A NEW VERSATILE CAPILLARY PRESSURE CORRELATION

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ABSTRACT

There are many aspects of accessing multi-phase flow performance within oil exploitation. One is the actual determination of multiphase flow properties from measured data, and another is the representation of the unknown functions for relative permeability and capillary pressure. It is essential that these functions have sufficient degrees of freedom to model the measured data whilst remaining straightforward and easy to communicate.

We propose a new analytical correlation as a possible replacement for currently adopted capillary pressure functions. As capillary pressure by definition approach infinite values at the residuals, this will cause problems for reservoir modelling and practical reservoir engineering calculations due to the singularities.

The new correlation exhibits the traditional capillary pressure shape, but a finite capillary pressure value is captured at both irreducible water saturation as well as residual oil saturation. For mixed-wet reservoirs, the spontaneous water saturation, where the capillary pressure is zero, is normally defined from imbibition tests in the laboratory. The new correlation uses this saturation value as one of the well defined parameters. This value, together with the finite capillary pressure at both residuals, makes it powerful to define hysteresis envelopes. A well defined threshold pressure is also captured for primary drainage experiments.

We demonstrate the strength of the new correlation by utilizing experiments performed on core samples from the Norwegian Continental Shelf. We utilize primary drainage experiments followed by imbibition performed by the porous plate technique, as well as imbibition experiments followed by secondary drainage performed by the centrifuge technique. The use of finite capillary pressure and the spontaneous saturation value is very well demonstrated, as well as defining the hysteresis envelope generated by the centrifuge tests. We have also used the correlation within a simulator to reconcile the production data from a multi-speed centrifuge experiment.

INTRODUCTION

Capillary pressure is routinely measured for input to full field reservoir models, as well as determination of wettability, saturation height relationships and pore size distribution. During the last decade, laboratory relative permeability is routinely determined by including the laboratory capillary pressure end-effect through simulations [6, 9, 10, 11, 12]. Figure 1 shows the basic capillary pressure curves for an oil-water system: *Primary* *drainage* is used for initializing the full field simulator and defining the pore-size distribution through mercury injection. *Secondary imbibition** is used for simulating aquifer influx in a full field simulation model, water injection in full field simulations, and for use in laboratory simulators to determine relative permeabilities from secondary imbibition flow experiments. The secondary imbibition curve is the lower boundary curve of the hysteresis envelope in an oil-water system. The *secondary drainage* curve for an oil-water system is located between the primary drainage curve and the secondary imbibition curve. It constitutes the upper boundary curve of the hysteresis envelope. The secondary drainage curve for an oil-water system models an oil flow in water filled saddle point between two oil accumulations. It is also used in laboratory simulators to determine relative permeabilities from secondary drainage flow experiments in oil-water systems. The saturation value where capillary pressure is zero is throughout this paper called *Spontaneous saturation*, see Figure 1. There exist a number of correlations to describe the primary drainage process. Burdine [5], Bentsen&Anli [3], Brooks & Corey [4] cover a selection. Both Jing & Wunnick [6] and Skjæveland et al. [13] extended the correlation by Brooks & Corey [4] to also honor the negative part. Masalmeh et.al [10] further extended Skjævelands correlation by an extra variable term (or function) to describe different curve shapes e.g. for bimodal pore size distribution in carbonate rocks.

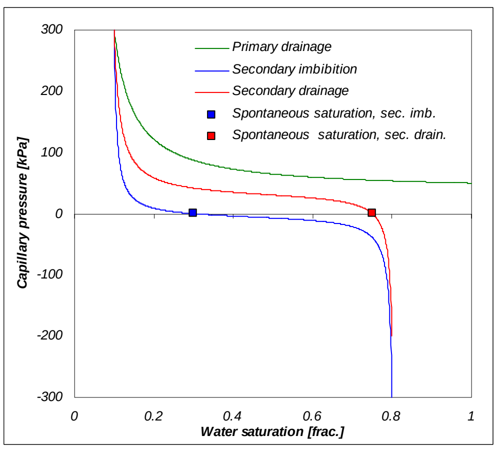

Figure 1: Basic capillary pressure curves

A number of correlations are currently in use, but to the authors knowledge none of them are finite and define the value of the spontaneous saturation. The new correlations are discussed and used for a variety of capillary pressure curves gained from various experimental approaches as well as used in the commercial core flow simulator Sendra [12] for history matching purposes of a multi-speed centrifuge experiment.

NEW CORRELATION – LET

The capillary pressure correlations for *primary drainage*, *secondary imbibition* and *secondary drainage* are described below. They all use the mathematical LET function as a building block to model a branch or a feature in a capillary pressure correlation. The structure of the mathematical LET function F(x) is:

$$F(S_{wn}) = \frac{S_{wn}^L}{S_{wn}^L + E(1 - S_{wn})^T} \quad (1)$$

* The term *secondary imbibition* is simply introduced to ensure proper counting for hysteresis scanning curves. It is equivalent to imbibition within conventional SCAL notation.

