EXPERIMENTAL INVESTIGATION OF THE INERTIAL EFFECTS OF FLOW THROUGH POROUS MEDIA

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This paper was prepared for presentation at the International Symposium of the Society of Core Analysts held in Halifax, Nova Scotia, Canada, 4-7 October, 2010

ABSTRACT

The Darcy Law and the Forcheimer equation have been widely used to describe Darcy and non-linear flow through porous media. However, the literature has not been clear as to which equation should satisfactorily describe the flow at the transition zone between the Darcy and non-linear flow regimes. This paper is aimed at providing some experimental evidence to help resolve this ambiguity. In this study, square arrays of circular acrylic rods are used to model real porous media. The rods are arranged in a square channel so as to achieve porosities ranging from 0.51 to 0.88. Using a refractive index matched viscous fluid as the working fluid, the flow rate is regulated so as to obtain pressure-driven flow through the porous medium at the regimes of flow pertaining to the Darcy, transition and non-linear flows. A high resolution particle image velocimetry technique is used to provide two-dimensional velocity data across various sections of the porous media. Differential pressure measurements are also obtained using an electronic pressure transducer. These two measurements provide a complete set of experimental data to characterize the flow. Results show that the Darcy's Law indeed extends somewhat beyond the Darcy flow regime to which it is usually limited. The data further determines the point of the transition zone that can be accurately described by the quadratic Forcheimer equation. The results of this experimental study provide insights into inertial effects on flow through porous media not found in the literature. This work is expected to serve as a fundamental basis for further experimental, theoretical or numerical studies of more complicated cases of porous media.

INTRODUCTION

The study of fluid flows through porous media finds wide engineering application in many areas such as oil and gas exploration, groundwater flows, and geothermal energy. The flow regimes associated with such flows are four, namely the Darcy, inertial, unsteady-laminar, and 'turbulent' flow regimes (Huang and Ayoubb, 2008). Studies have been conducted on the transitions between these flow regimes (*e.g.* Seguin, Montillet and Comiti, 1998; Noman and Kalam, 1990). Notable of such transitions of flow regimes is that between the linear Darcy and the non-linear inertial regimes. It is generally accepted that the Darcy's Law governs the flow when the local Reynolds number (*Re*) is less than 1, and that the onset of non-linear effects become apparent between 1 and 10 (Fourar *et al*

2004). However, there have been some uncertainties about the proper governing equation that adequately describes this transition. Authors have generally used Forcheimer's empirical quadratic equation to account for the non-linear inertia regime. This equation can be written as

$$-\frac{\Delta P}{L} = \frac{\mu}{K_f} + \beta\rho U^2 \tag{1}$$

Where ΔP, L, K_f are respectively the pressure drop across the sample, length of the sample and the intrinsic permeability; and μ, β, ρ, U are respectively the fluid dynamic viscosity, Forcheimer coefficient, fluid density and the seepage velocity. Others (*e.g.* Firdaous *et al* 1997) have proposed that the on-set of non-linearities are better represented by the following cubic law:

$$-\frac{\Delta P}{L} = \frac{\mu}{K_d} U + \frac{\gamma\rho^2}{\mu} U^3 \tag{2}$$

Where K_d is the intrinsic permeability of the medium and γ is a dimensionless parameter. In one notable effort to reconcile this apparent ambiguity, Fourar *et al* (2004) showed that these observations in the literature could be due to the differences in flow dimensionalization used by the authors. They concluded that equation (1) is a good description for a three-dimensional flow system undergoing a transition from Darcy to the inertia flow regime, because the flow dimension significantly reduced the transition zone between the flow regimes. For the case of a two-dimensional flow, equation (2) would best describe the governing equation as the transition zone is appreciable. Fourar *et al* (2004) also showed that the critical transitional *Re* occurs between *Re* = 2 and 4.

Although these results are suggestive in their own right, they still lack conclusive experimental proof. This paper therefore seeks to provide some experimental resolution to the above numerical results. We do this by exploring the results of flow through a two-dimensional physical system in the transition zone between the Darcy and the Forcheimer regimes. These kinds of experimental results are virtually non-existent in the literature (Fourar *et al* 2004). The results were achieved by modeling porous media flow in 2 dimensions using square arrays of circular acrylic rods to cover porosities ranging from 0.51 to 0.88. Velocity and pressure measurements were made using PIV and differential pressure gauges. This study represents the preliminary part of a larger experimental program aimed at providing comprehensive experimental data for flow through two- and three- dimensional model porous media.

