PERFECT CROSS FLOW MODEL FOR COUNTER-CURRENT SPONTANEOUS IMBIBITION

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ABSTRACT

In the present work, counter-current spontaneous imbibition is modeled in a bundle of tubes with perfect cross flow. The developed formulation was solved analytically with an iteration based method for a sample tube bundle and the results were compared with numerical simulation results of the governing equations of counter-current spontaneous imbibition using core and fluid properties analogous to the tube bundle model. It was observed that the proposed model is an exact analog of the modified Darcy Law. The tube bundle model provides a powerful tool with which to study and model porous media because it allows detailed prediction of flow behavior.

Particular attention was paid to the behavior of fluids at the imbibition face during a spontaneous counter-current imbibition process. The porous medium is assumed to be homogeneous and the fluids are assumed to be incompressible and immiscible in present study. The paper concludes that the neither tube bundle models nor numerical simulation provide information on the imbibition face saturation condition and further experimental work is necessary.

INTRODUCTION

A porous medium consists of different sized pores connected by means of different sized throats in a three dimensional structure. Displacement of fluids in this medium is a complex process which has been investigated using pore-network models; however, the pore structure of a sample often can be simplified by using an equivalent but less complex model (tube bundle models). Pore-network models are developed with essentially the same geometry as real porous media; tube bundle models use a much greater level of abstraction. Pore-network models are more sophisticated and more complex than tube bundle models and have higher flexibility to match the real porous medium. Blunt *et al* [3] presented a review of the physics and characteristics of pore-network models. Three dimensional pore-network models have been developed extensively since 1949 by many researchers [2].

The first tube bundle models did not introduce any interaction between the fluids flowing in neighboring tubes [9], which makes the models very simple but fails to predict the behavior of real porous media [7]. The deficiency of these simple models is more sensible in the case of complex two and three phase displacement processes. The interacting capillary bundle model was first described by Dong *et al* [4]. Another paper ([5]) by Dong *et al.* goes into details about the influence of viscosity and flow rate on a displacement process in tube bundle models. As shown by Ruth and Bartley [7], based on both theory and comparison with simulations, the "perfect cross flow (PCF)" model, as they term it, leads directly to the usual modified Darcy's law used extensively in analyzing multi-phase flow in porous media. Wang *et al*. [9], used triangular tubes instead of cylindrical in interacting uniform and serial type tube bundle models.

In the present study, a two-dimensional cylindrical tube bundle model was used to develop an analytical model for counter-current spontaneous imbibition (COUCSI) and the effect of the imbibition face boundary condition was investigated.

DEVELOPMENT OF THE PERFECT CROSS FLOW MODEL

Figure 1 shows the perfect cross flow model for the case under consideration. As shown in previous works, the wetting fluid (water in this case) advances fastest in the smallest tube, although a detailed analysis shows that most of the flow occurs in the larger tubes. The small tubes provide the driving force but the large tubes provide the least resistance.

It is generally believed that the non-wetting fluid will exit the sample through the largest tube. This assumption is equivalent to assuming essentially complete wetting of the open face in the COUCSI process in a natural porous medium. However, it has been observed in experiments that the open face saturation is closer to 50 % than 100% [6]. Arabjamaloei and Shadizadeh studied the inlet condition for COUCSI and proposed a method for determining its value [1]. Their prediction is that the imbibition face saturation corresponds to the value that minimizes the mobility variable defined as

$$M_w = \frac{k\, k_{rw}\, k_{rnw}}{\mu_w\, k_{rnw} + \mu_{nw}\, k_{rw}} \frac{dP_c}{dS_w} \tag{1}$$

Where P_c is the capillary pressure, μ is the viscosity, k is absolute permeability, S_w is wetting phase saturation. The subscripts w and nw denote the wetting and non-wetting phases respectively.These conflicting results are the reason that the effect of the imbibition face boundary condition was studied using the developed tube bundle model.

Formulation of the process

The wetting and non-wetting phase relative permeability expressions for a cylindrical tube bundle model (TBM) are presented by Wang *et al*. [9]. The basic development of the interacting capillary bundle model is given [7]. A similar approach was used to develop the formulations for COUCSI in TBM. (The development procedure will not be presented here because of space limitations). Three governing equations can be developed for TBM considering PCF as below (The largest tube which contributes to imbibition is denoted by the subscript *I*, and *NT* is the total number of tubes):

$$x_1^2 = 2\left\{\frac{M_{w1}\left(P_{c1} - P_{c2}\right)}{A_1}\left(\frac{1}{1-f_2}\right)\right\} t \tag{2}$$