In this case $F(S_{wn})$ is *0* at *0%* normalized water saturation, and *1* at *100%* normalized water saturation. The parameter *L* describes the lower part of the curve. The parameter *T* describes the top part of the curve in a similar way that the *L*-parameter describes the lower part of the curve. The parameter *E* describes the position of the slope of the curve, i.e. elevation. An *E*-value of one is neutral, and the position of the slope is governed by the *L*- and *T*-parameters. Increasing the value of the *E*-parameter pushes the slope towards the high end of the curve. Decreasing the value of the *E*-parameter pushes the slope towards the lower end of the curve. Experiences using the LET correlation indicate that $L, E, T \geq 0.01$ with no proper upper limitation.

An advantage of the correlation is that the structure shown in (1) can be extended to add new features to the capillary pressure when required. We will focus about the common capillary pressure curves; primary drainage, primary imbibition and secondary drainage, but a possible extension is sketched for hysteresis and scanning curves.

Primary drainage

When using capillary pressure within reservoir simulations, the input is a table with finite capillary pressure values, and it is sufficient that the table covers the capillary transition zone. A high oil column combined with a high capillary pressure curve may cause the simulator to run slower [1]. This is caused by the fact that the phase pressures and oil saturation are calculated dynamically.

Due to these reasons we have developed the finite capillary pressure correlation for primary drainage, and hence avoiding any inconvenient mathematical singularity. This is an approximation, and the aim is to deal with a handy correlation. The LET correlation honors a maximum capillary pressure as well as the threshold pressure. For a single-modal pore-neck distribution we have the overall structure

$$P_{cow}^{p} = (P_{cow}^{fp} - P_{cow}^{tp})F_{cow}^{fp} - P_{cow}^{tp}G_{rise}^{tp}F_{cow}^{tp} + P_{cow}^{tp} \qquad (2)$$

The first term is the forced part of the capillary pressure with LET function F_{cow}^{fp}. The second term is added to ensure that a gradual rise to the threshold pressure can be handled, but this term can be turned on or off by the flag, G_{rise}^{tp}. A flag can also be used to include or exclude a bimodal term, see Appendix A. Removing all special purpose components, we are left with the basic version of the LET correlation:

$$P_{cow}^{p} = \frac{(P_{cow}^{fp} - P_{cow}^{tp})(1 - S_{wx})^{L_{ow}^{fp}}}{(1 - S_{wx})^{L_{ow}^{fp}} + E_{ow}^{fp} S_{wx}^{T_{ow}^{fp}}} + P_{cow}^{tp} \qquad (3)$$

Although we strongly advocate the finite version of the correlation, we can easily turn it into an infinite version by using the flag G_{fin}^{fp}. See Appendix A for further details on the general correlation.

Through practical use, we have observed that the maximum capillary pressure, P_{cow}^{fp}, and the threshold pressure, P_{cow}^{tp} can be fixed. The *E*-parameter is adjusted to get a reasonable fit of interpreted capillary pressure data, e.g. analytical Hassler Brunner data [7]. The *T*-parameter is then adjusted to tune the fit of the steep part and the arc, while the L-parameter is adjusted to tune the plateau and also the arc. The *L*-parameter is partly used to prevent the E-parameter from becoming extremely large.

Secondary Imbibition

Secondary imbibition capillary pressure for an oil-water system contains of a positive and negative branch, hence we suggest combining two LET functions to cover this process. A third constant term is also added, and this is called a *threshold pressure* even though it is not an obvious label. It is used to rise or lower the slowly varying part, just like the constant threshold pressure term in primary drainage correlation. The new versatile LET correlation for secondary imbibition capillary pressure is:

$$P^{i}_{cow} = (P^{si}_{cow} - P^{ti}_{cow})F^{si}_{cow} + (P^{fi}_{cow} - P^{ti}_{cow})F^{fi}_{cow} + P^{ti}_{cow} \qquad (4)$$

The first term in Equation 4 describes the spontaneous imbibition part of the curve, F^{si}_{cow}. The intersection with the saturation axis, where capillary pressure is zero, is defined by the spontaneous water saturation S_{wsin}. This value is used to calculate the *E*-parameter of the spontaneous capillary pressure term as shown in Appendix B, Equations B6 and B7. Even though spontaneous water saturation does not cover all aspects of the wettability concept it is routinely measured with good accuracy in wettability tests in the laboratory [2].

