INTERPRETATION METHOD FOR RELATIVE PERMEABILITY WITH FULL ACCOUNT FOR DYNAMIC CAPILLARY PRESSURE

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ABSTRACT

The interpretation from un-steady state mathematic model without consideration of the effect of dynamic capillary pressure and end effect on experimental data cannot be consistent with the low permeability coreflooding. In this paper, Leverett function and its derivative and the second derivative are decided by an accurate analytical interpretation mathematical model with full account for the dynamic capillary force and end effect in immiscible displacement. Material balance equation is introduced to calculate water saturation gradient at the outlet of a core which can explain the effect of dynamic capillary force on the water and oil relative permeabilities. The novel mathematical model derived in the paper can be used for core flooding tests analysis involving constant injection rates and constant pressure differences.

INTRODUCTION

Relative permeability curves play a great role in reservoir engineering and numerical simulation especially in secondary and tertiary mode. The relative permeability-vs.-saturation data are usually obtained from coreflood in the laboratory. Two approaches have pointed out to calculate the relative permeability for experiments: (1) the steady state method where both of the immiscible fluids are injected into the upstream of the core simultaneously, and (2) the unsteady-state method in which one of fluids displaces the other. A steady state experiment is simple but it is time consuming. It usually takes couple of weeks to finish one experiment. On the contrary, the unsteady-state method can be implemented in a relatively short time. However, the interpretation of the data is more complex [1]. The objective of this paper is to derive a mathematic model with consideration of dynamic capillary pressure and end effect on interpretation of relative permeability.

Construction of Relative Permeability and Dynamic Capillary Pressure

Basic Equations

It is assumed that the core can be recognized as a homogeneous one dimensional system. The basic equations are used to describe water/oil displacement with uniform immobile initial water saturation. Distance along the core is measured from the inlet. Supposed that fluids flow through porous media comply with Darcy's law, where the phase of water velocity is expressed as below.

$$\frac{dP_w}{dx} = -\frac{u_w \mu_w}{KK_{rw}} + \rho_w g \sin\theta \qquad (1)$$

Assuming that the pressure of upstream and downstream is $P_1\big|_{x=0}$ and $P_2\big|_{x=L}$, respectively. Therefore, the differential pressure between two sides can be calculated using the following equation.

$$P_1 - P_2 = \Delta P = -\int_0^L \frac{\partial p_w}{\partial x} dx \qquad (2)$$

Where L is the length of the core sample,

According to Buckley-Leverett formation，the position, $x\big|_{S_w}$, of certain water saturation S_w at given time can be derived according to:

$$x\big|_{S_w} = \frac{Q_i(t)}{A\phi} f_w'\Big|_{S_w} \qquad (3)$$

Where $f_w' \equiv \frac{df_w}{dS_w}$； $Q_i(t) = \int_0^t q_i(t) dt$； $q_i(t) = Au_i(t)$

Because only the phase of water is injected at entrance of the core, therefore,

$$S_w\big|_{x=0} = 1 - S_{or}, \quad f_w\big|_{x=0} = 1, \quad f_w'\Big|_{x=0} = 0, \quad df_w'\Big|_{x=0} = 0 \qquad (4)$$

But both phases come out from the core outlet after breakthrough. Consequently

$$S_w\big|_{x=L} = S_{w_2}\ ,\quad f_w\big|_{x=L} = f_{w_2}\ ,\quad f_w'\Big|_{x=L} = f_{w_2}'\ ,\quad df_w'\Big|_{x=L} = df_{w_2}' \tag{5}$$

Substituting Equations 4, 5 and 3 in Equation 2 yields:

$$\Delta P_w = -\int_0^L \frac{\partial P_w}{\partial x}dx = \frac{u_w(t)Q_i(t)}{A\phi}\int_0^{f_{w2}'}\frac{1}{\lambda_w}df_w' + \frac{Q_i(t)}{A\phi}\int_0^{f_{w2}'}\rho g\sin\theta df_w' \tag{6}$$

If only considering the horizontal displacement and ignoring the effect of gravity, Equation 6 can be simplified as below,

$$\Delta P_w = -\int_0^L \frac{\partial P_w}{\partial x}dx = \frac{u_w(t)Q_i(t)}{A\phi}\int_0^{f_{w2}'}\frac{1}{\lambda_w}df_w' \tag{7}$$

