KOZENY'S EQUATION FOR BETTER CORE ANALYSIS

Pudji Permadi, Institut Teknologi Bandung, Indonesia
Andy Setyo Wibowo, PPPTMGB "Lemigas", Indonesia

This paper was prepared for presentation at the International Symposium of the Society of Core Analysts held in Napa Valley, California, USA, 16-19 September,2013

ABSTRACT

This paper examines the philosophy behind Kozeny's theory. A simple mathematical analysis has been made on his equation. The analysis has arrived at an equation that relates the equivalent hydraulic pore size to pore structure geometry and porosity. For ideal situations, it is found that the pore structure term has a geometrical exponent, b, of 0.5 and pore efficiency, E_P, through which fluid flows equal to 100%. This is physically obvious for any ideal Kozeny's model. The deviations from such ideal situation were investigated employing a number of geological rock samples, sandstones and carbonates. For each type of the geological rocks, routine core, SCAL, and petrographic analysis data were available and grouped according to similarity in petrographical features. Such grouping was made in the attempt to identify factors influencing the deviations.

Results of the analysis show that b and E_P values obtained are lower than those of the ideal model. The values of b are found to be diagenetic dependent and specific detailed rock fabrics and b relates to rock grouping by use of Leverett's J-Function. It is then obtained that permeability prediction is b dependent. Finally, a graphical analysis based on the equation derived demonstrates capability of identifying whether cores contain active micro-fractures or not.

INTRODUCTION

The well known Kozeny equation was derived on the basis of idealization of porous media by combining Poiseuille's and Darcy's equations [1, 2, 3, 4]. Gas flow experiments employing many porous materials demonstrated the weaknesses of the equation [2]. Further discussion about limitations of the equation is also available [3, 4]. However, Scheidegger [1] stated that various capillaric models and more elaborate ones do not give any better solution for practical purposes.

The present work is a refinement and extension of our previous work [5] by employing different sets of data in an effort to substantiate benefits from the Kozeny equation. A dimensional analysis of the equation enables us to apply a fractal theory [6] that provides the analysis a strong theoretical basis.

DIMENSIONAL ANALYSIS OF THE KOZENY EQUATION

One approach to arrive at the Kozeny equation is the use of hydraulic radius theory [1, 3]. For porous media, hydraulic radius m is defined as the ratio of pore volume V_p to surface area of the pore space S. Since m has unit of a length and permeability has unit of square of a length, the expression for permeability may be written as follows,

$$k = K_K \times m^2/f(\phi) \qquad (1)$$

where $K_K = 1/(F_s \times \tau)$ which is the well known Kozeny constant [3] in which F_s is shape factor and τ is tortuosity. For cylindrical tubes, $f(\phi) = 1/\phi$. Substituting m with V_p/S into Eq.(1) and multiplying by V_b/V_b where V_b is bulk volume gives us the well known Kozeny equation:

$$k = \phi^3/(\tau F_s S_b^2) \qquad (2)$$

Here the basis of the specific surface area S_b is bulk volume. Eq.(2) can be re-written in the following forms:

$$k/\phi^3 = 1/(\tau F_s S_b^2) \qquad (3)$$

or

$$(k/\phi)^{0.5} = \phi/(\tau F_s S_b^2)^{0.5} \qquad (4)$$

The term $(k/\phi)^{0.5}$ is a pore geometry term, frequently called the "mean hydraulic radius", and (k/ϕ^3) is simply called the "pore structure term" that accounts for all features of the internal structure (see Eq. (3)). Eventually, one gets a relation between pore geometry and pore structure as the following:

