

VERSATILE THREE-PHASE CORRELATIONS FOR RELATIVE PERMEABILITY AND CAPILLARY PRESSURE

Frode Lomeland and Einar Ebeltoft
Statoil ASA, Stavanger, Norway

This paper was prepared for presentation at the International Symposium of the Society of Core Analysts held in Napa Valley, California, USA, 16-19 September, 2013

ABSTRACT

There are at least two key aspects of simulating multi-phase flow experiments. One is the actual estimation of multiphase flow properties from measured data, and the other is the representation of the unknown functions for relative permeability and capillary pressure. It is essential that the representation of these functions has sufficient degrees of freedom to model the measured data whilst remaining straight forward and easy to communicate.

For two-phase flow, this has been achieved and reported in several publications the last decade. The situation for three-phase flow has so far not been that conclusive. The most widely used methods for three-phase flow are based on methods of combining two-phase relative permeability tables. At the outset this makes them very attractive and applicable for all kinds of two-phase relative permeability data. However, this generality and the lack of empirical parameters makes them less favourable when trying to reconcile and match experimental three-phase flow performance. A smooth and flexible correlation for three-phase relative permeability and capillary pressure has been developed and is presented as a replacement for currently adopted industry standards. This correlation has a sufficient, but limited number of three-phase flow parameters to match experimental three-phase data, and is straight forward to utilize for full-field applications.

The strength of the new correlation is demonstrated by the utilization of steady-state experiments performed at reservoir conditions on core samples from the Norwegian Continental Shelf. This also includes three-phase end-points for injection relative permeability and paths for residual oil and gas saturations in the three-phase saturation region.

INTRODUCTION

Two-phase relative permeability and capillary pressure for full-field application is quite well understood and is routinely determined from laboratory data at initial reservoir conditions. Even though a large number of well-acknowledged studies have been done within three-phase flow, they are not as conclusive as they are for two-phase flow, see i.e. [1] and its references. A common assumption when three-phase flow models are defined is that only oil relative permeability depends upon two saturations, while gas and water relative permeability are assumed to be only a function of their own saturations. This may be the case for strongly water-wet systems [2,3], but this assumption has been shown not to be valid for three-phase experiments performed at reservoir conditions [4,5] for weakly wetting media as well as for the theoretical considerations described in [6]. Hence, there is

a need to represent all relative permeabilities as functions of two saturations, preferably by being smooth and flexible with limited parameters.

This paper thus suggests a consistent representation of the three-phase flow properties depending on two saturations based upon several three-phase steady-state experiments performed during the last 5-6 years at reservoir conditions.

The approach for this work has been to design three-phase experiments based upon the actual field-situation [4,5] and then to utilize the presented correlation for providing three-phase surfaces as tables suitable for full field simulation by Eclipse. We do not claim that these relative permeability surfaces are unique or that they capture all possible hysteresis paths. However, the aim is to provide a pragmatic representation for full-field application based upon the experiments performed for the specific field situation.

NEW CORRELATION – LET

Although it is more common to use the ratio of relative permeabilities for gas/condensate [7] systems than for oil/water systems, we will model the ratio of relative permeability for oil/water as the ratio of their asymptotic behaviour, see Figure 1.

$$\frac{K_{rwo}}{K_{row}} = B = \frac{CS_{wn}^{L_{wo}}}{(1-S_{wn})^{L_{ow}}} \quad \text{and} \quad C = C(S_{wn}) > 0 \quad (1)$$

It is commonly accepted [8] that the sum of relative permeabilities is less or equal to one.

$$K_{row} + K_{rwo} = A = A(S_{wn}) \leq 1 \quad (2)$$

Solving for oil relative permeability gives:

$$K_{row} = \frac{A}{1+B} = \frac{A(1-S_{wn})^{L_{ow}}}{(1-S_{wn})^{L_{ow}} + CS_{wn}^{L_{wo}}} \quad (3)$$

A similar formula can be found for water relative permeability. This is suited to model oil relative permeability, except that the second term in the denominator contains the water parameter L_{wo} , and thus reduces the degrees of freedom of the formula instead of increasing it. However, the two functions A and C give sufficient freedom to replace the parameter L_{wo} with the oil relative permeability parameter T_{ow} . Realising this, we omit the water relative permeability and turn to the function $K_{rox} - K_{row}$, which sums to K_{rox} together with K_{row} , and which goes to zero as the normalized water saturation goes to zero. This simplifies the presentation and

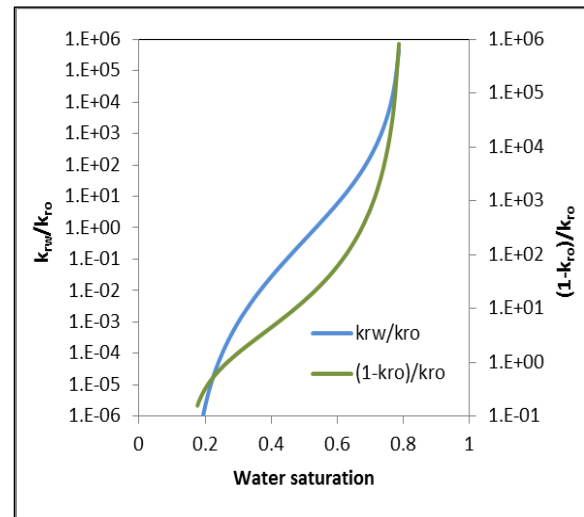


Figure 1: Ratio of relative permeabilities

further discussion. The simplest way to model such asymptotic behaviour, is to use a general power law. For oil relative permeability we get:

$$\frac{K_{rox} - K_{row}}{K_{row}} = \frac{E_{ow} S_{wn}^{T_{ow}}}{(1 - S_{wn})^{L_{ow}}} \quad (4)$$

This is the flexible and well-suited *LET* correlation [10] for oil relative permeability of a two-phase oil/water system. As the parameters L_{ow} and T_{ow} are real numbers, this is not a rational function, so we call this type of function a super-rational function. The logarithm of Eq. 4 is linear in the empirical parameters L , $\log(E)$ and T , and linear optimization can be used. This gives a linear system of equations for L , $\log(E)$ and T which can be solved explicitly and analytically.