Hence, differentiating Equation 7 with respect to time yields:

$$\frac{d(\Delta P_w)}{dt} = \frac{M}{A\phi}\frac{d}{dt}\left[u_w(t)Q_i(t)\right] + \frac{u_w(t)Q_i(t)}{A\phi}\frac{dM}{dt} \tag{8}$$

Where the function M is defined as [2] : $M = \int_0^{f_{w2}'}\frac{1}{\lambda_w}df_w'$

By differentiating function M with respect to time,

$$\frac{dM}{dt} = \frac{d}{dt}\left(\int_0^{f_{w2}'}\frac{1}{\lambda_w}df_w'\right) = \frac{1}{\lambda_w}\frac{df_w'}{dt} = -\frac{1}{\lambda_w}\frac{A\phi L}{Q_i^2(t)}\vec{q}_i(t) \tag{9}$$

Substituting Equation 9 in Equation 8:

$$\frac{d(\Delta P_w)}{dt} = \frac{\Delta P_w}{u_w(t)Q_i(t)}\frac{d}{dt}\left[u_w(t)Q_i(t)\right] - \frac{u_w(t)q_i(t)L}{Q_i(t)\lambda_w} \tag{10}$$

Assuming differential pressure is constant during the displacement.

$$\frac{d(\Delta P_w)}{dt} = 0 \tag{11}$$

Therefore, Equation 10 can be simplified as below.

$$K_{rw} = \frac{L}{\Delta P_w}\cdot\frac{u_w^{\,2}(t)q_i(t)}{Q_i(t)\frac{d\left[u_w(t)\right]}{dt} + u_w(t)q_i(t)}\cdot\mu_w \tag{12}$$

According to Darcy's Law, water fraction can be expressed by Equation 13 with consideration of dynamic capillary pressure effect and ignorance of the effect of gravity.

$$f_w = \frac{\lambda_w}{\lambda_t} + \frac{A\lambda_w}{q_t(1+\lambda_w/\lambda_o)}\left[\frac{\partial P_{static}^c(S_w)}{\partial S_w}\frac{\partial S_w}{\partial x} + \tau\frac{\partial}{\partial x}\left(\frac{\partial S_w}{\partial t}\right)\right] \tag{13}$$

The calculation of τ is given by S. Majid Hassanizadeh [3]. With the rearrangement of Equation 13, the mobility of the phase of oil can be obtained.

Then the relative permeability of oil phase can be obtained as below with constant differential pressure displacement.

$$K_{ro}=\frac{L(1-f_w)q_i(t)u_w{}^2(t)q_i(t)}{\Delta P_w q_i(t) f_w\left\{Q_i(t)\frac{d\left[u_w(t)\right]}{dt}+u_w(t)q_i(t)\right\}-Lu_w{}^2(t)q_i(t)A\nabla P_{dyn}^c}\cdot\frac{\mu_o}{K} \quad (14)$$

Moreover, assuming displacement with constant flow rate mode, Equation 10 can be derived:

$$\frac{d(\Delta P_w)}{dt}=\frac{\Delta P_w}{u_w(t)}\left[\frac{d\left[u_w(t)\right]}{dt}+\frac{u_w(t)}{t}\right]-\frac{u_w(t)L}{t\lambda_w} \quad (15)$$

With the rearrangement of Equation 15, the mobility of water phase can be derived and the relative permeability of water phase can be yields under constant flow rate.

$$K_{rw}=\frac{\mu_w}{Kt\Delta P_w}\cdot\frac{u_w^2(t)L}{\frac{d\left[u_w(t)\right]}{dt}+\frac{u_w(t)}{t}-\frac{u_w(t)}{\Delta P_w}\frac{d(\Delta P_w)}{dt}} \quad (16)$$

And also, the the relative permeability of oil phase can be yields under constant flow rate.

$$K_{ro}=\frac{(1-f_w)u_w^2(t)L}{f_w t\Delta P_w\left\{\frac{d\left[u_w(t)\right]}{dt}+\frac{u_w(t)}{t}-\frac{u_w(t)}{\Delta P_w}\frac{d(\Delta P_w)}{dt}\right\}-\frac{u_w^2(t)LA\nabla P_{dyn}^c}{q_i(t)}}\cdot\frac{\mu_o}{K} \quad (17)$$

Therefore, the new mathematical model of relative permeability with the consideration of dynamic capillary pressure is derived for low permeability coreflood. The relative permeability models at constant pressure are given in Equation 12 and 14, and Equation 16 and 17 show the relative permeability model under constant flow rate displacement.

