analysis of the model, combined with a new methodology to compute flow rates in the gaps, has been used in an existent straining theory. We adapted the theory to yield a scaling law, namely the dependence of straining rate on particle size. The scaling exponents varied largely when using different hypothesis. Two limiting cases were identified. In one, the probability of trapping in a given gap is assumed to be proportional to the flow rate through the gap. In the other, the probability is assumed proportional to the cross-sectional area in the gap where the particle could be trapped, independent of the local flow rate. The exponents in the scaling law obtained in these limiting cases bound the experimentally obtained exponent. Another two cases based in geometric arguments gave a closer prediction of the scaling of straining rate with particle size.

2. INTRODUCTION

There are two controlling mechanisms of colloid retention: filtration and straining. Filtration is a physicochemical mechanism controlled by electrostatic, chemical and van der Waals forces which lead to the attachment/detachment of particles to the filter media. Straining is a geometric mechanism. Particles are retained by straining when they arrive at constrictions in pore space too small to allow passage. This simple phenomenon of straining is still not well understood. Recent experiments (Ryan and Elimelech 1996, Bradford et al. 2003) suggest that filtration theory underestimates the extent of retention and also that the assumption that straining occurs only at pore throats is too restrictive. Bradford observed that the straining rate scales with the size of the strained particles d as

$(d/D)^{1.42}$ while Hall's geometric argument (1957) yields a dependence of $(d/D)^{1.5}$, D being the size of the soil grains. Our hypothesis is that straining occurs in small gaps between pairs of grains in addition to the centers of the pore throats between triplets of grains (Figure 1a). Our objective is to determine if the retention of particles in these gaps can explain anomalous straining described above. The geometric analysis of a dense random packing of equal spheres (Finney packing) will be key to reach this objective. The theory of Sharma and Yortsos (1987) is used to predict how the straining rate scales with the size of the strained particles.

3. PROCEDURE AND RESULTS

3.1 Geometric Characterization of Pore Space

We have calculated the number, frequency and density of near neighbors and point contacts for every sphere and the frequency distribution of gap widths in the Finney packing. The Delaunay tessellation identifies of groups of nearest neighbor spheres and thus identifies pore throats. The range of interest for gap sizes is close to the size of the particles being strained. We investigated gap widths between $0.01R$ and $0.1R$, where R is the radius of the soil grains. Gaps sizes bigger than $0.1R$ are considered part of the adjacent pore throat and smaller than $0.01R$ are considered point contacts. Making an analogy with sands, if the average diameter of a sand grain is 0.2 mm, particle size $0.1R$ (R=0.1mm) corresponds to 10 microns and particle size $0.03R$ corresponds to 3 microns (Figure 1b). The frequency of gaps in the Finney packing is shown in Figure 2 together with the frequency of pore throats. The empty range between $0.05R$ and $0.15R$ corresponds to portions of the pore space that are bigger than gaps in the range of interest but smaller than pore throats. The density of gaps whose width is between $0.01R$ and $0.1R$, was found to be 0.15 gaps/R^3; for comparison the density of small pore throats in the Finney packing is about 0.3 per R^3.

3.2 Calculation of Flow Rates through Gaps

The steady state flow of a single phase fluid through the Finney packing in pore throats was calculated using the approach of Bryant et al. (1993). The calculation provided the volumetric flow rate in each throat and the potential in each pore body. These potentials were used to compute the local gradient in potential in each gap. The velocities and therefore flows in gaps were calculated approximating the flow resistance of the gap by that of a slit of length equal to twice the range of capture a and width equal to the gap width w_{gap} (Figure 4). The range of capture is the distance from the minimum constriction where the particle can get trapped (Figure 1c) and introduces the dependence of the flow rate q_{gap} upon the size of the particle being strained (see below). Velocity u_{gap} varies with pressure gradient ∇P as $(-w_{gap}^2/12)(\nabla P/\mu)$. Because of the complex spatial distribution of gaps in the pore space this calculation is not straightforward. Figure 3 shows a 3D view of the gap between two spheres. The plane between the spheres is defined by the center of the gap and the centers of the Delaunay cells containing those two spheres. A transformation of the spatial coordinates of the centers of the gaps and the centers of the Delaunay cells containing the gaps was performed in order to find a correlation between spatial coordinates and pressure that allows an estimation of the pressure gradient. This local potential gradient was used to calculate the volumetric flow rate q through the gap appropriate to the particle being strained as $2aw_{gap}u_{gap}$. The results showed that the flow in the gaps is about three orders of magnitude smaller than the flow in throats.