The three-phase situation is more complex and regions of interest are sketched in Figure 2. Regions A + C will capture most immiscible saturation paths of a grid cell starting at the oil apex with S_{wir} present, i.e. an oil leg/field cell. Region B + C will capture most immiscible saturation paths of a grid cell starting at the gas apex with S_{wir} present, i.e. a gas cap/field cell. Region D contains grid cells with paleo-hydrocarbons. Region E is by definition excluded as the water saturation is regarded as truly immobile at S_{wir} . The blue line is S_{wir} while the green is the S_{or} -path and the red is the S_{gr} -path.

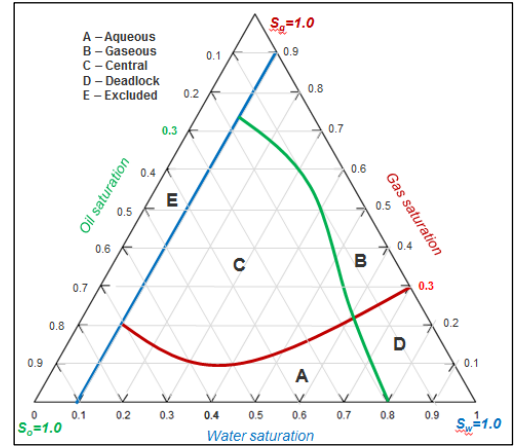


Figure 2: Three-phase regions

Equation (4) models the two-phase oil/water system along the water saturation axis in Figure 2. The same discussion for a two-phase gas/oil system will give a similar equation, and hence models gas/oil flow behaviour along the S_{wir} -axis. It is reasonable to demand that a three-phase model or function for oil relative permeability must reduce to its two-phase formulation when the normalized gas or water saturation approaches zero. This leads to the three-phase equation (5):

$$\frac{K_{rox} - K_{ro}}{K_{ro}} = \left[\frac{E_{ow} S_{wn}^{T_{ow}}}{(1 - S_{wn})^{L_{ow}}} \right]_{xt3} + \left[\frac{E_{og} S_{gn}^{T_{og}}}{(1 - S_{gn})^{L_{og}}} \right]_{xt3} + D_{int} \quad (5)$$

The subscript *xt3* means that the formula inside the parentheses should be extended to 3 phases. The term D_{int} is the interaction term that involves both gas and water saturation. As the denominators in both parentheses are describing the oil phase in their original two-phase context, the denominators are extended to an oil phase construction, B_{on} , similar to Stone 1 formulation as extended by Hustad & Holt [12]:

$$B_{on} = S_{on}^{L_{2o}} (1 - S_{wn})^{L_{ow} - L_{2o}} (1 - S_{gn})^{L_{og} - L_{2o}} \quad (6)$$

B_{on} is also a denominator in the interaction term. Considering the numerators which model the associate phases, the simplest three-phase extension is again a power law. However, now it contains the other associate phase which disappears in the two-phase reduction, and gives the following:

$$\frac{K_{rox} - K_{ro}}{K_{ro}} = \frac{E_{ow} S_{wn}^{T_{ow}} (1 - S_{gn})^{N_{3ow}}}{B_{on}} + \frac{E_{og} S_{gn}^{T_{og}} (1 - S_{wn})^{N_{3og}}}{B_{on}} + \frac{D_{on}}{B_{on}} \quad (7)$$

An interaction term must have its maximum impact on the K_{ro} formula when both the associated gas and water saturations have significant values. A simple function that shows this behaviour is a parabola-like construction which interacts with or involves both numerators in equation (7). The interaction term is:

$$D_{on} = D_{3o} 2\sqrt{E_{ow} E_{og}} * S_{wn}^{T_{3ow}} (1 - S_{gn})^{N_{3ow}/2} * S_{gn}^{T_{3og}} (1 - S_{wn})^{N_{3og}/2} \quad (8)$$

where

$$T_{3ow} = M_{3ow} * T_{ow} / 2 \quad \text{and} \quad T_{3og} = M_{3og} * T_{og} / 2 \quad (9)$$

The empirical parameters that are specific three-phase parameters have subscript 3, and there are altogether six parameters. They all have their merits and limits. However, a subset of the parameters can be optimized and combined with default values for the remaining ones. The N_3 parameters with $D_{3o} < 0$ tend to increase K_{ro} , while $D_{3o} > 0$, with softening or spreading parameters M_3 , tend to decrease K_{ro} . The Hustad & Holt parameter L_{3o} influences the lower part of oil relative permeability.

The two S_{or} paths initiated from either S_{orw} or from S_{org} do not generally coincide. However, it is generally accepted within the reservoir simulation community to use only one S_{or} path. The choice is often a combination of the path initiated at S_{orw} and the path initiated from S_{org} , or the path is just a continuation to S_{org} . This combined path is generally non-linear, and often has a minimum value S_{orm} lower than S_{org} . A simple function that is showing this behaviour is a parabola-like construction with two general powers, i.e. a super-parabola. A convenient correlation for the S_{or} path with 3-4 empirical parameters is presented in Appendix H.

If the flow in a core sample or grid cell starts in the oil apex and proceeds into the ternary region, it is not obvious what the residual oil saturation is or should be. We need to point at a residual oil saturation value when the saturations are at a general saturation point in the ternary diagram (red dot in Figure 3). A pointer is needed, and there are three candidates: 1) the simple pointer, 2) the ray pointer of Hustad & Hansen [13] and 3) the hysteric pointer.

The hysteric point is intuitively the most correct pointer. However, the S_{or} -value and thus the oil relative permeability value, is no longer unique, but dependent upon the saturation path because hysteresis occurs. One pragmatic drawback is that a table cannot be used to represent three-phase relative permeability as the S_{or} values depend upon the pointer used to generate the table. A functional representation like the LET-family of correlations

[9,10,11] must be implemented in the reservoir simulator code. However, we leave this topic, and just state that *the ray pointer* is further used in this work.

