PARTICLE DEPOSITION PROFILE IN THE RADIAL DEEP-BED FILTRATION OF INJECTION WATER

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ABSTRACT

Production of hydrocarbons is usually accompanied by the production of water. This produced water consists of formation water and/or water that has previously been injected into the formation. As more oil is produced, the amount of produced water increases. Unfortunately, the produced water is not a saleable product, hence, an operator must find ways to handle relatively large amounts of water in an environmentally-acceptable manner at the lowest cost. One way of managing this water is to re-inject it for disposal, pressure maintenance or enhanced oil recovery. An important and difficult task in the re-injection process is the prediction of the impact of water quality on well injectivity. This is mainly due to the poor understanding of the deposition mechanisms by which suspended solids and oil droplets present in the produced water are retained by the formation.

As reported earlier by Ali (2007), as part of an extensive study on formation damage, different concentrations of hematite particles suspended in water were injected into sandstone core samples, and the deposition measured in real-time using an X-ray apparatus. Analysis of the experiments, using deep bed filtration theory, showed that the rate of particle deposition is dependent on flowrate, being higher at low flowrates. This result is significant, since in radial injection (matrix injection) in field applications, fluid velocity decreases away from the injection face and therefore the deposition rate varies with distance into the formation. This effect has been modelled with a simple velocity step-change model, in which the rate of deposition changes at a critical velocity. The effect on the predicted deposition profile is determined and shown to be significant. The profile can be very different from that predicted without taking into account the velocity dependence.

INTRODUCTION

As reported by Ali (2007) and Ali et al (2005a,b, 2007), an on-line X-ray apparatus has been used to measure internal solid deposition during injection of hematite particles suspended in brine into sandstone cores. The experiments have been interpreted using deep bed filtration theory to model the retention of the particles inside the sandstone. The deep bed filtration equations are based on conservation of mass and a simple kinetic equation governing the rate of particle retention. The kinetic equation assumes that the rate of particle retention is directly proportional to the number of particles available to be captured by the porous medium. The number of particles available is equal to the product of the local suspension concentration and the interstitial velocity. The proportionality factor λ, called the filtration function, has unit m^{-1}.

The kinetic equation was first proposed by Iwasaki (1937), with the assumption that the filtration function λ is constant. Ives (1963, 1965) modified the Iwasaki equation and considered that the deposition of particle is highest at the beginning of injection where the probability of particle capture is high. Many authors have looked at ways of determining the filtration function. Most use the assumption that λ is constant. In this case λ can be determined, in principle, from the effluent concentration in core tests. Pang and Sharma (1994, 1997) used the classic deep bed filtration for constant λ but included a critical value of porosity at which deposition mechanisms change and an external filter-cake starts to grow.

Analysis of our experiments showed that λ is not a constant, but is a function of flowrate, being greater at lower flowrates (Ali 2007). This was predicted earlier by Veerapen et al. (2001), who suggested that particle deposition near pore throats will be reduced at higher velocities due to hydrodynamic shadowing and reduced surface entrainment.

This result can have significant consequences in radial injection flow, in which the local flowrate decreases with distance from the wellbore. It may result in less deposition near the wellbore, and more deposition deeper into the formation. Generally, injectivity decline will be lower when deeper deposition of solids takes place than with shallower deposition adjacent to the wellbore. As stated by Barkman and Davidson (1972) “If particulate matter in the injection water can be transported great distance from the wellbore, deposition will occur in a region of low gradient and the rate of impairment will be small.” A similar study conducted by Jorda (1987) confirmed the analysis of Barkman and Davidson, and suggested that deep invasion of solids beyond five well bore radii will cause injection decline to be much less severe than decline very close to the wellbore radius.

We therefore consider re-injection of water containing suspended solids via a well into a sandstone formation, assuming the injection pressure is below fracturing pressure. A very simple step-change model is assumed for the velocity dependence of the filtration coefficient λ, in order to highlight effects that can be expected.

DEEP-BED FILTRATION THEORY

We assume that the deposition of particles is governed by the Deep Bed Filtration theory. The radius of the well is r_w. We assume that the water is injected at constant injection flux $2\pi r_w u_w \phi$ per unit height of the injection interval, where u_w is the injection velocity at the wall of the well. Deeper into the formation, the approach velocity u at radius r is given by

$$u = r_w u_w / r \qquad \text{E.1}$$

In radial coordinates, the mass conservation equation of classic deep bed filtration theory is

$$\phi \frac{\partial c}{\partial t} + (1-\phi)\frac{\partial \sigma}{\partial t} + \frac{r_w u_w}{r}\frac{\partial c}{\partial r} = 0 \qquad \text{E.2}$$

The kinetic equation becomes

$$\frac{\partial \sigma}{\partial t} = \frac{\lambda r_w u_w}{r} c \qquad \text{E.3}$$

where λ is the filtration coefficient. The boundary and initial conditions and are

$$c(r_w, t) = c_0(t), \ \ \sigma(r,0) = 0, \ c(r,0) = 0 \qquad \text{E.4}$$

where $c_0(t)$ is the inflow concentration as a function of time at the wellbore radius $r = r_w$. We neglect the term $\phi \partial c / \partial t$ in E.1, since it is important only at very short time. This gives the simplified set of equations

$$\begin{aligned} &\frac{\partial c}{\partial r} + \lambda(1-\phi)c = 0 \\ &\frac{\partial \sigma}{\partial t} = \frac{\lambda r_w u_w}{r} c \end{aligned} \qquad \text{E.5}$$

The essential difference between these equations and the equations for linear flow lies in the radial dependence of the flux term in the kinetic equation.