3.3 Models for Particle Straining

The theory of Sharma and Yortsos (1987) establishes a mechanistic connection between pore scale straining events and macroscopic behavior considering size exclusion as the dominant mechanism for particle trapping. The size of the fine particles (strained particles) in this theory is comparable to the pore size. Continuum scale population balances have been formulated in terms of frequency distributions of pore throat sizes and particle sizes. The dimensional form of the population balance equation for single size particles is expressed as follows:

$$\frac{\partial C}{\partial t}+\frac{\partial C}{\partial x}=-\frac{1}{\mu}\frac{I(r_s)}{I(\infty)}C \qquad \text{Eqn.3.1}$$

where μ is the ratio of typical pore throat length to length of granular medium, r_s is the size of suspended particles, C is the concentration of suspended particles in number of particles by volume of injected carrier fluid, x is the distance along the length of the porous medium, and I is a cumulative local flow rate distribution. The cumulative flow rate distribution I is expressed as a function of the pore throat radius, r_p. In this case:

$$I(r)=\int_0^r r_p^2 u_R f_p dr_p \qquad \text{Eqn.3.2}$$

where u_R is the fluid velocity through a throat of radius r_p, and $f_p\ dr_p$ is the fraction of pore throats of radius r_p. Since the volumetric flow rate through a throat is proportional to $r_p^2u_R$ we substituted $r_p^2u_R$ by q in equation 3.2. In our work the flow distribution includes only gaps. (The inclusion of the pore throats will be the next step in future works after testing the theory for gaps.) The straining rate constant is derived from a dimensionless form of equation 3.1 as $I(1/A)/I(\infty)$, where A is the ratio of the average size of pore throats r_p to the average size of suspended particles r_s. In our case r_s/r_p is equivalent to d/D, hence the dependence of the straining rate constant on the size of the strained particle will be $I(d/D)/I(\infty)$.

The cumulative flow distribution $I(r)$ expresses the assumption that the probability of a particle entering a constriction is proportional to the flow rate into that constriction. We also compute the limiting case in which straining is taken to be independent of local flow velocity. This assumption reduces the integral I to the frequency distribution of the constrictions, and the integrand becomes f_p. Two other cases are also presented which test the influence of other geometric characteristics of the gaps. In one case the integrand in equation 3.2 was taken to be af_p (straining rate is proportional to range of capture) and in the other ar_pf_p (straining rate proportional to cross-sectional area of the gap). The application of this theory to obtain the straining constants is shown in Figure 5. The straining constant varies with the size of the strained particles; fitting the variation to a power law yields an exponent we term the scaling exponent. The scaling exponents obtained were: 4.22 (flow rate weighted), 0.72 (independent of flow rate), 1.32 (range of capture), and 2.02 (cross-section of capture). Figure 6 shows the variation of the scaling exponent for each case. A reason for the discrepancy in the scaling exponent may be derived from the use of the gap range of capture in the flow calculation. The range of capture is taken as the maximum range at which a particle of a given size can be strained but there is small chance that the particles get trapped at exactly the maximum range of capture. Probably the range of capture is smaller than the one considered. On the other hand, only an average velocity obtained from approximating gaps as slits has been considered, which provided a lower bound on the actual velocity and a conservative estimate of the contribution of gaps to particle straining. This approximation also means that point contacts between grains are not included as potential locations for straining. Neglecting the effect of previously strained particles and the separation of flow near gaps are other approximations that may explain why the scaling exponent obtained with the flow weighted theory differs from the ones reported in the literature.