$$f_n^2 \left\{\frac{M_{w1}\left(P_{c1} - P_{c2}\right)}{A_1}\left(\frac{1}{1-f_2}\right)\right\} = \left\{\frac{1}{A_n}\left(M'_{wn}\left(P_{cn} - P_{c(n+1)}\right)\frac{f_n}{f_n - f_{n+1}} - M'_{w(n-1)}\left(P_{c(n-1)} - P_{cn}\right)\frac{f_n}{f_{n-1} - f_n}\right)\right\} \tag{3}$$

and

$$f_I^2\left\{\frac{M_{w1}\left(P_{c1}-P_{c2}\right)}{A_1}\left(\frac{1}{1-f_2}\right)\right\}=\left\{\frac{1}{A_I}\left(M'_{wI}\left(P_{cI}-P_{c0}\right)-M'_{w(I-1)}\left(P_{c(I-1)}-P_{cI}\right)\frac{f_I}{f_{I-1}-f_I}\right)\right\} \quad (4)$$

Here x is the spatial coordinate, and δ is the tube diameter, A is the cross sectional area, σ is the interfacial tension between two fluids, and θ is the contact angle. f_n is the ratio of the length of imbibed water in a tube over the length of water in the smallest tube (for smallest tube $n=1$). The (n) is the index for a tube. Moreover, the variables M_w and λ_w are defined as bellow:

$$M'_{wI}=\frac{\lambda_{wI}\,\lambda_{nwI}}{\lambda_{wI}+\lambda_{nwI}} \quad (5)$$

$$\lambda_{wI}=\sum_{i=1}^{I}\frac{\pi\,\delta_{(i)}^4}{128\,\mu_w} \quad (6)$$

$$\lambda_{nwI}=\sum_{i=I+1}^{NT}\frac{\pi\,\delta_{(i)}^4}{128\,\mu_{nw}} \quad (7)$$

During the process wetting phase enters the system from smaller tubes and the non-wetting phase exits the system through the larger tubes. If the behavior of the fronts in the tubes is self-similar, then the ratios of any two distances must be constant with time. It follows that f_n is constant with time. As a consequence, the equations are algebraic and non-linear, but susceptible to solution by iteration. Once the fractions have been determined, Equation 2 may be used to determine x_1 (and subsequently the remaining x_n).

Solution of the Developed Model

The analytical solution for the above formulation can be obtained by guessing f_2 and then by using equations 3 and 4 the f_n values for each tube can be calculated. Regarding the backward calculation of S_w at different distances which starts at the front, by assuming a value for f_2, we would obtain different wetting phase saturations at the imbibition face. Therefore, given a saturation at the imbibition face, the corresponding value of f_2 may be obtained by an iterative approach. The numerical simulation of the COUCSI in a real porous medium by using continuity and Darcy's equations are given in many papers and will not be presented here ([8]).

Investigation of the effect of boundary condition

As mentioned above, the open face boundary condition is still a question without a conclusive answer. Arabjamaloei and Shadizadeh (2010) proposed that the wetting phase saturation at the inlet is the one which minimizes the mobility variable. This condition leads to nonzero capillary pressure at the inlet. In this paper the effect of inlet water saturation on the process was investigated. Using the sample data in this paper the minimum value of mobility variable corresponds to a wetting phase saturation at the inlet of about 0.8.

CASE STUDY

In this section the tube bundle model is compared with numerical simulation. A tube bundle consisting of 101 tubes with uniform distribution of tube sizes was chosen such that the smallest tube diameter was $1\ \mu m$ and the largest was $10.1\ \mu m$ with a linear increase in the diameter with tube number. The rest of the sample data are given in Table 1. By using different values for f_2, the effect of inlet boundary condition was investigated. The simulator uses the Darcy law, is based on the finite volume method and is one-dimensional.

DISCUSSION OF RESULTS

Figure 3 represents the effect of the value of f_2 on the saturation distribution obtained with TBM. In this figure, S_w is the sum of the areas of tubes occupied by water divided by the sum of the areas of all the tubes. As it is observed in Fig. 3, the chosen value of f_2 has a major effect on saturation distribution and the inlet saturation. Figure 4 represents the results of numerical simulation of the sample of porous medium and the equivalent tube bundle model for the case of $f_2=0.993$. As expected, both results are in agreement. This shows that the developed tube bundle model is an exact analogy of Darcy's Law. An interesting feature of Figure 3 is that, by increasing the value of f_2 above to 0.9935 (the dashed line in the figure), a plateau or a constant saturation zone is predicted by the tube bundle model. This means that water would move in all tubes except the widest one. It is impossible to obtain a constant saturation zone in simulation of COUCSI when using numerical methods which are based on Darcy's law, because a constant saturation zone in the numerical simulation means the flow rate is zero. Such a restriction does not apply for flow in a tube bundle model.