VELOCITY STEP-CHANGE FILTRATION FUNCTION

The velocity step-change model for the filtration function is introduced to simulate the experimental findings. It was found that the filtration coefficient was highest at low flow rates, and decreased with increasing flowrate. This decreasing filtration function is approximated by a simple step-change in the value of the filtration coefficient at a critical velocity u_c.

$$\lambda = \lambda_{high} \ \ for\ u > u_c, \ \ \lambda = \lambda_{low} \ \ for\ u \le u_c \qquad \text{E.6}$$

Assuming that the injection velocity is above the critical velocity $u_w > u_c$, there exists a critical radius r_c at which λ will change from λ_{high} to λ_{low}.

$$\begin{aligned} &\lambda = \lambda_{high} \ \ for\ \ r_w \le r < r_c, \ \ \ \lambda = \lambda_{low} \ \ for\ \ r \ge r_c, \\ &r_c = r_w u_w / u_c \end{aligned} \qquad \text{E.7}$$

From E.5, the solutions for the hematite concentration c and deposition σ are

$$\begin{aligned} &c = c_0 \exp\left\{-(1-\phi)\lambda_{high}(r_c - r_w)\right\}, \ \ r < r_c \\ &c = c_0 \exp\left\{-(1-\phi)\left[\lambda_{low}(r - r_c) + \lambda_{high}(r_c - r_w)\right]\right\}, \ \ r \ge r_c \\ &\sigma = \frac{\lambda_{high} r_w u_w}{r} \exp\left\{-(1-\phi)\lambda_{high}(r - r_w)\right\} \int_0^t c_0(s) ds, \ \ r < r_c \\ &\sigma = \frac{\lambda_{low} r_w u_w}{r} \exp\left\{-(1-\phi)\left[\lambda_{low}(r - r_c) + \lambda_{high}(r_c - r_w)\right]\right\} \int_0^t c_0(s) ds, \ \ r \ge r_c \end{aligned} \qquad \text{E.8}$$

By putting $\lambda_{low} = \lambda_{high}$ in E.8, it can be seen that for a constant filtration function, the concentration c decays exponentially with radial distance, and the deposition profile σ decreases monotonically. For the step-change model, the rate of decay of c increases from λ_{high} to λ_{low} at $r = r_c$. At this point, the rate of deposition with time increases. As shown in Figure 1 this implies that the deposition profile σ is not monotonically decreasing with radial distance, but has a step increase at $r = r_c$. We note from E.8 that the deposition just inside the borehole wall is given by σ_w, where

$$\sigma_w = \lambda_{high} u_w \int_0^t c_0(s)ds \qquad \text{E.9}$$

This is not the deposition due to any external filtercake forming, but due to filtration just inside the formation. We rewrite E.8 to give the normalised deposition profile σ/σ_w as

$$\begin{aligned}
\frac{\sigma}{\sigma_w} &= \frac{r_w}{r}\exp\left\{-(1-\phi)\lambda_{high} r_w\left(\frac{r}{r_w}-1\right)\right\}, \quad \frac{r}{r_w} < \frac{r_c}{r_w} \\
\frac{\sigma}{\sigma_w} &= \frac{\lambda_{low}}{\lambda_{high}}\frac{r_w}{r}\exp\left\{-(1-\phi)\lambda_{high} r_w\left[\frac{\lambda_{low}}{\lambda_{high}}\left(\frac{r}{r_w}-\frac{r_c}{r_w}\right)+\left(\frac{r_c}{r_w}-1\right)\right]\right\}, \quad \frac{r}{r_w} \geq \frac{r_c}{r_w}{}_c
\end{aligned} \qquad \text{E.10}$$

If we plot the normalised deposition profile as a function of the normalised radial distance r/r_w then the form depends on the following three parameters $(1-\phi)\lambda_{high} r_w$, $\lambda_{low}/\lambda_{high}$ and $r_c/r_w = u_w/u_c$.

RESULTS AND DISCUSSION

As an example of the deposition profile that can be obtained, we take the value 3 for $(1-\phi)\lambda_{high} r_w$, assuming a wellbore radius of 10cm, porosity of 23% and the values of λ_{high} of about $40m^{-1}$ from Ali et al (2005). We take the value of $\lambda_{low}/\lambda_{high}$ to be 5. And we take values 1.2, 1.5 and 2 for the ratio of the injection velocity to the critical velocity u_w/u_c. The results are shown in Figure 1. It can be seen that the deposition decreases with radial distance from the wellbore until the critical radius is reached. At this point there is a significant increase in the amount of deposition, and the amount of deposition may even exceed the deposition at the wellbore.