$$(k/\phi)^{0.5} = \phi \times (k/\phi^3)^{0.5} \qquad (5),$$

or

$$(k/\phi)^{0.5} = (V_p/V_b) \times (k/\phi^3)^{0.5} \qquad (6).$$

These two equations say that plotting $(k/\phi)^{0.5}$ against (k/ϕ^3) on log-log graph paper yields a straight line with a slope 0.5. When the straight line produces $V_p/V_b = \phi = 1$, the equations above are valid only for a single capillary tube with infinitely small wall thickness. This implies that fluid flows at constant rate for the entire flow path, i.e. volumetric flow efficiency (E_P) is 100%. Differently, in porous rocks that have many tiny crevices, non-uniform pore sizes, rough pore walls, and probably dead ends or blind pores as well, the flow would be dominated by connected large pores [4] and influenced by the Knudsen effect [2] in smaller pores, and even probably stagnant in the dead ends, resulting in $E_P < 100\%$.

Almost every complexity, such as porous rocks, may be described by use of a fractal theory saying that a power law behavior governs a relationship between variables of the properties involved [6]. In our case, $(k/\phi)^{0.5}$ is the dependent variable and (k/ϕ^3) or $[1/(\tau F_s S_b^2)]$ is the independent variable. Since the exponent 0.5 of the independent variable is valid only for cylindrical tubes (Poiseuille's Law), it is reasonable to expect that porous rocks would deviate from such an ideal model.

Therefore, one may re-write Eq. (6) in a more general form:

$$(k/\phi)^{0.5} = a(k/\phi^3)^b \qquad (7)$$

or, deducing from Eq. (3),

$$(k/\phi)^{0.5} = a[1/(\tau F_s S_b^2)]^b \qquad (8).$$

These last two equations are power law equations with a as a constant, and b as a power law exponent indicating a characteristic of pore structure of porous rocks.

For rock samples having the same values of each a, τ and F_s, the value of $(k/\phi)^{0.5}$ would depend only on S_b and b. Since both $(k/\phi)^{0.5}$ and $1/S_b$ are characteristics of a length, then $1/S_b$ is just the scale for the length $(k/\phi)^{0.5}$ measured and therefore b will be called as the "exponent of self-similarity" characterizing a similarity of the pore structure of all the samples.

APPLICATIONS

Two different data sets of rock samples were obtained from a reefal carbonate reservoir and a sandstone reservoir. The data cover the results of routine core analysis, petrographic analysis, and capillary pressure (SCAL). The permeability of all the core plugs employed is the equivalent liquid permeability. The porosity and permeability values range from 1.3 to 27.5% and 0.001 to 1,800 millidarcies, respectively, for the carbonate and from 1.6 to 18.8% and 0.007 to 16.9 millidarcies, respectively, for the sandstone. The petrographic analysis data were used to group the rock samples based on similarity in microscopic geological features [5] and identify factors that control deviation from the Kozeny equation. The SCAL data were used to analyze the J-Function in a comparison with the corresponding rock types or groups established.

RESULTS AND DISCUSSION

After establishing the rock groups based on the petrographic data (see **Tables 1** and **2**), the calculated k/ϕ^3 and $(k/\phi)^{0.5}$ for the samples of each rock group were plotted on the Type Curve previously published [5]. As we can see in **Figures 1** and **2,** each group of data points are distributed along a certain line below the thick solid line that belongs to cylindrical capillary tubes (a = 1.0 and b = 0.5). This work shows that the Type Curve valid not only for carbonates [5] but also for the sandstone employed and demonstrates the consistency of rock grouping based on similarity in microscopic geological features.

All the lines below the diagonal thick solid line have $a < 1.0$ and slope $b < 0.5$. The value of a, or E_P as described in the previous section, obtained ranges from 0.553 down to 0.242 for the carbonate and from 0.380 down to 0.180 for the sandstone. The exponent b ranges from 0.405 to 0.245 for the carbonate and from 0.345 to 0.225 for the sandstone. The lower the b value, the poorer the rock type quality is, yielding lower E_P.

Based on the methodology developed and the results obtained, it is concluded that the exponent b is not only an expression of similarity in pore structure but also similarity in environmental deposition and diagenesis for a group of rocks. Hence, rock grouping on the basis of dimensional analysis developed from the Kozeny equation is in line with Archie's rock type definition introduced initially more than 50 years ago.