The second term describes the forced and negative part of the curve. This may be regarded as the most important part as this frequently contains the data points obtained from centrifuge experiments and an appropriate analytical solution like Hassler-Brunner [7]. Data points are also obtainable for the spontaneous part, i.e. the positive part, through porous plate experiment. However, this kind of experiment may take more than a year to fulfil and are hence not frequently measured. Neglecting the term for threshold pressure, a simplified version of the LET correlation for the secondary imbibition process is:

$$P^{i}_{cow} = \frac{P^{si}_{cow}(1-S_{\mathrm{wn}})^{L^{si}_{ow}}}{(1-S_{\mathrm{wn}})^{L^{si}_{ow}} + E^{si}_{ow}S_{\mathrm{wn}}^{T^{si}_{ow}}} + \frac{P^{fi}_{cow}S_{\mathrm{wn}}^{L^{fi}_{ow}}}{S_{\mathrm{wn}}^{L^{fi}_{ow}} + E^{fi}_{ow}(1-S_{\mathrm{wn}})^{T^{fi}_{ow}}} \qquad (5)$$

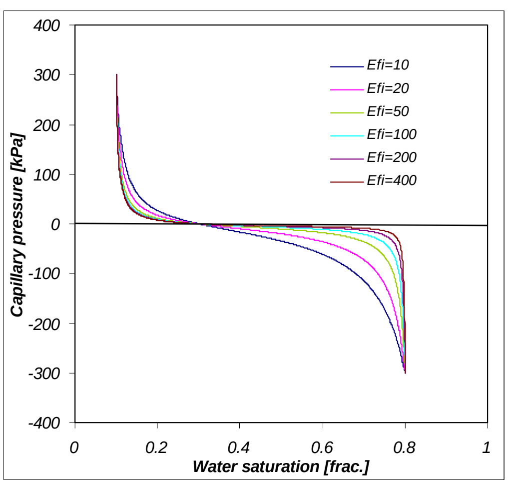

Figure 2: Variation in the forced E-parameter.

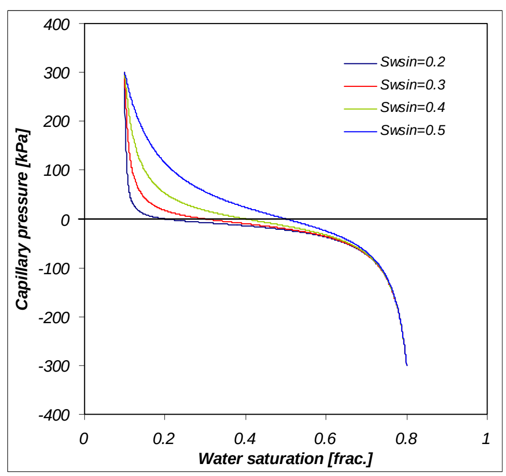

Figure 3: Variation in the value of spontaneous saturation.

Figure 2 and 3 (above) illustrates the effect of varying the forced *E*-parameter and the spontaneous water saturation. Figure 2 shows that we have great flexibility in the forced branch, with only minor changes in the spontaneous branch, when the spontaneous saturation is fixed and the forced *E*-parameter is varied. When the spontaneous saturation vary in Figure 3, the capillary pressure curve experiences considerable impact on the spontaneous branch, and minor changes on the forced branch.

Although the correlations for secondary imbibition can be extended to include a term for bimodal pore-neck distribution, a flag for infinite capillary pressure etc, such items are omitted for clarity. See Appendix B for further details on the general correlation.

Secondary drainage

The structure of the LET correlation for the secondary drainage capillary pressure is equal to the above described secondary imbibition capillary pressure, but the spontaneous branch and the forced branch are interchanged. Now, it is the spontaneous oil saturation, where the capillary pressure is zero, which is used to determine the empirical *E*-parameter of the spontaneous branch. The new versatile LET correlation for secondary drainage capillary pressure, without flags, is:

$$P_{cow}^{d} = \frac{(P_{cow}^{fd} - P_{cow}^{td})(1-S_{\mathrm{wn}})^{L_{ow}^{fd}}}{(1-S_{\mathrm{wn}})^{L_{ow}^{fd}} + E_{ow}^{fd} S_{\mathrm{wn}}^{\;T_{ow}^{fd}}} + \frac{(P_{cow}^{sd} - P_{cow}^{td}) S_{\mathrm{wn}}^{\;L_{ow}^{sd}}}{S_{\mathrm{wn}}^{\;L_{ow}^{sd}} + E_{ow}^{sd}(1-S_{\mathrm{wn}})^{T_{ow}^{sd}}} + P_{cow}^{td} \qquad (6)$$

See Appendix C for further details.