To further outline the benefits of the three-phase LET-formulation, the LETx-correlation [11] is considered. Water relative permeability is now extended to water saturation equal to one, and gas injection relative permeability is extended to gas saturation equal to one minus irreducible water saturation, see [11]. Using similar arguments as for LET above the following is achieved:

$$K_{rw} = \frac{K_{rwx} B_{wx}}{B_{wx} + E_{wx} + D_{wx}} \quad (10)$$

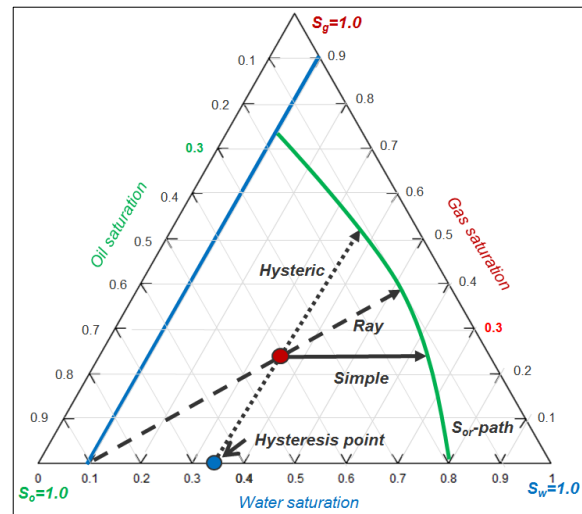


Figure 3: Pointers to residual oil saturation

This function, see Appendix C, covers the entire ternary diagram except the excluded zone E, see Figure 2, and the same goes for the extended gas injection relative permeability. The LETx version of oil relative permeability is identical to the LET version with standard saturation normalization for lower WAG cycle flow. For higher WAG cycles the LETx oil relative permeability is extended to the oil apex. For LETx it is only the relative permeability of the production phase that explicitly involves either oil or gas residual saturation. The endpoint values at the residual saturations, which for three-phase systems are curves on the relative permeability surface in the ternary diagram, are simply ordinary points on the injection relative permeability surfaces for LETx. This acknowledgement has a significant impact as it simplifies the modelling of three-phase flow considerably as well as simplifying the WAG modelling tremendously. The important issue is that the extended version of K_{rw} and injection K_{rg} is *independent of residual oil saturation* as well as the fact that the endpoint-values K_{rwr} and K_{rgr} are not present in the LETx correlation. As this is valid for both two- and three-phase systems, it follows that neither K_{rw} nor K_{rg} restrict any movement of the current saturation point in the entire ternary diagram. This means that crossing the S_{or} path is possible within the LETx formulation. For two-phase systems such as S_{or} crossing may occur due to oil vaporization, while for three-phase systems such crossing may also occur when changing the injection (or invasion) phase when S_{or} is reached. Note that the oil saturation remains at its residual value of the previous time step, but the change in saturation path of the grid cell moves the current saturation point behind the S_{or} path. The LETx formulation handles this behaviour just as easily as it handles an ordinary saturation point. Conventional formulations with standard saturation normalization must treat this behaviour with an extra-ordinary action such as shift to another functional formula or another model.

For completeness the water injection relative permeability for the LET version of K_{rw} with standard saturation scaling is derived. A grid cell, which is initially located at the oil apex, will never move immiscibly to the gas/water axis of the ternary diagram. We will therefore move the gas/water axis-boundary in the LETx formula to the S_{or} -path in the LET version, see green line in Figure 2. This is done by replacing the extended saturation normalization with a standard normalization, and hence the LET version of water injection relative permeability is achieved. The LET correlation of water relative permeability has the same mathematical structure as the LETx correlation, except for the saturation normalization. The maximum value of LET water relative permeability is now the endpoint value of the two-phase oil/water relative permeability. Along the residual oil path, the LET water injection relative permeability becomes the equation below.

$$\frac{K_{rwr}^{orw} - K_{rwr}}{K_{rwr}} = \frac{E_{wg} S_{gn}^{T_{wg}}}{(1 - S_{gn})^{L_{wg}}} \quad (11)$$

The three-phase end-point value K_{rwr} curve is not used as an *entity* in the LET correlation of K_{rw} . It is the numerator and the denominator of eq. (11) that is included non-linearly in the K_{rw} function. It is actually the two-phase oil/water end-point value K_{rwr}^{orw} that is also used for three-phase systems. Relative permeability formulas are shown in Appendix A-G.

In a reservoir simulation there are usually only a small or medium number of grid cells that enter deep into the interior of the ternary diagram. Also, the capillary pressure has generally only a small effect on the reservoir simulation results, and three-phase capillary pressure measurements are rare. We therefore recommend using two-phase capillary formulas with three-phase saturations replacing the two-phase saturations and no, or a minimum number of specific three-phase parameters. Two-phase capillary pressure increases or decreases steeply when approaching a residual (or irreducible) saturation, and three-phase capillary pressure should do the same along the corresponding residual paths. We generalize the finite LET capillary pressure functions [10] to three-phase, and follow the same hysteresis philosophy as for relative permeability, see Appendix I-K.

Being finite, the LET functions try to avoid very steep curves. Thus, the LET capillary pressure model is also less prone to giving numerical simulation problems. A situation that we expect to benefit from this gentle behaviour is the crossing of the S_{or} path. A LET model describing the behaviour after crossing the S_{or} path is presented in Appendix K. It is a gas apex model, with steep parts at the S_{gr} -path and the S_{wir} -path, supplemented with a special purpose oil apex term and saturation normalization for the capillary pressure after entering into the B region. An alternative model is to have hysteresis in the S_{or} -path (where you, for example continue with a constant residual oil saturation $S_{orym} \geq S_{orm}$) and rescale the oil apex P_{cw} (originating from P_{cow}) to the new S_{or} -path. You still have to consider the gas apex P_{cw} (originating from P_{cgw}) which becomes positive and steep towards the S_{wir} -path and negative and steep towards the S_{gr} -path. Another approximate model for the crossing into B region is a LETx model where the oil apex P_{cw} (originating from P_{cow}) is extended to the ternary gas axis. This model is continuous and smooth in the entire ternary diagram, and it can be used if crossing into B region is not a big issue for the simulation. The LETx model for the oil apex P_{cw} is presented in Appendix L.

INTERPRETATION OF THREE-PHASE SCAL EXPERIMENTS

To demonstrate the three-phase LET-formulation, it has been utilized on three-phase experimental data performed as steady-state at reservoir conditions [4,5] on a sandstone composite core sample from the North Sea. Core and fluid properties are given in Table 1 and the three-phase experimental trajectories are shown Figure 4. All experiments have been simulated with a two- and three-phase steady-state simulator corrected for any capillary end-effects [14]. Data from the S_{or} -path, S_{gr} -path and the two three-phase trajectories DDI-1 and DDI-2 have been used in the analysis to provide three-phase saturation tables applicable for Eclipse.

