TWO-PHASE SIMULATION AND CORRELATION OF NMR MAGNETIC DECAY IN EQUILATERAL TRIANGULAR PORES

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ABSTRACT

A simple representation of a porous rock is a bundle of straight tubes. If the tubes, or pores, have sharp corners, more than one phase can form stable configurations inside the pore. This model has been valuable in studying drainage and imbibition processes with multiple phases involved. In this paper we analyze the magnetic signal from two phases in tubes (i.e. pores) with triangular cross sectional area. The NMR signal from a fluid-filled porous rock is in general a complicated function of pore geometry, surface relaxation and properties of the fluids. The magnetic signal from a single pore can always be written as an infinite sum over exponential functions. At certain conditions, the magnetic signal simplifies greatly and follows a monoexponential curve. The composite magnetic signal from all the pores in the rock can then be expressed as a sum of exponential functions where there is one function for each pore size. This special case is called the Fast Diffusion Limit (FDL). The logarithm of the magnetic signal is then inversely proportional to the pore radius making it possible to find a pore-size distribution. We investigate (numerically) how the NMR signal for 2 immiscible phases depends on surface relaxivity, contact angle, and pore size. The fluid configuration is given by the Mayer-Stowe-Princen (MS-P) theory of immiscible displacement in angular geometries. From the numerical experiments, a correlation is developed for the magnetic decay of two immiscible phases inside a triangular tube. We test the correlation against the FDL approximation and find that the correlation gives a better representation of the magnetic decay. This can be due to the effects of corners and angles. These effects are incorporated in the correlation.

INTRODUCTION

In the context of wettability characterization NMR measurements are becoming very interesting because of the surface sensitive nature of NMR. Compared to conventional wettability measurements, NMR is faster and in addition it is non-invasive and can be performed in-situ. Previous work has been made to characterize the wettability by including a triangular pore model in the interpretation of the NMR measurements, Al-Mahrooqi et al. (2006). In pore modelling work, angular pores are used to model mixed-wet conditions, Øren et al. (1998), Helland and Skjæveland (2004a) and Helland and Skjæveland (2004b). When combining pore modelling with NMR simulations, some assumptions have been made that need further investigation. The magnetic signal from one phase in a single pore is given as a sum of exponential functions; see Brownstein and Tarr (1979) for details,

$$M_{\text{pore}}(t) = M_{\text{pore}}(0)\exp(-t/T_{2B})\sum_{i=0}^{\infty} I_i \exp(-t/T_i). \tag{1}$$

When the pore is within the FDL, the average time for a molecule to diffuse across the pore $\tau_D = a^2/D$ is much shorter than the average time for a molecule to relax $\tau_\rho = a/\rho$. We get that $\tau_D / \tau_\rho = \rho a/D = \gamma \ll 1$. The effect across the pore is that the magnetization is uniform and the magnetic decay is uniform and monoexponential and given by,

$$M_{\text{pore}}(t) = M_{\text{pore}}(0)\exp(-t/T_2), \tag{2}$$

where

$$1/T_2 = 1/T_{2B} + 1/T_0 . \tag{3}$$

In NMR interpretation, the FDL assumption is often valid and the magnetic decay caused by the surface relaxation $1/T_{2S}$ is monoexponential and given by

$$T_{2S} \equiv T_0 = V/(S\rho). \tag{4}$$

This assumption is based on calculations for a single phase in simple geometries such as circles, spheres, plates, squares, Brownstein and Tarr (1979), and recently we have published a solution for an equilateral triangle, Finjord et al. (2006). These geometries all have a certain radial regularity and symmetry. We wanted to see what happens when this symmetry is no longer present, for instance when the wetting phase placed in the corners of the triangle has contact with the pore wall on two sides of the pore and is constrained by the arched oil-water interface. We also wanted to explore what happens if the FDL parameter γ is larger than 1, i.e. outside the FDL interval. Does Eq. 3 still hold or has the decay become multiexponential for a single pore? From the results we wanted to find a correlation between surface relaxation T_0 and surface relaxivity ρ, contact angle θ and the fluid distribution. See Fig. 1 for image of triangular pore and Table 1 for relevant definitions.