Hysteresis

The finite LET correlations seem to be well suited for hysteresis formulation. As the endpoints are finite, the appropriate max / min value of the scanning curve is simply replaced by the value of the boundary curve at the turning point, see Figure 4. The threshold pressure of the scanning curve is a function of the turning point, and it is adjusted to match the scanning curves at higher initial water saturations or the scanning curves within the closed boundary curves in a multi-cyclic WAG process. As shown in Figure 3 the major features of the spontaneous branch of capillary pressure is governed by the spontaneous saturation (together with S_{wi} and S_{orw}), which in scanning curves are estimated by a development of the normalized spontaneous saturation, as shown in Figure 4. The hysteresis algorithm of Skjæveland et al. [13] can alternatively be used to handle a hysteresis behavior with the LET correlations in a flooding process. Failing to have experimental data for scanning curves, the hysteresis model can be matched to Skjæveland et al. or utilizing default values when established. For centrifuge experiments we generally don't have

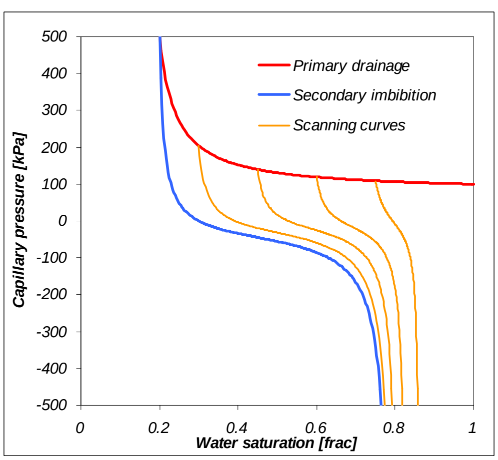

Figure 4: Basic hysteresis scanning curves

experimental data for the spontaneous part of the capillary pressure curve. However, the spontaneous branch of capillary pressure in IDC2[†] must be less than the forced branch of capillary pressure in DIC2 which must be less than the forced branch of capillary pressure in DIC1. In this way we can establish reasonable values of the spontaneous *L*- and *T*-parameters. We thus claim that we can establish entire curves for all boundary curves with a reasonable accuracy.

USE OF THE CORRELATION ON DATA FROM POROUS PLATE AND CENTRIFUGE EXPERIMENTS

To demonstrate the use of the new versatile LET-correlation we will show the behaviour on two samples from porous plate experiments and two samples from centrifuge experiments. All experiments are performed on samples of around 5 cm in length and 1.5” in diameter and they cover at least two capillary pressure cycles.

Porous plate experiments

The porous plate experiments have been performed at reservoir temperature and pressure utilizing dead oil on two sandstone samples from the Norwegian Sea.

Both samples where cleaned and the first cycle is primary drainage. When the maximum capillary pressure is achieved and initial water saturation is established the oil pressure is decreased. This allows water to imbibe into the core sample, and hence data is achieved along the spontaneous part of the secondary imbibition curve. The primary drainage cycle continued for 235 days, and the last pressure step for 75 days hence the restoration of the sample is regarded as fulfilled during this cycle without further aging.

Core A exhibits primary drainage cycle followed by secondary imbibition. The maximum value, the spontaneous water saturation and the minimum capillary pressure values are all kept fixed and the shape is well described by the parameters *L*, *E* and *T* as shown in Figure 5.

Core B exhibits three cycles: primary drainage cycle followed by secondary imbibition and finally secondary drainage, see Figure 6. There was no spontaneous oil imbibition for the secondary drainage curve; hence no negative part of this cycle is present. As for Core A, the shape for all cycles is well described by the parameters *L*, *E* and *T* when physical properties are fixed. These physical properties are maximum and minimum capillary pressure, the spontaneous saturations for the secondary imbibition and the secondary drainage and the threshold pressure for the primary drainage curve.

[†] The triplet-cycle (or WOG-triplet-cycle) is an easy bookkeeping system for the flooding processes whether it is separate experiments or a hysteresis sequence on a single core. It consists of a triplet and a cycle number. The positions in the triplet are reserved for water, oil and gas in that fixed sequence. Each position is occupied by a letter D, I or C meaning decreasing, increasing or constant phase saturation. Thus, DIC2 means secondary drainage for an oil-water system.

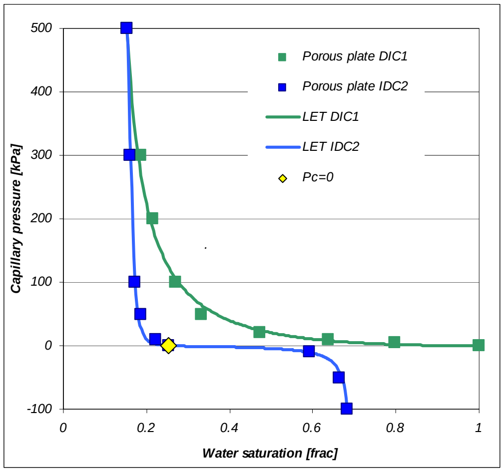

Figure 5: Primary drainage followed by secondary imbibition, Core A.

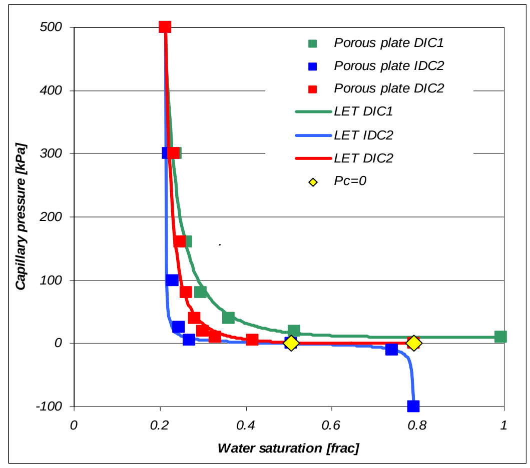

Figure 6: Primary drainage followed by secondary imbibition and secondary drainage, Core B.