3. CONCLUSIONS AND DISCUSSION

An independent description of the distribution of gap widths in the Finney packing has been made. The volumetric rate of a single phase flow through gaps has been calculated by means of a slit approximation of the gap shape. The volumetric flow in gaps in the Finney packing, appropriate to the particle being strained, was found to be about three orders of magnitude smaller than the volumetric flow in pore throats obtained from a steady state flow calculation. A direct measurement of the distribution of gap widths and the volumetric flow through them had not been reported before. This information will be useful in evaluating transport phenomena influenced by these constrictions.

The assumption that the rate of particle straining is proportional to the flow rate through the gaps yields a greater sensitivity of straining rate to particle size than observed experimentally. On the other hand, the assumption that the rate of straining is independent of flow rate through gaps, and depends only on the frequency of gaps of the appropriate size, yields a weaker sensitivity to particle size than observed. This suggests that the straining rate does depend on flow rate through gaps, but the dependence is weaker than first order. When the range of capture was considered the scaling exponents were close to the ones reported from Bradford (2003) and Hall (1957). Hall's geometric model considered crevices between spheres in point contact (analogous to the gaps in this work) to be associated with the pore throats, while we considered gaps to be independent of throats. Moreover, the model that assumed straining depends only on the frequency distribution of gaps was implemented in such a way that the probability of a gap straining a particle was independent of particles size (as long as the particle was larger than the gap width). Thus this model establishes a lower bound on the scaling behavior of the straining rate with particle size.

These observations suggest that the probability of straining in a gap depends on flow rate through the gap, but the dependence is weaker than first order. We are led to propose that straining in a gap cannot be treated correctly without reference to the throat associated with the gap.

4. ACKNOWLEDGMENTS

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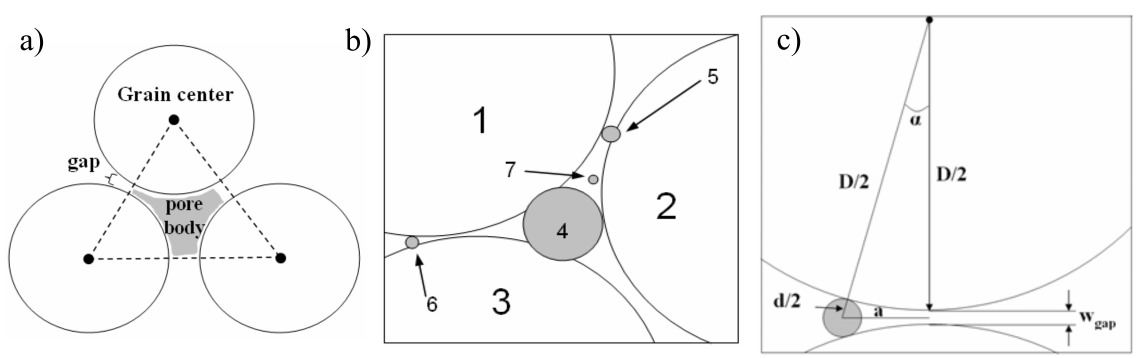

Figure 1: a) Cross section of a pore body. A gap is defined as the void space between the centers of two neighboring grains **b)** Trapping of particles smaller than pore throats. Spheres 1, 2 and 3 have equal radius and represent soil grains. Sphere 4 is is retained in the pore throat. Spheres 5, 6 and 7 are too small to be trapped in the pore throat; nevertheless particles 5 and 6 are strained in gaps. Flow is assumed to be normal to the plane of the paper**. c)** Scheme of the range of capture. The particle of diameter *d* is moving perpendicular to the plane of the paper. It will be trapped if it enters the gap, which has width w_{gap}, within a distance *a* of the center of the gap.