In the case of two or more phases inside the pore, no analytical solution has been published, so the magnetic relaxation decay problem needs to be solved numerically. We will use Random Walk for obtaining the numerical solution, see Finjord et al. (2006) for details on the Random Walk algorithm in an equilateral, triangular pore. We use a triangular grid with 301 x 301 points for the triangle and $5 \cdot 10^5$ walkers for each phase. The geometry of the fluid phases is calculated using the theory of Mayer-Stowe-Princen for immiscible fluid in triangular pores, Mason and Morrow (1991). We simulated cases for 6 different pore sizes. The size of each pore was determined by choosing the appropriate ratio n_w of the length of the wetting wall a_w and the length of the pore a. In addition the choice of the diffusion parameter γ determined the pore size. The resulting decay curves for each phase were normalized and fitted to a sum of two exponential functions,

$$M(t) = \sum_{\alpha=o,w} I_{0\alpha}\exp(-t/T_{0\alpha}) + (1 - I_{0\alpha})\exp(-t/T_{1\alpha}), \tag{5}$$

where $I_{0\alpha}$,, $T_{0\alpha}$, and $T_{1\alpha}$ are correlated with relaxivity, contact angle and fluid distribution. The result is compared with the original decay curves, which also was compared with the original decay curve in the FDL regime.

RESULTS

Fitting the decay curves from the oil and the water to a biexponential sum gives the curves for $I_{0\alpha}$, $T_{0\alpha}$ and $T_{1\alpha}$,. The intensities $I_{0\alpha}$ appear to be only weakly dependent on the contact angles θ and the correlation for the intensities becomes a function of contact length a_α along the pore wall, diffusion coefficient D_α and surface relaxivity ρ_α. See Fig. 2 and Eqs. 10 and 13.

We observe in Fig. 3 an example of how the relaxation times $T_{0\alpha}$ and $T_{1\alpha}$ depend on contact angle θ, relaxivity ρ and fluid geometry. We find that the relaxation time for the non-wetting phase decreases and the corresponding relaxation time for the wetting phase increases as the wetting angle increases. The correlations for $T_{0\alpha}$ and $T_{1\alpha}$ are given in Eqs. 8, 9, 12, and 13 and are a sum of the relaxation times for the fast and the slow diffusion regime for both water and oil seen in Table 2.

Comparing the magnetic decay from the simulations with the magnetic decay calculated from $T_{2S} = V/S\rho$, showed that this assumption becomes inaccurate as the relaxivity increases and at low contact angles, see Fig. 4 for an example. We observe from Fig. 4 that the fit is overall in accordance with the simulated results. This favours the use of the presented correlation. The resulting equation can be used for modelling NMR decay for two phases in an equilateral triangular pore by assuming a pore size distribution $P(R_n)$ and generate magnetic decay curves based on the NMR parameters: ρ_o, ρ_w, D_o, D_w, $T_{2B,w}$, $T_{2B,o}$, and the wetting parameters ϕ and a_w. Changing the wetting parameters changes the decay curves correspondingly. We get that

$$M(t) = \sum_{n=1}^{m} P(R_n) M_n(t), \tag{6}$$

where M_n is M_{pore} for a given pore size R_n. and M_{pore} is given by

$$M_{\text{pore}}(t) = \sum_{\alpha=o,w} V_\alpha \left[I_{0\alpha} \exp(-t/T_{0\alpha} - t/T_{2B\alpha}) + (1 - I_{0\alpha}) \exp(-t/T_{1\alpha} - t/T_{2B\alpha}) \right]. \tag{7}$$