Multi-speed centrifuge experiments

The centrifuge experiments have been performed at ambient conditions utilizing laboratory oil on two sandstone samples from the North Sea (core C) and the Norwegian Sea (core D). Core C was flushed with brine and laboratory oil before S_{wi} was established. Core D were cleaned by solvents and saturated *100%* with brine before drained to S_{wi}.

When utilizing standardized multi-speed centrifuge experiments, it is not possible to obtain capillary pressure data through the spontaneous part. At S_{wi} the samples are placed in an Amott cell for spontaneous imbibition of water. This defines the spontaneous water saturation. When the production ceases and the capillary pressure is assumed to be zero, the samples are centrifuge to residual oil, S_{orw} to achieve the negative branch of the capillary pressure curve. When the production ceases, the samples are again placed in an

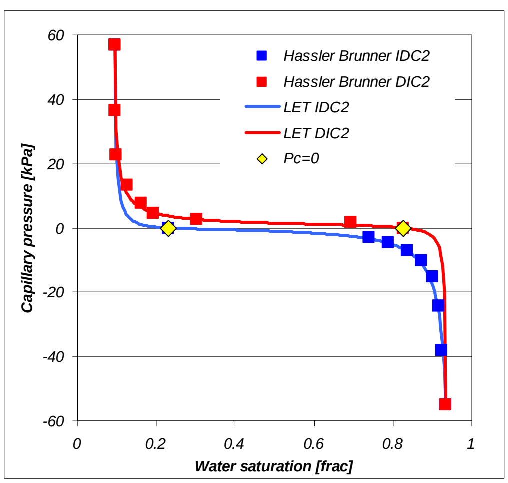

Figure 7: Primary drainage followed by secondary imbibition, Core C.

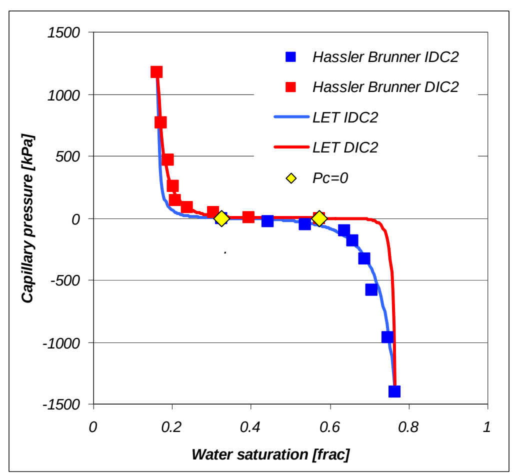

Figure 8: Primary drainage followed by secondary imbibition, Core D.

Amott cell for spontaneous imbibition of oil. This defines the spontaneous oil saturation for the secondary drainage curve. Finally, the samples are centrifuged to irreducible water saturation, S_{wir}, to achieve the positive branch of the secondary drainage capillary pressure curve.

Figure 7 and 8 (above) shows that the capillary pressure points determined by the Hassler Brunner method [7] are well described by the LET-correlation. Also for these experiments, the maximum and minimum capillary pressure values are fixed at the appropriate water saturation values. The spontaneous saturation values are also kept fixed for the secondary imbibition and secondary drainage. Keeping these points fixed in the analytical data, the shape of the capillary pressure curves – secondary imbibition and secondary drainage - are well defined by the parameters *L*, *E* and *T*.

INTERPRETATION OF CENTRIFUGE EXPERIMENTS BY USE OF A CORE FLOW SIMULATOR

To further demonstrate the new correlation it has been included in a commercial core flow simulator for history matching of production data from a centrifuge imbibition experiment. Interpretation of capillary pressure and reconciliation of the experimental production data have been performed by utilizing the optimization routine in the commercial core flow simulator Sendra [12].

Traditional analytical interpretation of centrifuge data [7] may introduce uncertainties for the intermediate saturation values, however the end-point values as well as the intersection with the saturation axis is well determined through material balance. Hence, these saturation values and corresponding capillary pressure is fixed throughout the determination of the capillary pressure that reconciles the experimental production data.

The experiment is performed on a single restored state plug from the Norwegian Sea performed at ambient pressure at elevated temperature in a traditional Beckman centrifuge. Core and fluid properties are given in Table 1 together with the experimental conditions. Fluids used are dead reservoir oil and synthetic formation water at equilibrium.

Table 1: Core and fluid properties for the centrifuge experiment.

