APPLICABILITY OF CARMAN-KOZENY EQUATION FOR WELL CONSOLIDATED SAMPLES

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ABSTRACT

The Carman-Kozeny equation has been and is being used extensively by scientists and industry to estimate permeability from grain size distribution data. It is well attested in literature that the aforementioned model was developed on sand packs. To observe the reliability of the model on well consolidated sand, this paper reports on a study of 25 samples from an offshore Ghana formation of Turonian age. Grain size distribution data along with other petrophysical properties were used to estimate permeability using the Carman-Kozeny equation. Different statistical methods were used to compare different permeability estimations and correlate them with grain size distribution data. The results show significant scatter and predicted values that highly deviate from measured permeability; this is contrasted with the excellent correlation introduced by Ruth *et. al.* [7] using the same set of samples. It is believed that the poor prediction could be a result of inapplicability of the model, or to the nature of the grain size distribution for well consolidated sand which may be highly dependent on the amount of energy expended in breaking samples prior to grain size analysis.

INTRODUCTION

It has long been recognized that permeability is related to the grain size distribution of granular porous media [1]. There have been numerous empirical formulae, capillary models, statistical models and hydraulic radius theories proposed for estimating hydraulic conductivity (permeability) of a rock sample [2,3]. A well-known relationship correlating permeability with properties of pores and grain size is known as the Carman-Kozeny (C-K) equation (proposed by Kozeny in 1927 and later modified by Carman in 1937) although Carman and Kozeny never published together [2,4]. The principal limitation of the C-K equation is the fact that all geometrical features of the sample are combined into the C-K factor [5]. Carman [6] made an attempt to introduce more microstructural features into the equation, writing the C-K factor in terms of its components, namely the pore shape factor (S) and tortuosity (τ) and introduced the following equation:

$$k = \frac{d_m^2}{36\tau^2 S}\frac{\phi^3}{(1-\phi)^2} \qquad \text{Eq.1}$$

in which k is the permeability of the sample, d_m is effective grain diameter, ϕ is porosity (in fraction), τ is the tortuosity of the sample and S is the shape factor defined by Carman [6] for various sectional shapes. Carman experimentally found that a value of $\sqrt{2}$ should be chosen for tortuosity and 2.5 for shape factor. However, Ruth *et. al.* [7], while using Mercury Injection Porosimetry (MIP) data to estimate permeability, demonstrated that a tortuosity dependent on formation factor, which itself depends on porosity, such that:

$$\tau^2 = \frac{a}{\phi^{m-1}} \qquad \text{Eq.2}$$

leads to good correlations. It is also shown in other literature that tortuosity can range from 1.7 to 4 in beds packed with non-uniform spheres, depending on factors such as packing, arrangement, media homogeneity and channel shape [5]. In this study, the tortuosity factor proposed by Ruth *et. al.* [7] is used. In addition to tortuosity, effective grain diameter has also been of interest to researchers. Different researchers have used different effective diameters. As an example, Odong [3] used D_{10} (which represents the grain diameter for which 10% of the sample is finer) as the effective grain diameter. Chapius and Aubertin [2] used the hydraulic mean diameter of grain sizes to calculate effective diameter. Mastera [8] believes using hydraulic mean diameter would result in a systematic under estimation of permeability when the volume of coarser pores is significantly more than can be occupied by finer sediments. Kolterman and Gorelick [9] and Kamann *et. al.* [10] showed that permeability varies non-linearly as a function of the volume fraction of the finer component (demonstrated in figure 1). They developed an empirical formula for estimating k using the C-K equation, considering whether or not the volume of the finer-grain fraction has the potential to fill the volume of pores in the coarser grain fraction. Using the same concept, a recursive method was later developed by Mastera [8] to calculate the effective grain-size diameter for sediment mixtures with more than three grain sizes.

In this study different approaches to the C-K equation are examined on a data set for well consolidated samples to investigate the reliability of permeability estimations made using the C-K equation for such samples.

THE DATA SET

In this study a set of 25 samples are used, the same set as used in Ruth *et al.* [7]. As Ruth claims, to date this data set has been used by the operator to populate permeability-porosity transforms and supplemented by drill stem tests, to calibrate effective flow properties. The samples come from three different pools, one which produces oil and two which produce gas condensate. Because electrical data is not available for all the samples, Ruth [7] suggested using 2 for the cementation factor and 1 for a, which will reduce equation 2 to simply:

$$\tau^2 = \frac{1}{\phi} \qquad \text{Eq. 3}$$

The porosity ranges from 3% to 28% and this results in a tortuosity range of 5.5 to1.9. Grain size data also cover a wide range of grain sizes from sand size grains (0.5 mm) to clay size grains (1 µm). Figure 2 shows a sample grain size distribution curve.