With the obtained fitting parameters for wetting (w) and non-wetting (o) phases, we get,

$$T_{0w} = r_w/\rho_w + r_w^2/D_w, \tag{8}$$

$$T_{1w} = 0.125 \cdot r_w/\rho_w + 0.00125 \cdot r_w^2/D_w, \tag{9}$$

$$I_{0w} = 1 - \left[0.0019 \cdot (\rho_w a_w/D_w)^2 + 0.0024 \cdot (\rho_w a_w/D_w)\right], \tag{10}$$

$$T_{0o} = r_o/\rho_o + 9R^2/4D_o\pi^2 - 0.001 r_o^2/D_o, \tag{11}$$

$$T_{1o} = 0.5 \cdot r_o/\rho_o + 0.051 \cdot r_o^2/D_o, \tag{12}$$

$$I_{0o} = \left(1.2 + \left((\rho_o a_o/D_o)/25\right)\right)^{-6.3} + 0.685, \tag{13}$$

where r_o and r_w are given by Eq. 19. This correlation is only valid when two phases are present in the pore. See Helland and Skjæveland (2004a) for details on capillary entry pressure and Finjord et al. (2006) for magnetic decay for one phase in triangular pore.

DISCUSSION AND CONCLUSIONS

When interpreting the NMR decay curves and the T_2 distribution, it is common to assume monoexponential decay for each pore. Recently, there has been an increased interest in angular pore shapes, where it is possible to model coexisting two or three phases in the same pore. We find that the FDL-assumption for these angular pores is inaccurate, especially at low contact angles or high surface relaxivity. This is probably caused by the inaccessibility of the pore wall close to the fluid-fluid meniscus. The observed low surface area could interfere with calculations of the pore size distributions or surface relaxivity. In addition the magnetic decay becomes multiexponential earlier for these irregular fluid geometries. This might lead to the signal from the higher intensities being interpreted as smaller pores or bound fluid.

For added accuracy in modelling one can use a biexponential sum for the surface relaxation in these angular pores. We have found a biexponential correlation for the magnetic decay for two phases in a triangular pore. The correlation is valid for primary drainage for contact angles in the range 0–55 degrees, relative pore sizes ranging from 1–50, and relative surface relaxivity of 1–10 when two phases are present. The correlation function can be used as a biexponential sum or it is an option to use a monoexponential decay curve with the corresponding relaxation times $T_{0\alpha}$.

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NOMENCLATURE

a	= Length of side of triangle
a_α	= Length of fluid-pore contact
D_α	= Diffusion coefficient
I	= Intensity
M	= Magnetization
$P(R_n)$	= Pore size distribution
r	= Radius of curvature
r_α	= Calculated fluid radius = V_α/S_α
R	= Inscribed radius of pore
S_α	= Length of fluid-pore contact
t	= Time
$T_{i\alpha}$	= Decay time for fluid
T_2	= Total decay time of pore
T_{2B}	= Bulk decay time of fluid
T_{2S}	= Surface decay time of fluid
V	= Fluid volume
γ	= $R\rho/D$, diffusion regime parameter
θ	= Contact angle
ρ	= Surface relaxivity
τ_D	= Average diffusion time
τ_ρ	= Average relaxation time

Subscripts

i	= Relaxation mode, i = 0, 1
α	= o for non-wetting phase, w for wetting phase

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TABLES

Table 1: Governing equations for the fluid configurations.

a) *Wetting phase in corners of pore*	
$a_w = 2r\cos(\theta + \pi/6)$	(14)
$V_w = 3a_w^2 f(\theta)/4$	(15)
$S_w = 6a_w$	(16)
b) *Non*-wetting *phase in the centre of pore*	
$V_o = (\sqrt{3}a^2/4) - (3a_w^2 f(\theta)/4)$	(17)
$S_o = 3a - 6a_w$	(18)
Definitions for wetting and non-wetting phase.	
$r_\alpha \equiv V_\alpha / S_\alpha$	(19)
$\gamma_\alpha \equiv r_\alpha \rho_\alpha / D_\alpha$	(20)

$\tau_D \equiv r_\alpha^2 / D_\alpha$	(21)
Where:	
$f(\theta) = \left(\theta - \frac{\pi}{3} + \cos(\theta)\left[\sqrt{3}\cos(\theta) - \sin(\theta) \right] \right) \Big/ \cos^2\left(\theta + \frac{\pi}{6} \right)$	(22)

Table 2: Table from Finjord et al. (2006) giving the analytical solution for magnetic decay in a triangular pore.