EXPERIMENTAL APPARATUS AND PROCEDURE

A square test channel of length 500 mm, and a span and depth of 120 mm was used. The channel was made up of a section for flow conditioning, followed by a porous medium section, as shown in **Fig. 1**. The dimensions of the last section were modified so as to cover a flow area of depth and span 82 mm and 109.5 mm respectively, fully filled by the porous medium. To facilitate optical access, the channel and porous media models were made from transparent acrylic material of refractive index 1.47. The porous media models were constructed by inserting circular rods into holes drilled through the side plates. The spacing between rods l for rod diameter d , and porosity ϕ was determined from the relation: $l = d/2\sqrt{[\pi/(1-\phi)]}$. Rods of length covering the entire span of the channel were arranged in square arrays to achieve porosities of $\phi = 0.51$, 0.78 and 0.88. The streamwise, wall-normal and spanwise directions are respectively denoted by x,Y and z. The location $x = 0$ coincides with the center of the most upstream columns of rods. For the wall-normal direction $Y = 0$ is at the bottom of the lower wall. The location $z = 0$ is fixed at the channel mid-span.

A Cargille Immersion Liquid (Code 5040) of refractive index of 1.47 was used as the working fluid. The flow was seeded with silver-coated hollow glass spheres of mean diameter 10μm and specific gravity 1.4. A centrifugal single-speed pump of maximum flow rate, 8 - 29 gallon/minute was used to deliver a non-pulsating pressure flow into the channel. The flow field was illuminated with a Nd-YAG, 120 mJ/pulse thin sheet laser beam of 532 nm wavelength. The laser sheet was positioned in such a way that its plane was perpendicular to the camera. A HiSense 4M digital camera using a charged-couple device of 2048 pixel × 2048 pixel chip, and pitch 7.4 μm, was used to capture images of the flow field. The captured images were stored continuously through a buffer system onto a desktop computer, and then post-processed using the adaptive-correlation option of DynamicStudio v.2.30 commercial software developed by Dantec Dynamics. We estimate that the errors of velocities (u), are about 2% for the case of 0.88 porosity porous medium, and 8% and 11% respectively for the case of 0.78 and 0.51 porosities (as in Arthur *et al,* 2009); the error increases with decreasing porosity due to optical distortions resulting from a highly packed medium. Taps were constructed through the models and the test channels for pressure measurement. It is to be noted that due to the very low pressure differences (of the order of 0.1 inches of water), we resorted to using a 4000 Series Capsuhelic pressure gauge supplied by Dwyer Instruments with accuracy is about ±2%. The gauge was zeroed before each round of experiment.

A summary of the test conditions are provided in Table 1. Seepage velocity (U) is defined as the area averaged streamwise velocities measured within the porous medium. This was obtained by comparing the values for one or more unit cells in a region within the porous medium where the flow is not affected by the top or bottom wall (a unit cell is the representative elementary volume per unit span of the flow section; it is shown in **Fig. 2**). These velocities were generally of a maximum of 8% deviations. However, for the 0.51 porosity porous media, deviations were relatively higher. Nonetheless, the trends demonstrated in the results were similar. As indicated in the table, the Reynolds number is here defined by the seepage velocity, the diameter of the rods, and the kinematic viscosity of the working fluid. Viscosity values were determined at room temperature.

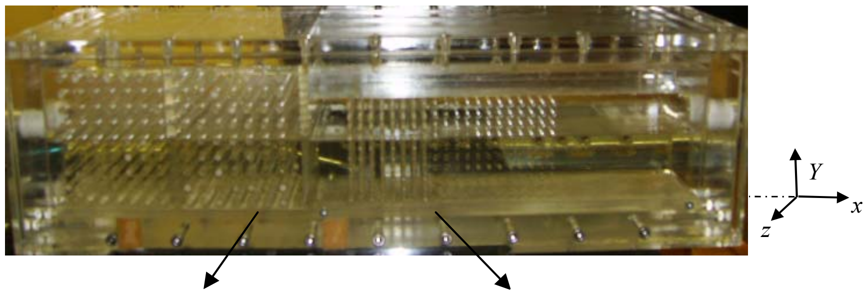

Fig. 1: Picture of the test channel with various sections. In this picture, 88% porous medium is installed in the porous medium section.

Table 1: Test Conditions

ϕ	ΔP (in. of water)	$\Delta P/L$ (Pa/m)	U (m/s)	d (m)	Re= Ud/ν
0.88	0.015	74.727	0.006697	0.00318	1.06
0.88	0.02	99.636	0.009616	0.00318	1.53
0.88	0.02	99.636	0.015379	0.00318	2.45
0.88	0.025	124.545	0.023084	0.00318	3.67
0.78	0.02	184.510	0.004754	0.00318	0.76
0.78	0.02	184.510	0.007444	0.00318	1.18
0.78	0.025	230.638	0.012678	0.00318	2.02
0.78	0.0275	253.702	0.017296	0.00318	2.75
0.51	0.0675	622.722	0.011506	0.00476	2.74
0.51	0.055	507.403	0.005956	0.00476	1.42
0.51	0.0325	299.830	0.005155	0.00476	1.23
0.51	0.02	184.510	0.00416	0.00476	0.99
0.51	0.015	138.383	0.002523	0.00476	0.60
0.51	0.015	138.383	0.000977	0.00476	0.23