Table 1: Core and fluid properties for the SCAL experiments

Core properties		Fluid properties	
Length, core [cm]	28.78	Viscosity, water [cP]	0.332
Diameter, core [cm]	3.66	Viscosity, oil [cP]	0.293
$k_o(S_{wi})$ [mD]	4981	Viscosity, gas [cP]	0.030
S_{wi} [frac.]	0.119	Temperature [$^{\circ}$ C]	90.5
Porosity [frac.]	0.31	Pore pressure[kPa]	29 000

The three-phase trajectories have been compared with standard correlations for Stone-1 and Baker. Figure 5 shows the model of the S_{or} -path. The LET-formulation matches the experimental saturation data very well. The path is provided as a base-case with optimistic and pessimistic cases measured by eye. Similar behaviour is experienced for the S_{gr} -path.

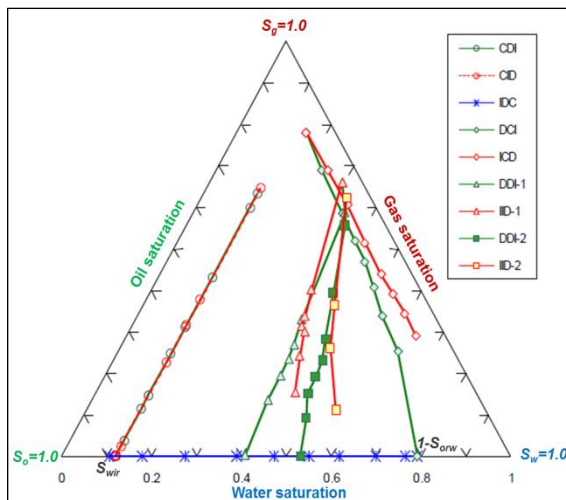


Figure 4: Three-phase trajectories

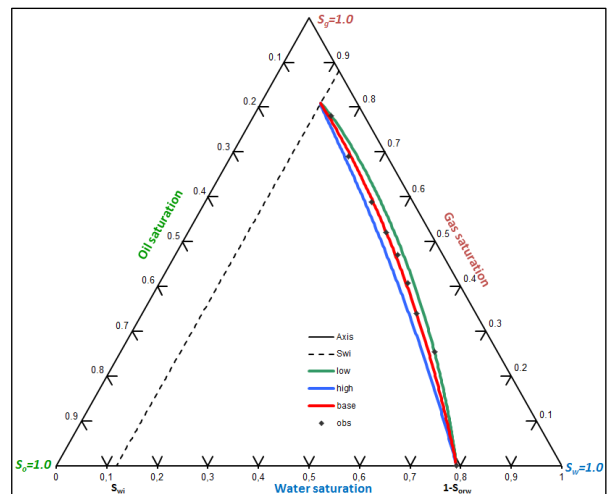


Figure 5: Match of the S_{or} -path

The relative permeability to water, oil and gas has been determined through three-phase simulation of the two three-phase trajectories in Figure 4. Standard correlations like Stone-1 and Baker calculate the three-phase relative oil-permeability through methods of interpolating two-phase relative permeabilities (represented by tables) in the ternary region. Hence, the prediction by these correlations should preferably mimic the three-phase relative permeabilities determined in the three-phase region. The results from Stone-1 and

Baker are compared with the experimental data together with the LET and LETx formulation that takes into account the actual three-phase information covered by the experiments.

Stone-1, when implemented for a water-wet scenario, assumes that water- and gas relative permeability only depend upon their own saturation, while Baker assumes that his saturation weighted arithmetic interpolation between the two-phase boundary versions of water- and of gas- relative permeabilities are sufficiently accurate. This may be the case for