	Fast Diffusion Limit: $\gamma \ll 1$	Slow Diffusion Limit: $\gamma \gg 1$
T_0	$R/(2\rho)$	$9R^2/(4D\pi^2)$
T_1	$9R^2/(4D\pi^2 i^2)$	$9R^2/(4D\pi^2(i+1)^2)$

FIGURES

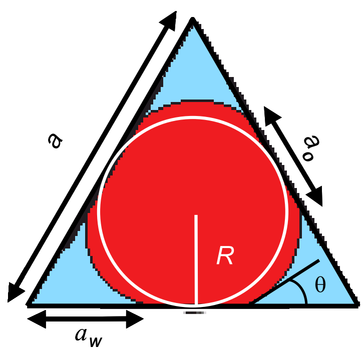

Figure 1: Fluid configuration for the triangular pore.

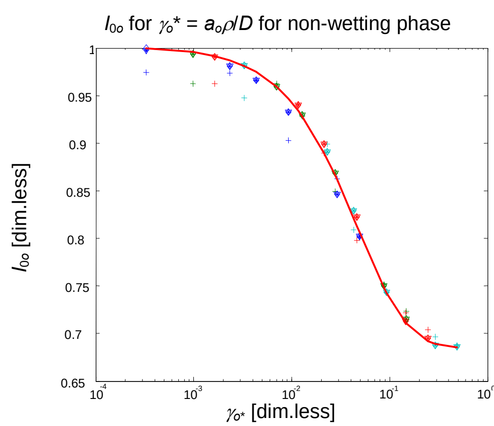

Figure 2: Intensity for non-wetting phase. Simulated data (Markers), Correlation data (Line).

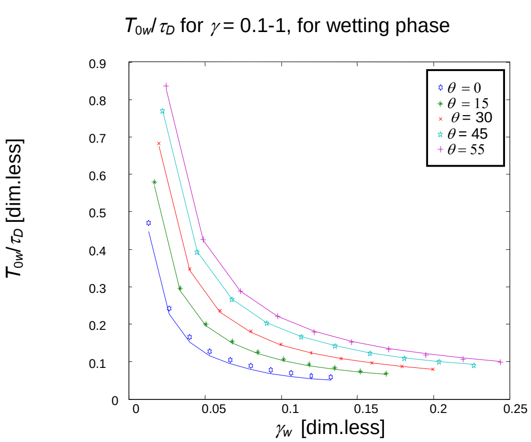

Figure 3: Relaxation time for wetting phase. Simulated data (Markers), Correlation data (Lines).

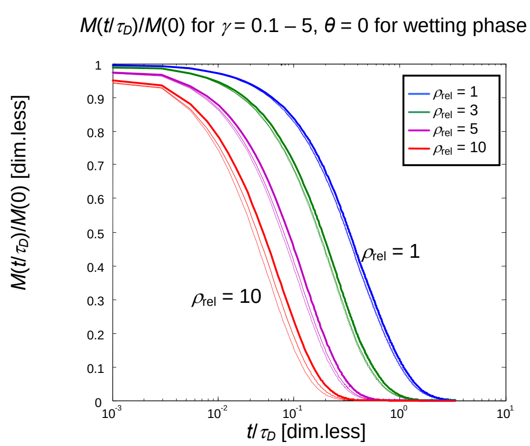

Figure 4: Magnetic decay for wetting phase. Simulated data (Thick line), Correlation data (Thin line), FDL data (Broken line).