RESULTS AND DISCUSSION

As the present focus was on plane flow, preliminary experiments were undertaken to ensure that the flow was indeed two- dimensional. As visualized in **Fig. 2**, even at Re ~ 1, inertia forces are still low compared with the viscous forces. As a result, the streamlines tend to follow the surface of the rods. However, at Re = 1.53, the symmetry is somewhat skewed, indicating the increased importance of inertial forces. This leads to the clearer changes seen in Re > 2 where some of the lower streamlines are found to be separated and attached to the top lines. It must be pointed out that even until at the Re, there are no indications of any formation of eddies which are associated with the Forcheimer regime (Fourar *et al*. 2004). This could be an indication of the evolution of weak inertial effects. If the preliminary observations of Arthur *et al* (2009) are anything to go by, this indicates that, at such a Reynolds number, any inertial effects through and over the porous medium will be negligibly small if only velocity measurements are considered. But turning to the effects within the porous medium, it may be pointed out that as shown in **Fig. 3**, the inertial effects within the porous medium are more apparent when the differential pressure measurements are measured. This shows in the deviation of the points of the 0.88 porosity model from a linear horizontal profile, as expected in a Darcy flow. The 0.78 and 0.51 model suggest somewhat similar pattern if the first and fourth points of the latter are considered to be largely affected by distortions due to optical effects in measurements, which in turn tended to affect the values obtained for seepage velocities. Although these plots are preliminary and provide inconclusive results, the linear deviation of the 0.88 model indicates that the Forcheimer equation could be used even for ranges of weak inertial effects in two-dimensional porous media models.

(a) (b) (c) (d)

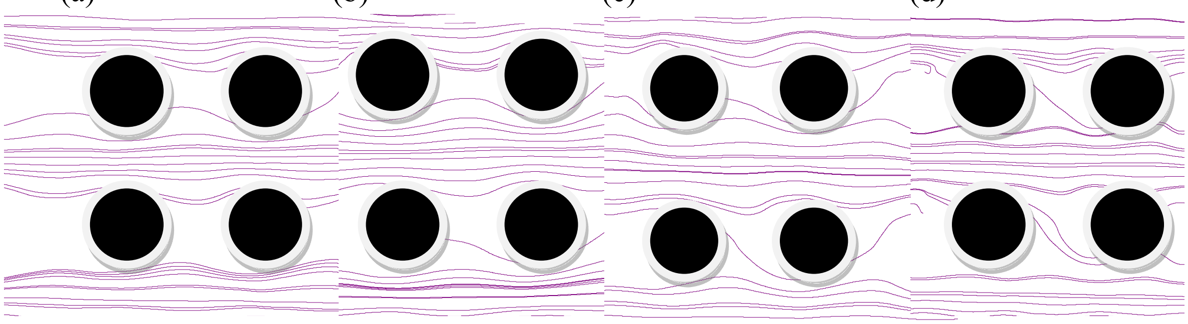

Fig. 2: Streamlines of a sample unit cell. The 4 black circles represent a unit cell of rod positions. These results are for a porous media of 0.78 porosity, and Reynolds numbers, Re: (a) 1.06 (b) 1.53 (c) 2.45 (d) 3.67.

CONCLUSIONS

The following are a summary of conclusions that can be drawn from this preliminary study:

1) For a two-dimensional porous medium, the gradual transition of flow from the Darcy regime to the Forcheimer regime begins at Re < 1. Weak inertia effects are visible when streamlines are considered, and are even more apparent when pressure measurements are taken.

2) There are suggestions in the literature that two-dimensional porous media are more susceptible to the use of the cubic law. However present results indicates that the Forcheimer equation may be valid at lower Reynolds number flow regimes.

The above results will also serve as a basis for a more detailed research at the weak inertial flow regime over a wider range of Reynolds numbers.

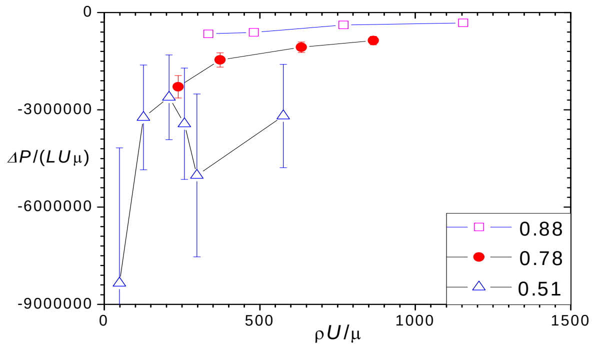

Fig. 3: Plot of $\Delta P/\ LU\mu$ against $\rho U/\mu$ (in SI units) for the various models.

ACKNOWLEDGEMENT

This work was supported by a grant from the Natural Sciences and Engineering Council of Canada

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