Calculation of Water Saturation Gradient at the downstream of Cores

According to relative permeability model, the water saturation gradient with respect to time and certain position at the outlet of a core needs to be derived. The water saturation gradient at the outlet of a core can be expressed as:

$$\frac{\partial S_w(x,t)}{\partial x}=-\frac{\partial S_w(x,t)}{\partial t}\bigg/\frac{dx}{dt} \quad (18)$$

By substituting Equation Buckley-Leverett in Equation 18

$$\frac{\partial S_w(x,t)}{\partial x} = -\frac{A\phi}{q_i(t)\, f_w{}'} \frac{\partial S_w(x,t)}{\partial t} \tag{19}$$

The average water saturation of the core in the rage of [0, L] can be calculated as the following equation with the combination to material balance equation.

$$S_w(x,t) = \overline{S_w}(x,t) - t\frac{\partial \overline{S_w}(x,t)}{\partial t} \tag{20}$$

By differentiating the Equation 20,

$$\frac{\partial S_w(x,t)}{\partial t} = -t\frac{\partial^2 \overline{S_w}(x,t)}{\partial^2 t} \tag{21}$$

According to material balance equation, the average water saturation of the core can be expressed as following [4].

$$\overline{S_w}(x,t) = S_{wi} + \frac{\left(1 - f_{w_2}\right) Q_i(t)}{Ax\phi} \tag{22}$$

By differentiating Equation 22 with respect to time，

$$\frac{\partial \overline{S_w}(x,t)}{\partial t} = \frac{-1}{Ax\phi}\left[q_i(t)\, f_w(x,t) + Q_i(t)\, f_w{}'(x,t) - q_i(t) \right] \tag{23}$$

At constant flow rate, by differentiating Equation 23, and substituting to Equation 21 with the combination of Equation 5 yields.

$$\left.\frac{\partial S_w(x,t)}{\partial t}\right|_{x=L} = \frac{-Q_i(t)}{AL\phi}\left[2\frac{\partial f_{w_2}(L,t)}{\partial t} + t\frac{\partial^2 f_{w_2}(L,t)}{\partial t^2} \right] \tag{24}$$

And: $$\left.\frac{\partial S_w(x,t)}{\partial x}\right|_{x=L} = \frac{tQ_i(t)}{A\phi L^2}\left[2\frac{\partial f_{w_2}(L,t)}{\partial t} + t\frac{\partial^2 f_{w_2}(L,t)}{\partial t^2} \right] \tag{25}$$

At constant differential pressure, by differentiation of Equation 23, and substituting to Equation 21 with the combination of Equation 5,

$$\left.\frac{\partial S_w(x,t)}{\partial t}\right|_{x=L} = \frac{t}{AL\phi}\left[\frac{d\left[q_i(t)\right]}{dt}\left[f_w(L,t) - 1 \right] + 2q_i(t)\frac{\partial f_w(L,t)}{\partial t} + Q_i(t)\frac{\partial^2 f_w(L,t)}{\partial t^2} \right] \tag{26}$$

Substituing Equation 26 to 19 with the combination of equation 5,

$$\left.\frac{\partial S_w(x,t)}{\partial x}\right|_{x=L} = -\frac{tQ_i(t)}{q_i(t)\, A\phi L^2} \cdot \left[\frac{d\left[q_i(t)\right]}{dt}\left[f_w(L,t) - 1 \right] + 2q_i(t)\frac{\partial f_w(L,t)}{\partial t} + Q_i(t)\frac{\partial^2 f_w(L,t)}{\partial t^2} \right] \tag{27}$$

Equation 20 can be also applied at the outlet of a core as $x = L$, hence, the water saturation at the downstream of a core can be expressed as below,

$$S_{w_2} = \overline{S_w} - t\frac{\partial \overline{S_w}}{\partial t} \tag{28}$$

Where, $\overline{S_w}$ is the total average water saturation in the entire core.

Therefore, all of the equations needed for water saturation gradient with respect time time and position x, and relative permeability of both oil and water phases have been derived with full account for dynamic capillary pressure and end effect. Due to space limitation, two examples will be provided in the poster.

CONCLUSIONS

The novel mathematical model derived in the paper could be used for core flooding tests analysis involving constant injection rates and constant pressure differences. The future works will be conducted to evaluate the developed formalisms using both a synthetic case and real experimental data

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