Length, core [cm]	4.69	*Viscosity, water [cP]*	0.527
Diameter, core [cm]	3.79	*Viscosity, oil [cP]*	0.919
$k_o (S_{wi})$ *[mD]*	92.4	*Temperature [^{o}C]*	70
S_{wi} *[frac.]*	0.135	*Centrifuge speeds [RPM]*	500; 700; 1000; 1500; 2300; 3500; 5000
Porosity [frac.]	0.197		

Figure 9 shows the capillary pressure determined through the optimization routine. The flexibility to the new correlation is sufficient when the maximum- and minimum capillary pressure as well as the value of the spontaneous saturation are fixed. There is also a good match of the capillary pressure curve determined through simulations and the *independently* obtained capillary pressure data points from the analytical Hassler-Brunner interpretation [7].

Figures 10 show the experimental data and the simulated data through the optimisation procedure. As can bee seen, the experimental data are well reconciled by use of the new capillary pressure correlation shown in Figure 9.

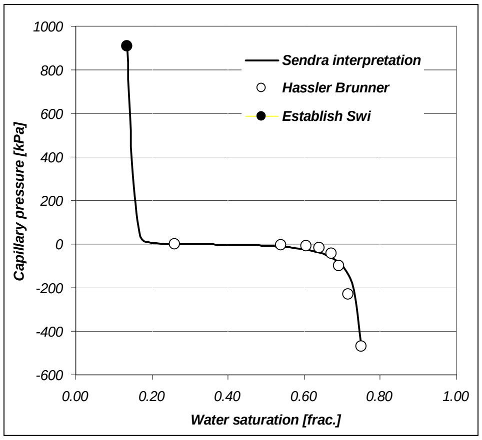

Figure 9: Capillary pressure

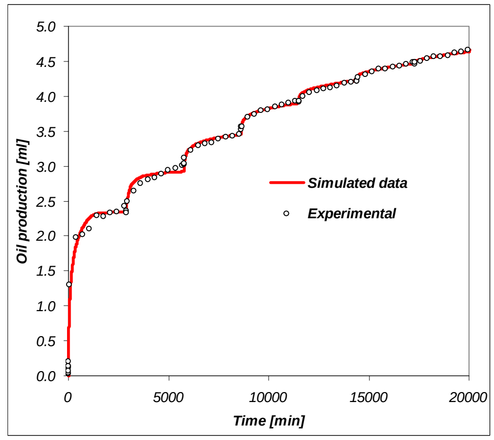

Figure 10: History match of production

CONCLUSION

- A new analytical capillary pressure correlation has been developed.
- By using a finite correlation for the capillary pressure one avoids the mathematical problems of an infinite correlation without a significant sacrifice in accuracy when modelling the capillary transition zone.
- Using measurable key properties of the capillary pressure and the LET function as building blocks, the branches and features of the capillary pressure are modelled in a natural way.
- Using a reasonable number of empirical and measurable parameters the correlation is able to describe a span of capillary pressure curves and also reconcile production data through simulations of a multi-speed centrifuge experiment.
- Despite the addition of parameters, the LET correlation remains easily accessible and applicable for full field reservoir simulations and engineering.

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REFERENCES

1. Aziz K., and Settari A.: “Petroleum Reservoir Simulation”, Applied Science Publishers, 1983, p30.
2. Amott, E: “Observations relating to the wettability of porous rock”, Petroleum transactions of AIME 1959.
3. Bentsen R.G., Anlie J.: “Using Parameter Estimation Techniques to Convert Centrifuge Data into a Capillary Pressure Curve”, Trans., AIME, 1977, 56.

4. Brooks R.H. and Corey A.T.: ‘‘Hydraulic Properties of Porous Media,’’ Hydraulic Paper No. 3, Colorado State U., 1964.
5. Burdine N.T.: “Relative Permeability Calculations from Pore Size Distribution”, Trans., AIME, 1953, **198**, pp. 71-78.
6. Jing X.D., and Wunnik J.N.M. van, “A Capillary Pressure Function for Interpretation of Core-Scale Displacement Experiments”, SCA 9807.
7. Hassler, G.L., Brunner, E.: ”Measurement of capillary pressure in small core samples”, Trans, AIME (1945), Vol. 160, 114.
8. Lomeland F., Ebeltoft E. and Hammervold Thomas W.: “A New Versatile Relative Permeability Correlation”, SCA 2005-32
9. Maas, J.G., Schulte, A.M.: ”Computer Simulation of Special Core Analysis (SCAL) Flow Experiments Shared on the Internet”, Reviewed Proceedings of the 1997 International Symposium of the SCA, Calgary, Canada, September 7-10, 1997.
10. Masalmeh S.K., Abu Shiekah I. and Jing X.D. “Improved Characterization and Modeling of Capillary Transition Zones in Carbonate Reservoirs”, SPE Reservoir Evaluation & Engineering, Vol. 10 (2): 191-204 APR (2007), SPE paper 109094
11. Masalmeh S.K., and Jing X.D., “Capillary Pressure Characteristics of Carbonate Reservoirs: Relationship between Drainage and Imbibition Curves, SCA 2006-16.
12. Sendra, “User guide”, Reslab Integration AS, 2007.
13. Skjæveland S.M., Siqveland L.M., Kjosavik A., Hammervold Thomas W.L., Virnovsky G.A.: “Capillary Pressure Correlation for Mixed-Wet Reservoirs” SPE 60900 / SPE 39497 India oil and Gas Conference and Exhibition, New Dehli, India, 17-19 Feb, 1998 / SPE 60900 SPE Reservoir Eval. & Eng. 3 (1), Feb 2000