The data points of the SCAL samples (solid triangle) shown in Figures 1 and 2 are located on or close to rock-type line Nos. 5, 6, 7, 8, 9, and 10 for the carbonate and Nos. 8, 9, 10, 12, and 13 for the sandstone. The corresponding J-Functions exhibited in **Figures 3** and **4**, respectively, for the carbonate and the sandstone are clearly distinguishable among the rock types. This means that the dimensional analysis developed from the Kozeny equation closely matches with the theory of J-Function for rock groupings.

The relations between S_{wirr} and permeability k (see **Figures 5** and **6** for the carbonate and sandstone, respectively) give high correlation coefficients, exhibiting that such relationships are independent of rock type. However, since each rock type has its own equation with certain a and b values for Eq. (7) and $S_{wirr} = ck^{-d}$ is a general type correlation for S_{wirr} against k, it is then easy to combine such equations to obtain a general equation $k = A\times[\phi^{p}/(S_{wirr})^{q}]$, where $A = \left(\frac{1}{a}\right)^{\frac{1}{b}} \times (c)^{\frac{0.5}{bd}}$, $p = (3 - \frac{0.5}{b})$, and $q = \left(\frac{0.5}{bd}\right)$. This implies that permeability prediction should be rock type dependent.

The method developed can also be utilized to identify whether micro-fractures that naturally developed within a rock are active or not. Micro-fractures defined here are fractures that can only be seen via thin sections or SEM. Active micro-fractures are those that improve hydraulic conductivity of the rock. Figures 1 and 2 present the data points of rock samples having micro-fractures, designated with asterisk symbol. It is observed that the existence of micro-fractures has no influence on rock grouping. For a given rock type, rock samples with active micro-fractures have lower porosity but higher permeability than those of the samples having no or less/non-active micro-fractures. It has been identified for all the rock types for both the carbonate and the sandstone that the samples with active micro-fractures mostly have k/ϕ^3 above 8×10^3. Figures 1 and 2 depict the dividing line separating the samples with active micro-fractures from those having no or less/non-active micro-fractures. It should be noted, however, that samples giving $k/\phi^3 > 8\times10^3$ do not necessarily contain active micro-fractures, particularly for high quality rock types.

CONCLUSIONS

1. Dimensional analysis of the Kozeny equation has resulted in a power law equation.
2. The exponent b is a strong indicator of similarity resulted from similar geological processes. The method established for rock typing is not only for carbonates but also for sandstones and in line with Archie's rock type definition.
3. Rock typing here conforms to the Leverett's J-Function principle. It is recommended that rock typing be made first prior to selection of core samples for SCAL tests.
4. It is shown that permeability prediction should be rock type dependent.
5. The method of analysis developed may be used to identify whether a core plug contains active micro-fractures or not. Overall, the method may help to improve diagenetic studies.

REFERENCES

1. Scheidegger, A.E. (1959): *The Physics of Flow Through Porous Media*, University of Toronto Press, Canada, pp. 112 – 133.
2. Kraus, G., J. W. Ross, and L. A. Girifalco (1953): Surface Area Analysis by Means of Gas Flow Methods, *J. Phy. Chem.*, **vol**. 57, March, pp. 330 – 333.
3. Harmsen, G. J. (1955): The Concept of Hydraulic Radius in Porous Media, *Transaction AIME*, **vol**. 204, 274 – 277.
4. Wyllie, M. R. J. and G. H. F. Gardner (1958): The Generalized Kozeny-Carman Equation: Part I – Review of Existing Theories, *World Oil*, March, 121 – 128.
5. Wibowo, A. S. and P. Permadi (2013): A Type Curve for Carbonate Rock Typing, *Paper IPTC-16663-MS presented at the IPTC* held in Beijing, China, 26 – 28 March.