As a start, the C-K equation was used to estimate the permeability of samples, without including the tortuosity values, using a C-K factor of 5, and D_{10} as the effective grain diameter. The results were compared with another case in which tortuosity was calculated using the Ruth *et al* approach [7]. A shape factor of 2.5 was used along with D_{10} as the effective grain diameter. Trend lines are drawn for each case and compared in terms of slope and R^2. The results show good improvement when tortuosity is calculated using porosity values and included in the C-K equation to estimate permeability (Fig 3). Therefore, in all further investigations tortuosity is included in the calculations. For the next step, the effect of effective grain diameter on permeability estimation was investigated. Effective grain diameter was calculated using three different approaches mentioned in the literature: D_{10}, harmonic mean, and the method developed by Mastera [8]. In all calculations tortuosity was calculated using the Ruth *et al* approach and a shape factor of 2.5 was used. Again trend lines are drawn for the three different approaches and compared in terms of slope and R^2. While harmonic mean and Mastera's method show better R^2, estimations made using D_{10} seem to have better proximity with the measured values, because the other two trend lines have a different slope, and seem not to be following the right trend. Finally, estimations using C-K, with D_{10} as the effective grain diameter, while including tortuosity, were compared with predictions of Ruth *et. al.* on the same set of data using MIP data [7] (Ruth *et. al.* excluded one sample because they believed it was an erroneous one; therefore, to compare the two methods, that sample was also excluded from the C-K estimations). In their paper, Ruth *et al.* assume that flow through a porous media can always be modeled as flow through a series of identical representative elemental volumes (REV) containing a single tortuous tube. Then the Darcy law in applied to the REV to correlate permeability with porosity, tube diameter and tortuosity. Finally, using capillary pressure concepts, tube diameter is replaced with the equation introduced by Purcell, correlating permeability with mercury injection porosimetry (MIP) data as follows:

$$\mathrm{k} = \frac{\phi^{m}(\sigma\cos\theta)^2}{2\,a}\int\frac{\mathrm{d}S_v}{(P_c)^2} \qquad \text{Eq. 4}$$

where a and m are found from Archie's equation, σ is the interfacial tension, θ is the contact angle and P_c and S_v are capillary pressure and the saturation of the vacuum in the MIP experiment, respectively[7]. Comparing the two methods in terms of slope and R^2 (Fig 5), the Ruth *et al* method seems to be much more promising. One reason for that could be because it uses pore throat concepts to estimate permeability whereas C-K uses grain surface area and grain diameter.

DISCUSSION AND CONCLUSON

So far both methods (Carman-Kozeny and Ruth *et al* [7]) show promising results provided proper assumptions are made such as effective grain diameter for the C-K and threshold pressure for the Ruth *et al* method. As Ruth *et al.* mentioned in their paper, the data set was not originally intended to produce a predictive model for permeability. Therefore more work should be done on different data sets to analyze the predictive capability of the aforementioned methods. As Sheppard [11] indicates, the fundamental relationship between the sedimentologic properties of sedimentary rocks and flow through them is poorly understood. It is also worth mentioning that Dvorkin [12] declared that these methods cannot be used to replace physical or digital measurements, but can rather serve for quality control of physical and digital data.

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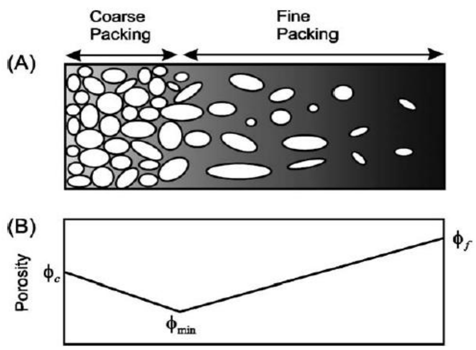

Figure 1 Ideal packing model. (a) for a binary mixture of grains in which either fine grains fill the space between coarse grains or coarse grains are dispersed among fine sediments (b) porosity as a function of fine and coarse grains. Minimum porosity occurs when the amount of fine grains is equal to the porosity of coarse grains [9].