NOMENCLATURE

E_{xy}^{z}	Empirical parameter for phase x and phase y and other information z	S_{orw}	Residual oil saturation after water injection
F_{cow}^{xy}	LET oil-water capillary function for x = f,s,t and y = p,i,d	S_{osd}	Spontaneous oil saturation in the secondary drainage process
G_{xy}^{z}	User flag with information z for feature xy	S_{wi}	Irreducible water saturation
L_{xy}^{z}	Empirical parameter for phase x and phase y and other information z	S_{wn}	Water saturation, short range normalization
P_{cow}^{z}	Oil-water capillary pressure for z = primary drainage (p), secondary imbibition (i) and secondary drainage (d)	S_{wsi}	Spontaneous water saturation in the secondary imbibition process
P_{cow}^{fz}	Max or min forced (f) capillary pressure value for z = p,i,d	S_{wx}	Water saturation, long range normalization
P_{cow}^{sz}	Max or min spontaneous (s) capillary pressure value for z = p,i,d	T_{xy}^{z}	Empirical parameter for phase x and phase y and other information z
P_{cow}^{tz}	Threshold (t) pressure value for z = p,i,d		

APPENDIX A Oil-water primary drainage capillary pressure

We will now list the LET capillary pressure correlation for oil-water primary drainage. Let us first define the long range normalized water saturation

$$S_{\mathrm{wx}} = \frac{S_{\mathrm{w}} - S_{\mathrm{wi}}}{1 - S_{\mathrm{wi}}} \quad \text{(A1)}$$

The overall structure of the LET correlation for primary drainage capillary pressure is

$$P_{cow}^{p} = (P_{cow}^{fp} - G_{bim}^{bp}(P_{cow}^{bp} - P_{cow}^{tp}) - P_{cow}^{tp})F_{cow}^{fp} + (P_{cow}^{bp} - P_{cow}^{tp})G_{bim}^{bp}F_{cow}^{bp} - P_{cow}^{tp}G_{rise}^{tp}F_{cow}^{tp} + P_{cow}^{tp} \quad \text{(A2)}$$

The LET function for the forced term is

$$F_{cow}^{fp} = \frac{(1 - S_{\mathrm{wx}})^{L_{ow}^{fp}}}{G_{fin}^{fp}(1 - S_{\mathrm{wx}})^{L_{ow}^{fp}} + E_{ow}^{fp} S_{\mathrm{wx}}^{T_{ow}^{fp}}} \quad \text{(A3)}$$

The LET function for the bimodal term (or branch) is

$$F_{cow}^{bp} = \frac{(1 - S_{\mathrm{wx}})^{L_{ow}^{bp}}}{(1 - S_{\mathrm{wx}})^{L_{ow}^{bp}} + E_{ow}^{bp} S_{\mathrm{wx}}^{T_{ow}^{bp}}} \quad \text{(A3)}$$

The LET function for a gradual rise to the threshold pressure is

$$F_{cow}^{tp} = \frac{S_{\mathrm{wx}}^{L_{ow}^{tp}}}{S_{\mathrm{wx}}^{L_{ow}^{tp}} + E_{ow}^{tp}(1 - S_{\mathrm{wx}})^{T_{ow}^{tp}}} \quad \text{(A4)}$$

We notice that

$$P_{cow}^{fp} > P_{cow}^{tp} \geq 0 \quad \text{(A5)}$$

A simplified version is

$$P_{cow}^{p} = \frac{(P_{cow}^{fp} - P_{cow}^{tp}) * (1 - S_{\mathrm{wx}})}{G_{fin}^{fp}(1 - S_{\mathrm{wx}}) + E_{ow}^{fp} S_{\mathrm{wx}}} + P_{cow}^{tp} \quad \text{(A6)}$$

Another simplified infinite version is

$$P_{cow}^{p} = \frac{(P_{cow}^{fp} - P_{cow}^{tp}) * (1 - S_{\mathrm{wx}})}{E_{ow}^{fp} S_{\mathrm{wx}}^{T_{ow}^{fp}}} + P_{cow}^{tp} \quad \text{(A7)}$$

or

$$P_{cow}^{p} = \frac{A_{ow}^{fp} * (1 - S_{\mathrm{wx}})}{S_{\mathrm{wx}}^{T_{ow}^{fp}}} + P_{cow}^{tp} \quad \text{(A8)}$$