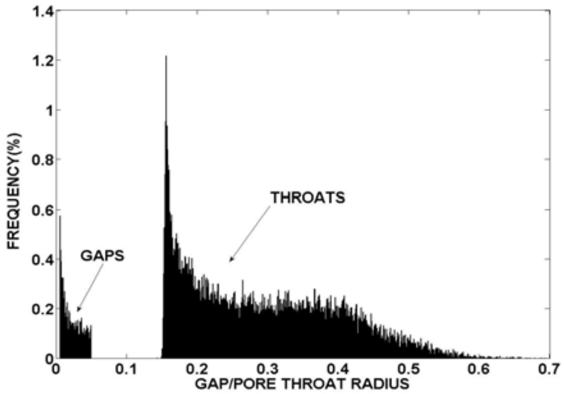

Figure 2: Frequency distribution of gaps in the size range of interest for anomalous straining and pore throats in Finney packing. Gap radius equals half gap width.

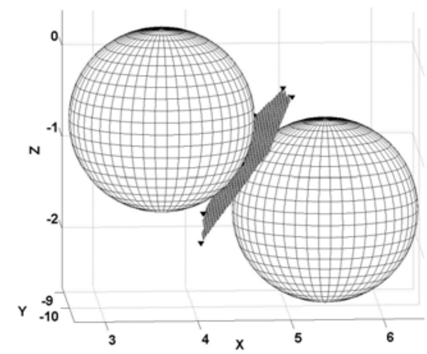

Figure 3: Spatial view of two spheres in Finney packing. The centers of the Delaunay cells (▲) in which the spheres are contained and the center of the gap between the two spheres lie in the same plane.

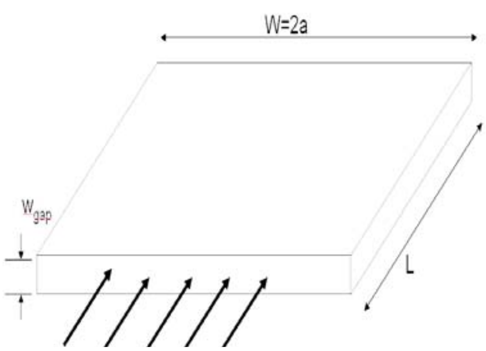

Figure 4: Flow through a gap represented by a slit. Arrows represent the direction of the flow. The range of capture *a* is shown in Fig. 1c.

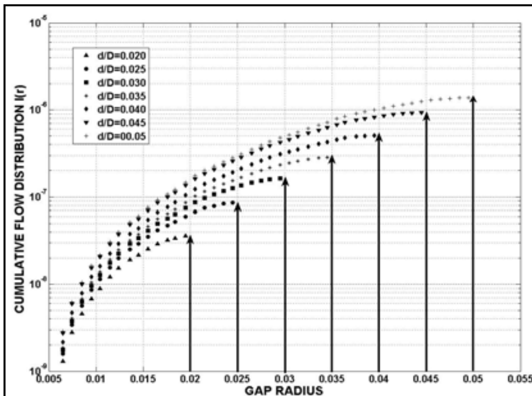

Figure 5: Example of cumulative flow distribution in the gaps region. The arrows indicate the value of $I(r)$ for different values of radius of strained particles (r_s).

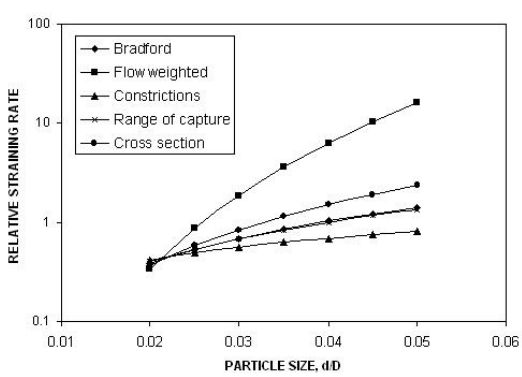

Figure 6: Correlation between straining constant and particle size calculated with five different methods. The flow-weighted method gives the largest deviation with respect to the other methods.