6. Peitgen, H. O., H. Jürgens, and D. Saupe (2004): *Chaos and Fractals New Frontiers of Science*, 2nd Edition, Springer-Verlag New York, Inc., pp. 182-191.

Table 1. Samples grouping based on Petrographic analysis data for the carbonate.

RT	**Microscopic geological features**
5	Packstone, **grain supported**, medium hard-hard , **medium-coarse grain**, **moderate-well sorted**, **point>floating ,** medium-high abraded , coral , slightly foram , slightly algae, slightly molusk, slightly moldic, slightly detrital clay, quartz, motteld vug, mold, intra particle, Inter crystalline, pinpoint, slightly cemented, calcite sparite, slightly micrite, **slightly siderite**, slightly pyrite, **equant /slightly blocky clay.**
6	Wackestone-packestone, **mud-grain supported**, **medium hard-hard** , **fine-medium grain**, **poorly-medium sorted**, **planar>point>floating**, medium-high abraded, coral , **slightly foram**, slightly algae, slightly molusk, Slighly moldic, slighly detrital clay, quartz, **pinpoint-mottled vug** , mold , **intraparticle, intercrystalline,** stylolite , **slightly cemented** , calcite sparite**, slightly micrite**, slightly pyrite, **equant/ blocky clay.**
7	Wackestone-packestone, mud-grain supported, **hard**, fine-medium grain , poorly-moderate sorted , planar>point>floating , medium abraded , coral, **Foram** , slightly algae , quartz, pinpoint-mottled vug, slightly moldic, **slightly intraparticle**, **slightly intercrystalline**, stylolite , **slightly-moderate cemented**, slightly calcite sparite, **micrite** , **equant /blocky clay.**
8	Wackestone-packstone, mud-Grain supported, **hard-very hard** , **mud-medium grain**, **poorly sorted**, **planar>point>floating** , medium abraded, coral, foram , slightly algae, **slightly carbonate mud** , quartz, **pinpoint-mottled vug** , sliightly intra particle, **trace stylolite**, **slightly-moderate cemented**, slightly calcite sparite, micrite , **equant /blocky clay.**
9	Wackestone-packstone, mud-Grain supported, hard-very hard , **fine-medium grain** , **poorly-moderate sorted** , **point>planar**, **medium-high abraded** , slightly coral, foram , slightly algae, **carbonate mud,** quartz, **slightly pinpoint-mottled vug**, stylolite, slightly-moderate cemented , slightly calcite sparite, micrite , slightly pyrite , **equant /blocky clay.**
10	Wackestone-packstone, mud-grain supported, **hd-very hard** , **mud-medium grain**, poorly-moderate sorted , **planar>point** , **medium abraded** , slightly coral , foram, slightly algae, slightly molusk, **slightly moldic**, **slightly Carb mud**, **quartz,** slightly pinpoint vug, **slightly intraparticle**, stylolite, **slightly-moderate cemented** , slightly calcite sparite, **micrite** , slightly pyrite , **equant /blocky clay.**
11	Wackestone-packstone, mud-grain supported , **very hard** , mud-medium grain, poorly-moderate sorted, **planar>point>floating**, medium abraded , coral , **slightly foram** , slightly algae , slightly molusk, **carbonate mud**, **micro crystalline** , **slightly quartz** , **slightly moldic**, stylolite, **moderate cemented** , slightly calcite sparite, **slightly micrite**, slightly pyrite .
12	Wackestone-packstone, mud-grain supported , very hard**,** mud-medium grain, poorly-moderate sorted , **planar** , medium abraded, slightly coral , slightly foram, slightly algae, molusk , carbonate mud, micro crystalline, slightly quartz , **moldic**, styl olite, moderate cemented , slightly calcite sparite, slightly micrite , slightly pyrite.
13	Wackestone-packstone, mud-grain supported, very hard , mud-medium grain, poorly-moderate sorted, planar **,** medium abraded, coral , slightly foram, slightly algae, molusk , **slightly carbonate mud** , microcrystalline , **pinpoint vug**, stylolite, moderate cemented , slightly calcite sparite, slightly pyrite.