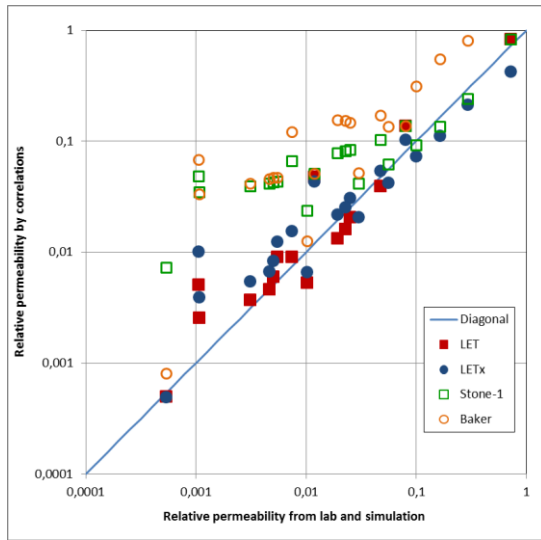


Figure 6: Water relative permeability

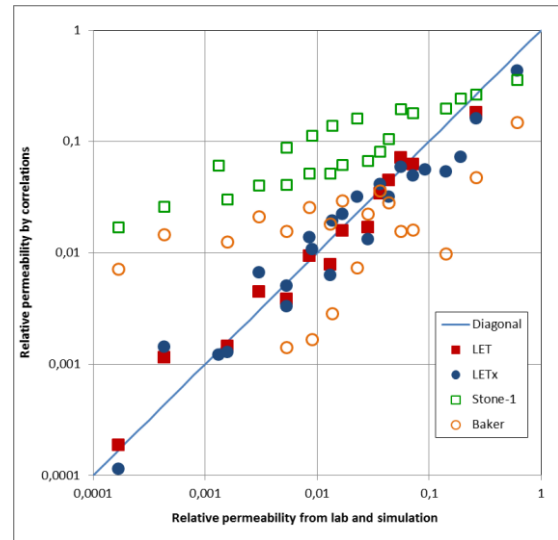


Figure 7: Gas relative permeability

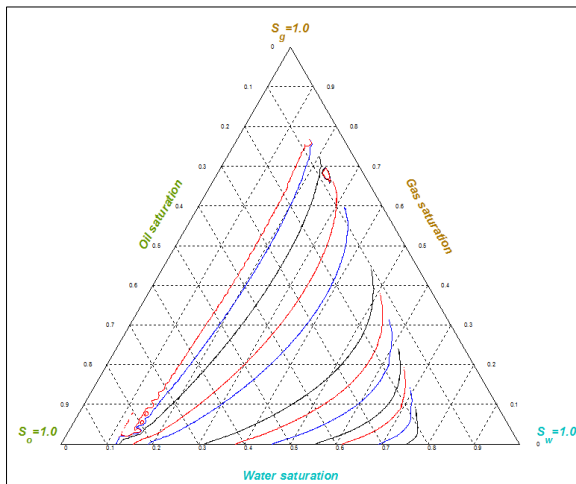


Figure 8: Water relative permeability isoperms

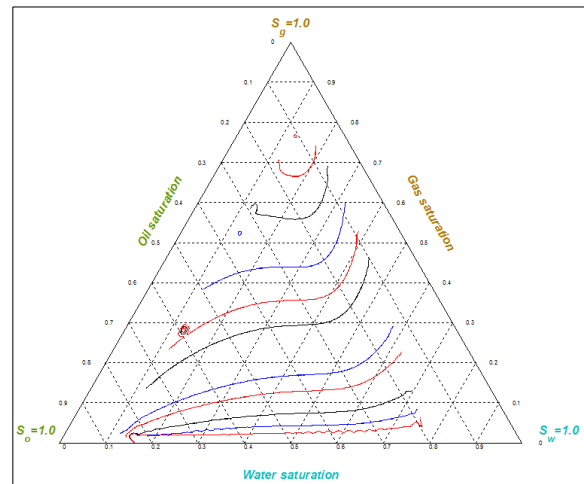


Figure 9: Gas relative permeability isoperms

strongly water-wet samples, but Figure 6 and Figure 7 demonstrate that this assumption is not valid as neither Stone-1 nor Baker predict the measured three-phase water- and gas relative permeabilities. The LET and LETx correlations that assume water- and gas relative permeability depend upon two saturations, and actually use the information in the three-phase region, match the experimental data very well. Relative permeability isoperms by

using the three-phase LET formulation are shown in Figure 8 and Figure 9 for water- and gas relative permeability respectively. These figures show that there are significant internal structures in both water and gas relative permeability surfaces that are captured in the LET model. The relative permeability surfaces are further provided as the saturation tables SWF32D, SOF32D, SGF32D suited for Eclipse.

CONCLUSIONS

- A new analytical correlation for three-phase relative permeability has been developed; the LET function with an extended version LETx
- Using six parameters specific to three-phase systems, the function is able to describe a span of relative permeability surfaces
- We have demonstrated that the three-phase LET family of relative permeability functions gives an excellent match to laboratory results
- Despite the addition of parameters, the LET correlation remains easily accessible and applicable for full field reservoir simulations and engineering
- A flexible and convenient saturation function for three-phase residual oil and gas saturation has been presented
- A pragmatic and convenient three-phase capillary pressure model has been presented

ACKNOWLEDGEMENT

Statoil is acknowledged for giving permission to publish the experimental results. Vilgeir Dalen is acknowledged for valuable contributions and advice during the work on this paper. Eimund Gilje is acknowledged for valuable discussions and Eirik Vik for providing the ternary diagrams.

REFERENCES

1. Sorbie K. S., van Dijke, M.I.J.: "Fundamentals of Three-Phase Flow in Porous Media of Heterogeneous Wettability", Institute of Petroleum Engineering, Heriot-Watt University, Edinburgh, Scotland, UK, September 2004.
2. Stone, H.L.: "Probability Model for Estimating Three-Phase Relative Permeability," Trans. SPE of AIME, 249, JPT (Feb. 1970) 214-218.
3. Baker, L.E.: "Three-Phase Relative Permeability Correlations," paper SPE/DOE 17369 in proceedings of the SPE/DOE Symposium on Enhanced Oil Recovery, Tulsa (April 17-20, 1988) 539-553.
4. Nordtvedt, J.E., Ebeltoft, E., Iversen, J.E., Urkedal, H., Vatne, K.O., Watson, A.T.: "Determination of Three-phase Relative Permeabilities from Displacement Experiments", SPE 36683, Proceedings of the 1996 SPE technical Conference and Exhibition, Denver, Colorado, USA, October 6-9, 1996.
5. Petersen E.B.Jr., Lohne A., Vatne K.O., Helland J.O., Virnovsky G. and Øren P.E.: "Relative permeability for two- and three-phase flow processes relevant to the depressurization of the Statfjord field". Paper SCA2008-23, International Symposium of the Society of Core Analysts, Abu Dhabi, UAE, 29 October – 2 November, 2008.

6. Hui, M.-H. and Blunt, M.J.: 2000, "Pore-Scale Modeling of Three-Phase Flow and the Effects of Wettability", SPE59309, Proceedings of the SPE/DOE Improved Oil Recovery Symposium, Tulsa, OK, April 2000.
7. Fevang, Ø, Whitson, C.H.: Modeling Gas-Condensate Well Deliverability, SPE Reservoir Engineering 11, no. 4 (November 1996): 221-230.
8. Stone, H.L.: "Estimation of Three-Phase Relative Permeability and Residual Oil Data", J. Can. Pet. Tech., vol. 12, no. 4 (Oct.-Dec. 1973), page 53-61.
9. Lomeland F., Ebeltoft E. and Thomas W.H.: "A New Versatile Relative Permeability Correlation". Paper SCA 2005-32, International Symposium of the Society of Core Analysts held in Toronto, Canada, 21-25 August, 2005.
10. Lomeland F. and Ebeltoft E.: "A New Versatile Capillary Pressure Correlation". Paper SCA 2008-08, International Symposium of the Society of Core Analysts, Abu Dhabi, UAE, 29 October – 2 November, 2008.
11. Lomeland F., Hasanov B., Ebeltoft E. and Berge M. 2012. A Versatile Representation of Up-scaled Relative Permeability for Field Applications. Paper SPE 154487-MS presented at the EAGE Annual Conference & Exhibition incorporating SPE Europec held in Copenhagen, Denmark, 4-7 June 2012.
12. Hustad, O.S. and Holt, T.: "Gravity Stable Displacement of Oil by Hydrocarbon Gas after Water flooding," paper SPE/DOE 24116 in proceedings of the SPE/DOE Eighth Symposium on Enhanced Oil Recovery, Tulsa (April 22-24, 1992) 131-146.
13. Hustad, O.S. and Hansen, A.G.: "A Consistent Formulation for Three-phase Relative Permeabilities and Phase Pressures on Three Sets of Two-Phase Data", paper in the book RUTH, A Norwegian Research Program on Improved Oil Recovery, Program Summary, Norwegian Petroleum Directorate, Stavanger (1996) 183-194; See also Proc.; Eight European Symposium on Improved Oil Recovery, Vienna (1995) 289-298
14. Virnovsky, G, Mykkelteviet, J., Nordtvedt J.E.: F.: "Application of a Steady-state Three-phase Simulator to Interpret Flow Experiments". SCA 1996-35. International Symposium of the Society of Core Analysts, Montpellier, France, September, 1996.