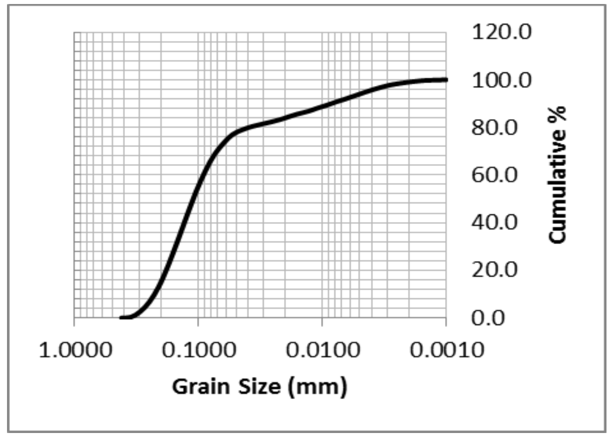

Figure 2 A typical grain size distribution curve of the data set

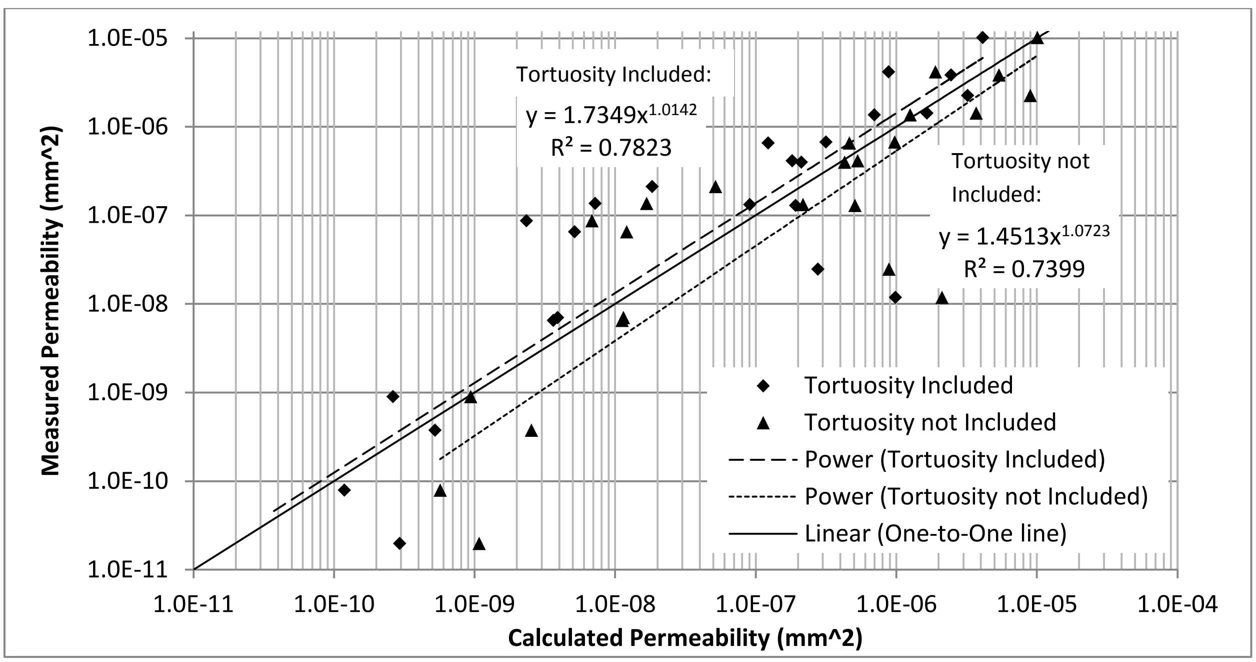

Figure 3 Comparing permeability estimation using CK equation, with and without tortuosity effect