A typical injection rate in a Kuwait water-injection well is between 5000 to 10,000 bbl/day, ($800\text{-}1600m^3$/day). This gives an injection velocity u_w in the range 250 - 500 m/day. With increasing flowrate, the velocity at the wellbore increases, and hence the critical distance increases. Figure 2 shows the normalized radial velocity $u = Q/2\pi\phi h r_w$ of a well with rate Q 10,000 bbl/day and injection height h of 5 meter, and reservoir porosity of 23%. We can see that the normalized velocity decreases by 95% of its original value in the first 2 meters, because of the $1/r$ behaviour.

For the sake of demonstration, let us take a critical velocity of 26.7 m/day, and the experimentally-determined values for the filtration coefficient, λ_{high} = $45m^{-1}$ above the critical velocity and λ_{low} = $400m^{-1}$ below the critical velocity (Ali 2007, Ali et al.2005a). Let us assume we are injecting above the critical velocity (constant flow-rate). As shown in Figure 2, the velocity will decay rapidly with the radial distance. We can calculate the critical radius $r = r_c$ from equation E.7. The filtration coefficient is λ_{high} below the critical radius and λ_{low} above the critical radius (lower velocity). This is shown in Figure 3. At high injection flowrate (high wellbore velocity), there is a larger distance from the wellbore within which the filtration coefficient stays at the value λ_{high}. And as we decrease the injection flowrate, the critical radius becomes smaller and closer to the wellbore radius. From this figure it is seen that a very high flow-rate of 10,000bb/day (1600m^3/day) gives a wellbore velocity of 533m/day, and the critical distance at which the filtration coefficient changes is at 2.25 meter radius.

CONCLUSION

Velocity-dependent behaviour of the filtration coefficient in Deep Bed Filtration theory has been modelled with a simple velocity step-change model. In radial injection, there is a critical radius at which the velocity drops below the critical velocity. Beyond the critical radius the filtration function is higher than within the critical radius. The deposition decreases with radial distance from the wellbore until the critical radius is reached. At this point there is a significant increase in the amount of deposition, and the amount of deposition may even exceed the deposition at the wellbore. This is very different from the predictions of the constant filtration coefficient model.

NOMENCLATURE

c	Concentration of suspended solids (volume of suspended solids per volume of fluid)
c_o	Inlet concentration (concentration at the face of the formation, after passing through the external filter cake)
h	Reservoir height, m
r	Radial distance, m
r_c	Critical radius at which critical velocity is attained, m
r_w	Wellbore radius, m
Q	Injection rate, m^3/s
t	Injection time, s
u	Fluid velocity m/s
u_c	Critical fluid velocity, m/s
u_w	Fluid velocity at the wellbore wall
λ	Filtration coefficient m-1
λ_{high}	Filtration coefficient above the critical velocity
λ_{low}	Filtration coefficient below the critical velocity
ϕ	porosity
σ	Concentration of deposited solids (volume of deposited solid per volume of sand)

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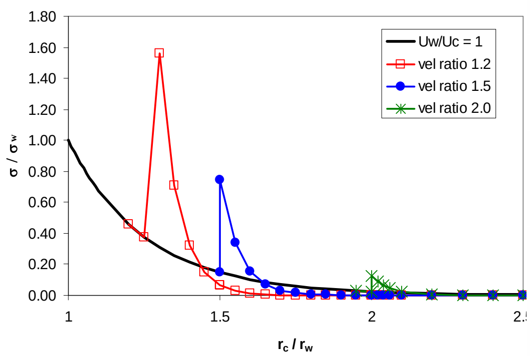

Figure 1: Illustration of the normalised deposition profile as a function of the normalised radial distance

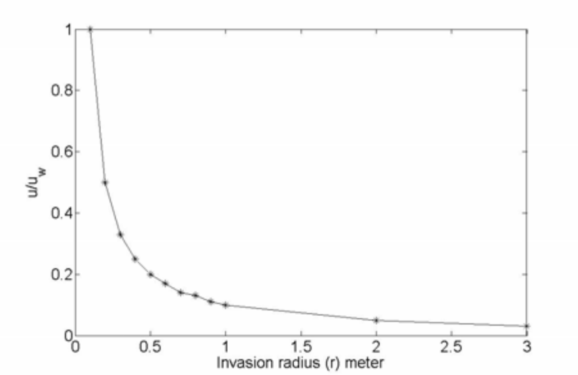

Figure 2: Normalised velocity in radial flow injection

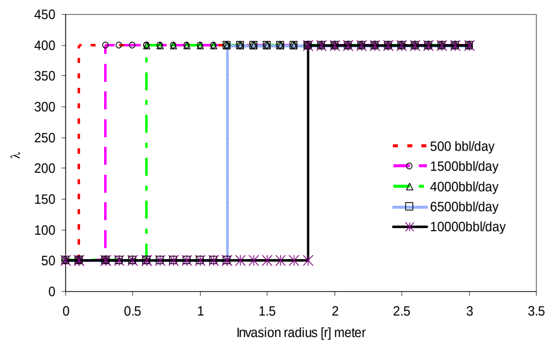

Figure 3: Radius at which the step-change occurs in the filtration coefficient