NOMENCLATURE

A-E	Parameter, Function, Region	P	Capillary pressure
F	Shape function	R	Parameter, Function
K	Relative permeability	S	Saturation
L-N	Parameter	T	Parameter

NOMENCLATURE SUBSCRIPT AND SUPERScript

a	Boundary saturation	q	Extended normalization for gas production
b	Boundary saturation	r	Residual, Relative
c	Capillary	s	Spontaneous
f	Forced	t	Threshold
g	Gas	u	Upper
ir	Irreducible	w	Water
m	Minimum	x	Maximum, Extended normalization
n	Standard normalization	y	Turning point for hysteresis
o	Oil	z	"Boundary" saturation where P_c is zero
p	Extended normalization oil	3	Three-phase

APPENDIX A: Miscellaneous Information

The general LET functional form for three-phase relative permeability is

$$K_{ry} = \frac{AB}{B + D + E} \quad \text{where } y = w, o, g \quad (\text{A1})$$

The elements A, B, D and E will be described in each K_{ry} appendix. Parameters or functions with subscript 3, among other subscript letters, are three-phase empirical parameters, physical parameters or functions.

APPENDIX B Kro production LETx

K_r function for oil out-flow with water and gas inflow in a cell in region A+C, and no oil flow in region B.

$$S_{wp} = \frac{S_w - S_{wir}}{1 - S_{wir} - S_{or}} \quad \text{and} \quad S_{op} = \frac{S_o - S_{or}}{1 - S_{wir} - S_{or}} \quad \text{and} \quad S_{gp} = \frac{S_g}{1 - S_{wir} - S_{or}} \quad (\text{B1})$$

$$A = K_{rox} \quad \text{and} \quad T_{3ow} = M_{3ow} * T_{ow} / 2 \quad \text{and} \quad T_{3og} = M_{3og} * T_{og} / 2 \quad (\text{B2})$$

$$B = S_{op}^{L_{3o}} (1 - S_{wp})^{L_{ow} - L_{3o}} (1 - S_{gp})^{L_{og} - L_{3o}} \quad (\text{B3})$$

$$D = D_{3o} 2\sqrt{E_{ow}E_{og}} * S_{wp}^{T_{3ow}} S_{gp}^{T_{3og}} (1 - S_{gp})^{N_{3ow}/2} (1 - S_{wp})^{N_{3og}/2} \quad (\text{B4})$$

$$E = E_{ow} S_{wp}^{T_{ow}} (1 - S_{gp})^{N_{3ow}} + E_{og} S_{gp}^{T_{og}} (1 - S_{wp})^{N_{3og}} \quad (\text{B5})$$

APPENDIX C Krw injection LETx

K_r function for water inflow with potential gas and oil out-flow in a cell in region A+C+B.

$$S_{wx} = \frac{S_w - S_{wir}}{1 - S_{wir}} \quad \text{and} \quad S_{ox} = \frac{S_o}{1 - S_{wir}} \quad \text{and} \quad S_{gx} = \frac{S_g}{1 - S_{wir}} \quad (\text{C1})$$

$$A = K_{rwx} \quad \text{and} \quad T_{3wo} = M_{3wo} * T_{wo} / 2 \quad \text{and} \quad T_{3wg} = M_{3wg} * T_{wg} / 2 \quad (\text{C2})$$

$$B = S_{wx}^{L_{3w}} (1 - S_{ox})^{L_{wo} - L_{3w}} (1 - S_{gx})^{L_{wg} - L_{3w}} \quad (\text{C3})$$

$$D = D_{3w} 2\sqrt{E_{wo}E_{wg}} * S_{ox}^{T_{3wo}} S_{gx}^{T_{3wg}} (1 - S_{gx})^{N_{3wo}/2} (1 - S_{ox})^{N_{3wg}/2} \quad (\text{C4})$$

$$E = E_{wo} S_{ox}^{T_{wo}} (1 - S_{gx})^{N_{3wo}} + E_{wg} S_{gx}^{T_{wg}} (1 - S_{ox})^{N_{3wg}} \quad (\text{C5})$$

APPENDIX D Krg injection LETx

K_r function for gas inflow with potential water and oil out-flow in a cell in region A+C+B.

$$S_{wx} = \frac{S_w - S_{wir}}{1 - S_{wir}} \quad \text{and} \quad S_{ox} = \frac{S_o}{1 - S_{wir}} \quad \text{and} \quad S_{gx} = \frac{S_g}{1 - S_{wir}} \quad (\text{D1})$$

$$A = K_{rgx} \quad \text{and} \quad T_{3gw} = M_{3gw} * T_{gw} / 2 \quad \text{and} \quad T_{3go} = M_{3go} * T_{go} / 2 \quad (D2)$$

$$B = S_{gx}^{L_{3g}} (1 - S_{wx})^{L_{gw} - L_{3g}} (1 - S_{ox})^{L_{go} - L_{3g}} \quad (D3)$$

$$D = D_{3g} 2\sqrt{E_{gw} E_{go}} * S_{wx}^{T_{3gw}} S_{ox}^{T_{3go}} (1 - S_{wx})^{N_{3go}/2} (1 - S_{ox})^{N_{3gw}/2} \quad (D4)$$

$$E = E_{gw} S_{wx}^{T_{gw}} (1 - S_{ox})^{N_{3gw}} + E_{go} S_{ox}^{T_{go}} (1 - S_{wx})^{N_{3go}} \quad (D5)$$