Table 2. Samples grouping based on petrographic analysis data for the sandstone.

RT	**Microscopic geological features**
8	Sandstone; Light gray-gray, hard**, fine-coarse grain**, sub angular, poor-well sorted, planar>point, calcite, slightly argillaceous, less otassium&lagioclase-feldspar, **slightly mica, slightly pyrite**, slightly fossil, bioturbation, quartz-**ankerite**-dolomite cement, dissolution porosity, fracture, cemented.
9	Sandstone; Light gray-gray, hard, **very fine-fine/coarse grain**, sub angular, **poor-well sorted**, **planar>point**, calcite, **chlorite** (2/26), **slightly argillaceous**, less potassium&plagioclase feldspar**, slightly fossil**, bioturbation, **quartz-dolomite cement**, dissolution porosity, fracture, **cemented** .
10	Sandstone; Light gray-gray, hard, very fine-fine/coarse grain, sub angular, m**oderate-well sorted**, **point>planar**, **calcite, chlorite (4/18),** less potassium&plagioclase feldspar**, pyrite**, bioturbation, quartz-dolomite cement, dissolution porosity (fracture), **well cemented** .
11	Sandstone; Light gray-gray, hard, **very fine-fine grain**, sub angular, moderate-well sorted, point>planar, calcite, less potassium&plagioclase feldspar, **chlorite (4/14)**, **pyrite**, bioturbation, **quartz-trace dolomite cement**, dissolution porosity, fracture, well cemented.
12	Sandstone; Light gray-gray, hard, very fine-fine grain, sub angular, well sorted, point>planar, **slightly argillaceous**, **slightly mica**, calcite, chlorite (2/13) less potassium&plagioclase feldspar, bioturbation, **quartz cement**, dissolution porosity, fracture, well cemented.
13	Sandstone; **Light gray**, hard, **very fine grain**, **sub angular-sub.rounded**, well sorted, point>planar, less potassium&plagioclase feldspar, calcite, **chlorite (5/11)**, **pyrite**, bioturbation, quartz cement, dissolution porosity, fracture, well cemented.
14	Sandstone**; Light gray-gray**, hard, very fine grain**, sub angular**, well sorted, pint>planar, **slightly argillaceous, slightly mica**, less potassium&plagioclase feldspar, calcite, bioturbation, quartz cement, dissolution porosity , fracture, well cemented.

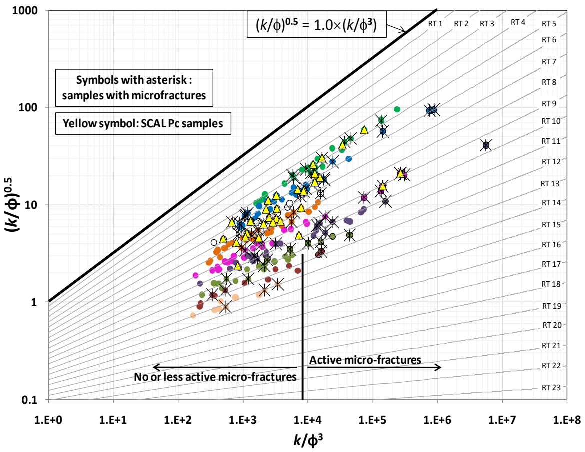

Figure 1. Rock grouping for the Carbonate.

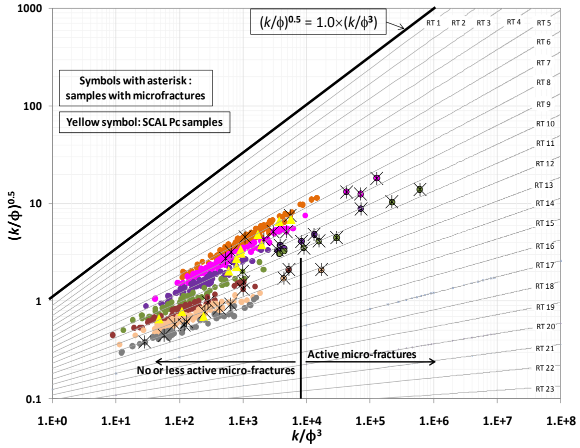

Figure 2. Rock grouping for the Sandstone.