APPENDIX B Oil-water secondary imbibition capillary pressure

We will now list the LET capillary pressure correlation for oil-water secondary imbibition. Let us first define the (short range) normalized water saturation

$$S_{\mathrm{wn}} = \frac{S_{\mathrm{w}} - S_{wi}}{1 - S_{wi} - S_{\mathrm{orw}}} \quad \text{(B1)}$$

The overall structure of the LET correlation for secondary imbibition capillary pressure is

$$P_{cow}^{i} = (P_{cow}^{si} - P_{cow}^{ti})F_{cow}^{si} + (P_{cow}^{fi} - P_{cow}^{ti})F_{cow}^{fi} + P_{cow}^{ti} \quad \text{(B2)}$$

The LET function for the spontaneous term is

$$F_{cow}^{si} = \frac{(1-S_{\mathrm{wn}})^{L_{ow}^{si}}}{(1-S_{\mathrm{wn}})^{L_{ow}^{si}} + E_{ow}^{si} S_{\mathrm{wn}}^{T_{ow}^{si}}} \tag{B3}$$

The LET function for the forced term is

$$F_{cow}^{fi} = \frac{S_{\mathrm{wn}}^{L_{ow}^{fi}}}{S_{\mathrm{wn}}^{L_{ow}^{fi}} + E_{ow}^{fi}(1-S_{\mathrm{wn}})^{T_{ow}^{fi}}} \tag{B4}$$

We notice that

$$P_{cow}^{si} \geq 0 \quad and \quad P_{cow}^{fi} < P_{cow}^{ti} \leq 0 \tag{B5}$$

We also notice that

$$P_{cow}^{i}(S_w = S_{wsi}) = 0 \quad \Rightarrow E_{ow}^{si} \tag{B6}$$

giving

$$E_{ow}^{si} = -\left[\frac{(P_{cow}^{si} - P_{cow}^{ti}) * (S_{w\sin}^{L_{ow}^{fi}} + E_{ow}^{fi}(1-S_{w\sin})^{T_{ow}^{fi}})}{P_{cow}^{fi} S_{w\sin}^{L_{ow}^{fi}} + P_{cow}^{ti} E_{ow}^{fi}(1-S_{w\sin})^{T_{ow}^{fi}}} + 1\right] * \frac{(1-S_{w\sin})^{L_{ow}^{si}}}{S_{w\sin}^{T_{ow}^{si}}} \tag{B7}$$

APPENDIX C Oil-water secondary drainage capillary pressure

We will now list the LET capillary pressure correlation for oil-water secondary drainage. Let us first define the (short range) normalized water saturation

$$S_{\mathrm{wn}} = \frac{S_{\mathrm{w}} - S_{\mathrm{w}i}}{1 - S_{\mathrm{wi}} - S_{\mathrm{orw}}} \tag{C1}$$

The overall structure of the LET correlation for secondary drainage capillary pressure is

$$P_{cow}^{d} = (P_{cow}^{fd} - P_{cow}^{td})F_{cow}^{fd} + (P_{cow}^{sd} - P_{cow}^{td})F_{cow}^{sd} + P_{cow}^{td} \tag{C2}$$

The LET function for the forced term is

$$F_{cow}^{fd} = \frac{(1-S_{\mathrm{wn}})^{L_{ow}^{fd}}}{(1-S_{\mathrm{wn}})^{L_{ow}^{fd}} + E_{ow}^{fd} S_{\mathrm{wn}}^{T_{ow}^{fd}}} \tag{C3}$$

The LET function for the spontaneous term is

$$F_{cow}^{sd} = \frac{S_{\mathrm{wn}}^{L_{ow}^{sd}}}{S_{\mathrm{wn}}^{L_{ow}^{sd}} + E_{ow}^{sd}(1-S_{\mathrm{wn}})^{T_{ow}^{sd}}} \tag{C4}$$

We notice that

$$P_{cow}^{fd} > P_{cow}^{td} \geq 0 \quad and \quad P_{cow}^{sd} \leq 0 \tag{C5}$$

We also notice that

$$P_{cow}^{d}(S_o = S_{osd}) = 0 \quad \Rightarrow E_{ow}^{sd} \tag{C6}$$

giving

$$E_{ow}^{sd} = -\left[\frac{(P_{cow}^{sd} - P_{cow}^{td}) * (S_{osdn}^{L_{ow}^{fd}} + E_{ow}^{fd}(1-S_{osdn})^{T_{ow}^{fd}})}{P_{cow}^{fd} S_{osdn}^{L_{ow}^{fd}} + P_{cow}^{td} E_{ow}^{fd}(1-S_{osdn})^{T_{ow}^{fd}}} + 1\right] * \frac{(1-S_{osdn})^{L_{ow}^{sd}}}{S_{osdn}^{T_{ow}^{sd}}} \tag{C7}$$