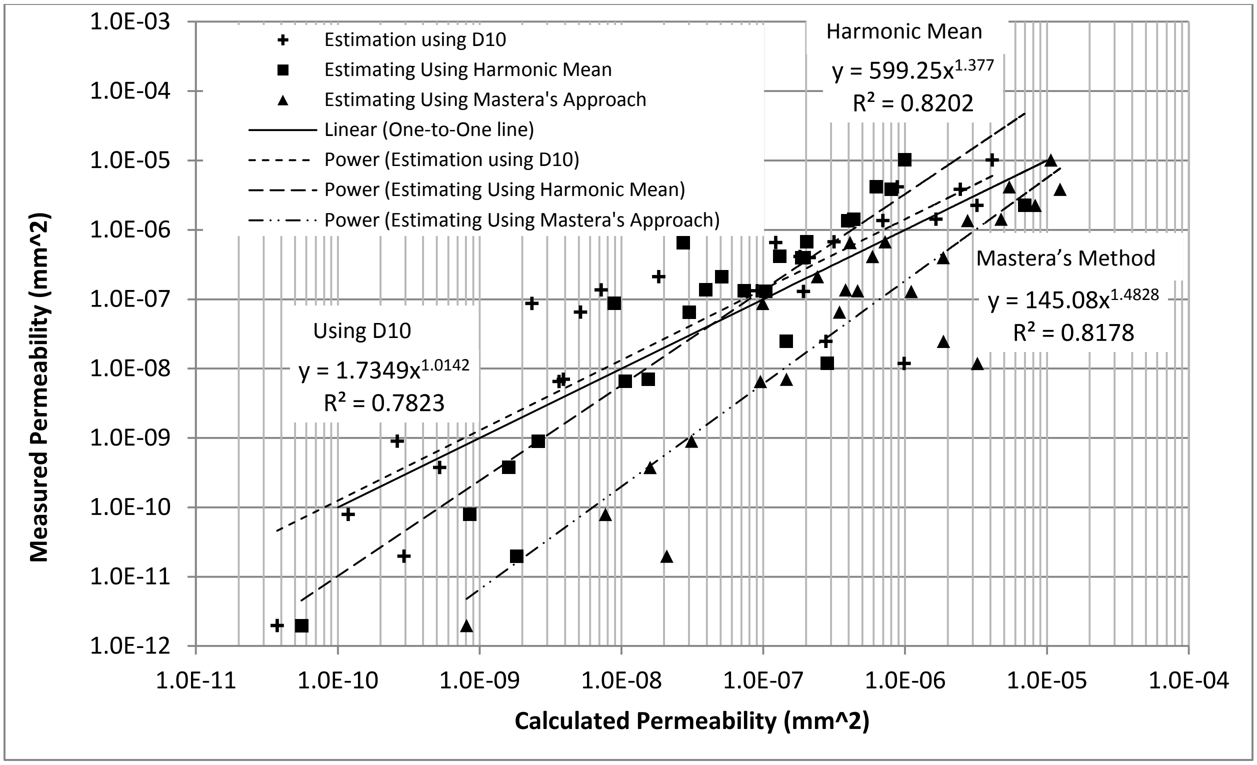

Figure 4 Comparing the effect of using different effective grain diameters on permeability estimation, using CK equation

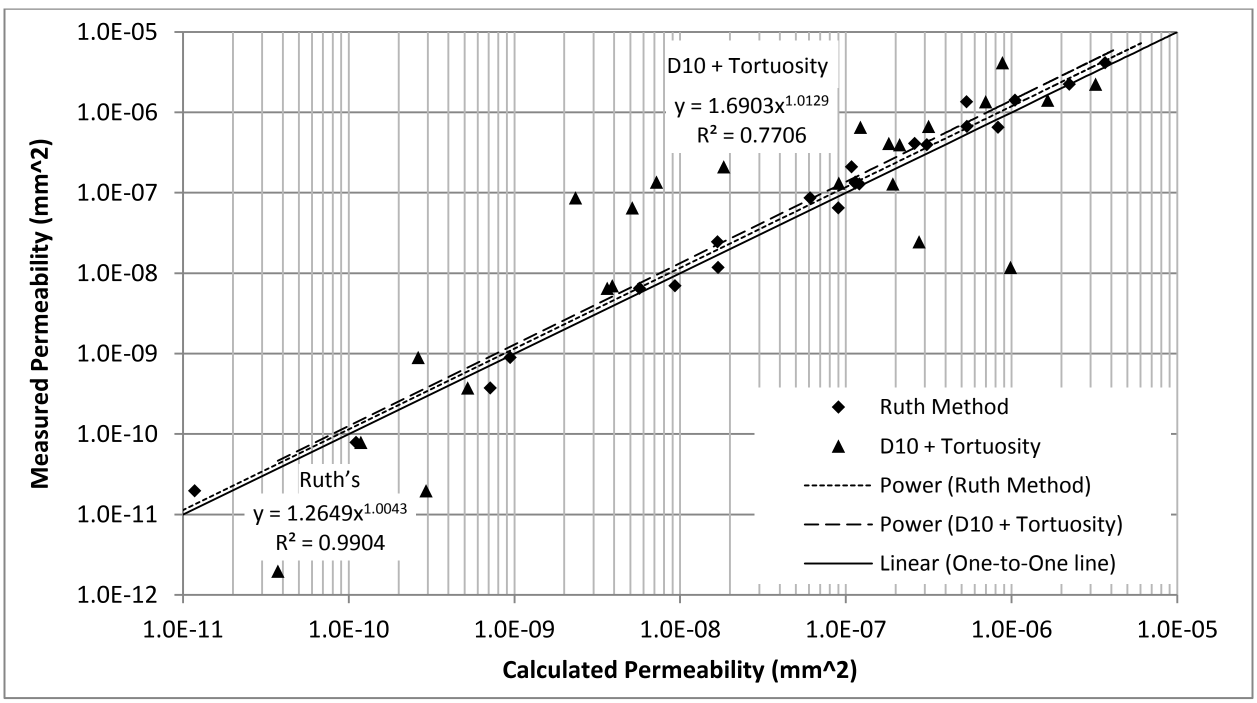

Figure 5 Comparing permeability of the data set estimated using CK and method introduced by Ruth *et al*