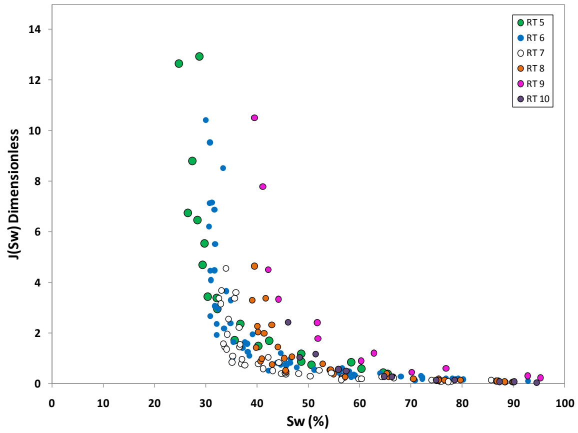

Figure 3. J-Function for the Carbonate.

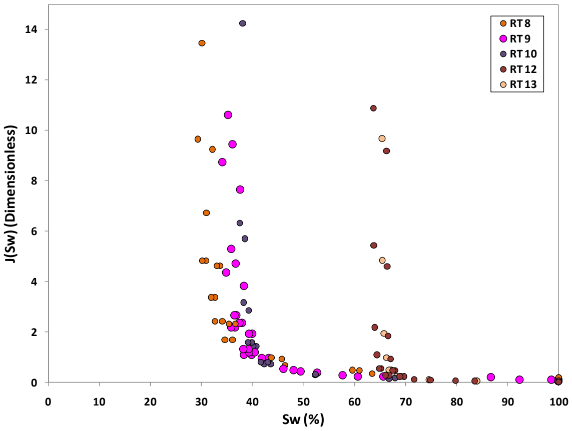

Figure 4. J-Function for the Sandstone.

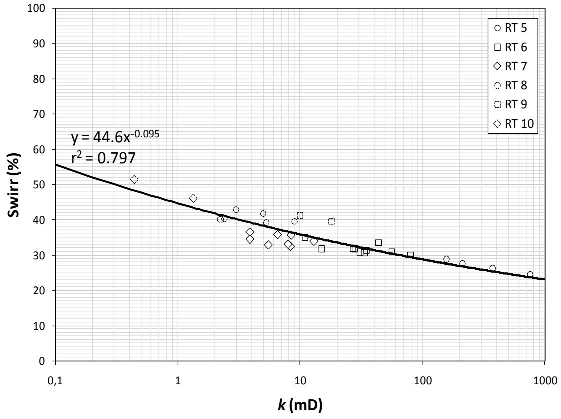

Figure 5. *Swirr* vs. *k* for the Carbonate.

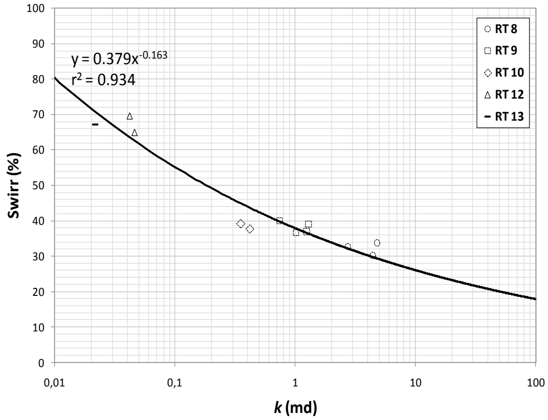

Figure 6. *Swirr* vs. *k* for the Sandstone.