APPENDIX E Krw injection LET

K_r function for water inflow with potential gas and oil out-flow in a cell in region A+C. Residual saturation S_{gr} is applied in cases of hysteresis.

$$S_{wn} = \frac{S_w - S_{wir}}{1 - S_{wir} - S_{or} - S_{gr}} \quad \text{and} \quad S_{on} = \frac{S_o - S_{or}}{1 - S_{wir} - S_{or} - S_{gr}} \quad \text{and} \quad S_{gn} = \frac{S_g - S_{gr}}{1 - S_{wir} - S_{or} - S_{gr}} \quad (E1)$$

$$A = K_{rwr}^{orw} \quad \text{and} \quad T_{3wo} = M_{3wo} * T_{wo} / 2 \quad \text{and} \quad T_{3wg} = M_{3wg} * T_{wg} / 2 \quad (E2)$$

$$B = S_{wn}^{L_{3w}} (1 - S_{on})^{L_{wo} - L_{3w}} (1 - S_{gn})^{L_{wg} - L_{3w}} \quad (E3)$$

$$D = D_{3w} 2\sqrt{E_{wo} E_{wg}} * S_{on}^{T_{3wo}} S_{gn}^{T_{3wg}} (1 - S_{gn})^{N_{3wo}/2} (1 - S_{on})^{N_{3wg}/2} \quad (E4)$$

$$E = E_{wo} S_{on}^{T_{wo}} (1 - S_{gn})^{N_{3wo}} + E_{wg} S_{gn}^{T_{wg}} (1 - S_{on})^{N_{3wg}} \quad (E5)$$

APPENDIX F Krg injection LET

K_r function for gas inflow with water and oil out-flow in a cell in region A+C. Residual saturation S_{gr} is applied in cases of hysteresis.

$$S_{wn} = \frac{S_w - S_{wir}}{1 - S_{wir} - S_{or} - S_{gr}} \quad \text{and} \quad S_{on} = \frac{S_o - S_{or}}{1 - S_{wir} - S_{or} - S_{gr}} \quad \text{and} \quad S_{gn} = \frac{S_g - S_{gr}}{1 - S_{wir} - S_{or} - S_{gr}} \quad (F1)$$

$$A = K_{rgr}^{org} \quad \text{and} \quad T_{3go} = M_{3go} * T_{go} / 2 \quad \text{and} \quad T_{3gw} = M_{3gw} * T_{gw} / 2 \quad (F2)$$

$$B = S_{gn}^{L_{3g}} (1 - S_{on})^{L_{go} - L_{3g}} (1 - S_{wn})^{L_{gw} - L_{3g}} \quad (F3)$$

$$D = D_{3g} 2\sqrt{E_{go} E_{gw}} * S_{on}^{T_{3go}} S_{wn}^{T_{3gw}} (1 - S_{wn})^{N_{3go}/2} (1 - S_{on})^{N_{3gw}/2} \quad (F4)$$

$$E = E_{go} S_{on}^{T_{go}} (1 - S_{wn})^{N_{3go}} + E_{gw} S_{wn}^{T_{gw}} (1 - S_{on})^{N_{3gw}} \quad (F5)$$

APPENDIX G Krg production gas apex LETx

K_r function for gas out-flow with potential water and oil inflow in a cell in region B+C.

$$S_{wq} = \frac{S_w - S_{wir}}{1 - S_{wir} - S_{gr}} \quad \text{and} \quad S_{oq} = \frac{S_o}{1 - S_{wir} - S_{gr}} \quad \text{and} \quad S_{gq} = \frac{S_g - S_{gr}}{1 - S_{wir} - S_{gr}} \quad (G1)$$

$$A = K_{rgx} \quad \text{and} \quad T_{3gw} = M_{3gw} * T_{gw} / 2 \quad \text{and} \quad T_{3go} = M_{3go} * T_{go} / 2 \quad (G2)$$

$$B = S_{gq}^{L_{3g}} (1 - S_{wq})^{L_{gw} - L_{3g}} (1 - S_{oq})^{L_{go} - L_{3g}} \quad (G3)$$

$$D = D_{3g} 2\sqrt{E_{gw}E_{go}} * S_{wq}^{T_{3gw}} S_{oq}^{T_{3go}} (1 - S_{wq})^{N_{3go}/2} (1 - S_{oq})^{N_{3gw}/2} \quad (G4)$$

$$E = E_{gw} S_{wq}^{T_{3gw}} (1 - S_{oq})^{N_{3gw}} + E_{go} S_{oq}^{T_{3go}} (1 - S_{wq})^{N_{3go}} \quad (G5)$$

APPENDIX H Saturation paths MRN

Three normalized boundary saturations for use in different boundary functions

$$S_{gbn} = \frac{S_{gb}}{1 - S_{wir} - S_{org}} \quad \text{and} \quad S_{gzn} = \frac{S_{gz}}{1 - S_{wir} - S_{org}} \quad \text{and} \quad S_{wan} = \frac{S_{wa} - S_{wir}}{1 - S_{wir} - S_{grw}} \quad (H1)$$

Super-parabolic Sor path function with normalized gas boundary saturation

$$S_{or} = S_{orw} (1 - S_{gbn})^{M_{or}} + S_{org} S_{gbn}^{N_{or}} + R_{or} 2\sqrt{S_{orw} S_{org}} * (1 - S_{gbn})^{M_{or}/2} S_{gbn}^{N_{or}/2} \quad (H2)$$

Alternative super-parabolic version with S_{orm} included is

$$S_{or} = (S_{orw} - S_{orm})(1 - S_{gbn})^{M_{or}} + (S_{org} - S_{orm}) S_{gbn}^{N_{or}} + R + S_{orm} \quad (H3)$$

$$R = R_{or} 2\sqrt{(S_{orw} - S_{orm})(S_{org} - S_{orm})} * (1 - S_{gbn})^{M_{or}/2} S_{gbn}^{N_{or}/2} \quad (H4)$$

Spontaneous water saturation S_{ws} path with normalized gas “boundary” saturation

$$S_{ws} = S_{wso} (1 - S_{gzn})^{M_{ws}} + S_{wir} S_{gzn}^{N_{ws}} + R_{ws} 2\sqrt{S_{wso} S_{wir}} * (1 - S_{gzn})^{M_{ws}/2} S_{gzn}^{N_{ws}/2} \quad (H5)$$

Super-parabolic S_{gr} path function with normalized water boundary saturation

$$S_{gr} = S_{gro} (1 - S_{wan})^{M_{gr}} + S_{grw} S_{wan}^{N_{gr}} + R_{gr} 2\sqrt{S_{grw} S_{gro}} * (1 - S_{wan})^{M_{gr}/2} S_{wan}^{N_{gr}/2} \quad (H6)$$

APPENDIX I Pcw oil production oil apex

P_{cw} function for oil out-flow and potential water and gas inflow in a cell in region A+C.

$$S_{wp} = \frac{S_w - S_{wir}}{1 - S_{wir} - S_{or}} \quad \text{and} \quad S_{op} = \frac{S_o - S_{or}}{1 - S_{wir} - S_{or}} \quad \text{and} \quad S_{gp} = \frac{S_g}{1 - S_{wir} - S_{or}} \quad (I1)$$

$$P_{cw} = (P_{cir} - P_{ct})F_s + (P_{cor} - P_{ct})F_f + P_{ct} \quad \text{and} \quad P_{cw}(S_{ws}) = 0 \Rightarrow E_s \quad (I2)$$

$$F_s = \frac{S_{op}^{L_{sow}}}{S_{op}^{L_{sow}} + E_s S_{wp}^{T_{sow}}} \quad \text{and} \quad F_f = \frac{S_{wp}^{L_{fow}}}{S_{wp}^{L_{fow}} + E_{fow} S_{op}^{T_{fow}}} \quad (I3)$$

APPENDIX J Pcw gas or water production gas apex

P_{cw} function for gas out-flow and potential water and oil inflow in a cell in region B, or alternatively for water out-flow and potential gas and oil inflow in a cell in region B.

$$S_{wq} = \frac{S_w - S_{wir}}{1 - S_{wir} - S_{gr}} \quad \text{and} \quad S_{oq} = \frac{S_o}{1 - S_{wir} - S_{gr}} \quad \text{and} \quad S_{gq} = \frac{S_g - S_{gr}}{1 - S_{wir} - S_{gr}} \quad (\text{J1})$$

$$P_{cw} = P_{cwg} = (P_{cir} - P_{cwt})F_{swg} + (P_{cgr} - P_{cwt})F_{fwg} + P_{cwt} \quad \text{and} \quad P_{cw}(S_{wsg}) = 0 \Rightarrow E_s \quad (\text{J2})$$

$$F_{swg} = \frac{S_{gq}^{L_{sgw}}}{S_{gq}^{L_{sgw}} + E_s S_{wq}^{T_{sgw}}} \quad \text{and} \quad F_{fwg} = \frac{S_{wq}^{L_{fgw}}}{S_{wq}^{L_{fgw}} + E_{fgw} S_{gq}^{T_{fgw}}} \quad (\text{J3})$$

APPENDIX K P_{cw} after crossing S_{or}

P_{cw} function for gas out-flow and potential water and oil inflow in a cell in region B, or alternatively for water out-flow and potential gas and oil inflow in a cell in region B. The saturation path has reached S_{or} path and is continuing in a new direction behind S_{or}. The turning point at the S_{or} crossing has saturations denoted (S_{wy}, S_{oy}, S_{gy}).

$$S_{wxy} = \frac{S_w - S_{wir}}{S_{or}} \quad \text{and} \quad S_{oxy} = \frac{S_o}{S_{or}} \leq 1 \quad \text{and} \quad S_{gxy} = \frac{S_g + S_{wir} + S_{or} - 1}{S_{or}} \quad (\text{K1})$$

$$P_{cw} = P_{cwg}(S_{wq}, S_{oq}, S_{gq}) + P_{cor}F_{foy} - P_{oywg} \quad \text{and} \quad P_{cwg}(S_{wy}, S_{oy}, S_{gy}) = P_{oywg} \quad (\text{K2})$$

$$F_{foy} = \frac{S_{oxy}^{L_{fow}}}{S_{oxy}^{L_{fow}} + E_{fow}(1 - S_{oxy})^{T_{fow}}} \quad (\text{K3})$$

APPENDIX L P_{cw} oil production oil apex LET_x

P_{cw} function for oil out-flow and potential water and gas inflow in a cell in region A+C with proposed extension to include region B, especially parts of B where S_o ≥ S_{orm}.

$$S_{wx} = \frac{S_w - S_{wir}}{1 - S_{wir}} \quad \text{and} \quad S_{ox} = \frac{S_o}{1 - S_{wir}} \quad \text{and} \quad S_{gx} = \frac{S_g}{1 - S_{wir}} \quad (\text{L1})$$

$$P_{cw} = (P_{cir} - P_{ct})F_s + (P_{cwu} - P_{ct})F_f + P_{ct} \quad \text{and} \quad P_{cw}(S_{ws}) = 0 \Rightarrow E_{sx} \quad (\text{L2})$$

$$P_{cwu} = C_{wu} * P_{cor} \quad \text{and} \quad P_{crr} = C_{orw} * P_{cor} \quad \text{and} \quad \text{default} \quad N_{3ex} = 2 \quad (\text{L3})$$

$$\log(E_{fx}) = \left(\frac{1 + S_{orx}}{1 + S_{orwx}} \right)^{N_{3ex}} * \log(E_{fowx}) \quad \text{and} \quad S_{orwx} = \frac{S_{orw}}{1 - S_{wir}} \quad (\text{L4})$$

$$T_{fx} = \frac{L_{fowx} \log(S_{worx}) - \log(E_{fx}) + \log\left(\frac{P_{cwu} - P_{crr}}{P_{crr} - P_{ct}}\right)}{\log(S_{orx})} \quad \text{and} \quad S_{orx} = \frac{S_{or}}{1 - S_{wir}} \quad (\text{L5})$$

$$F_s = \frac{S_{ox}^{L_{sowx}}}{S_{ox}^{L_{sowx}} + E_{sx} S_{wx}^{T_{sowx}}} \quad \text{and} \quad F_f = \frac{S_{wx}^{L_{fowx}}}{S_{wx}^{L_{fowx}} + E_{fx} S_{ox}^{T_{fowx}}} \quad (\text{L6})$$