In conclusion, we have two solution methods that agree with each other over a wide saturation range; the TBM provides additional model which can't be simulated using numerical simulation (*i.e.* the constant saturation zone). However, neither of the solution methods lead to any prediction of the saturation at the inlet face, and in fact they allow essentially 100% water saturation at the inlet. But 100% wetting phase saturation is not supported experimentally. This finding emphasizes that future progress and understanding of COUCSI will depend on determining what the wetting phase saturation at the boundary is and what controls it. This in turn calls for experimentation to determine this saturation condition.

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Table 1. Data for the case study

Parameter	***Value***	***Parameter***	***Value***
Permeability	*5.795 μm^2*	*Cross sectional area*	*0.01095 μm^2*
Oil viscosity	*1 cp*	*Water viscosity*	*1 cp*
Interfacial tension (σ)	*6 mN/m*	*Contact angle (θ)*	*0*
Porosity	*1*	*Sample length*	*50 cm*

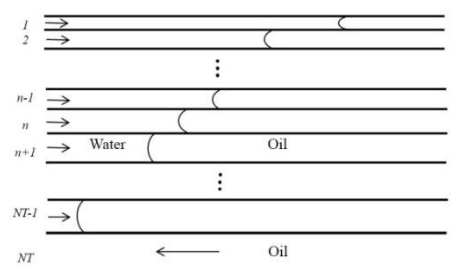

Figure 1. Schematic of Tube Bundle Model

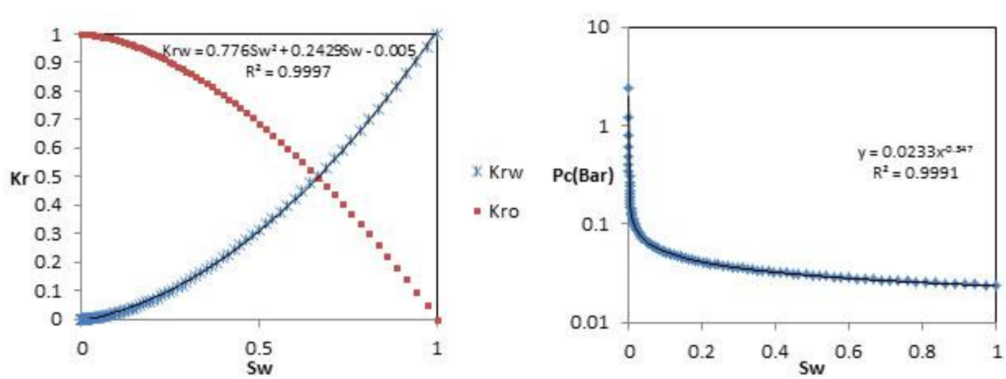

Figure 2. Relative permeability and capillary pressure functions for tube bundle model

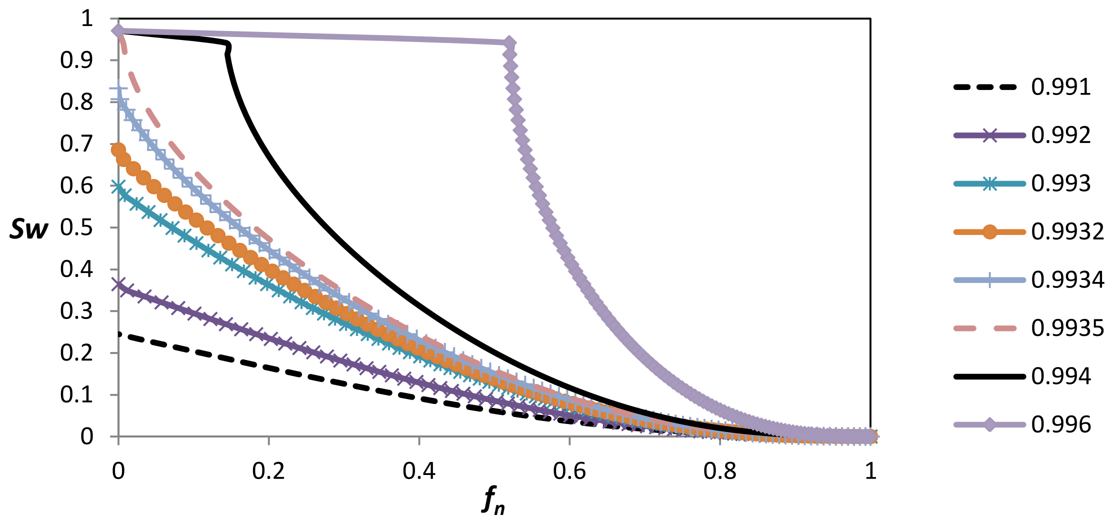

Figure 3. Water saturation distribution corresponding to different guess values for f_2 (S_w shows the average cross sectional wetting phase saturation and f_n is the ratio of x over x_n)

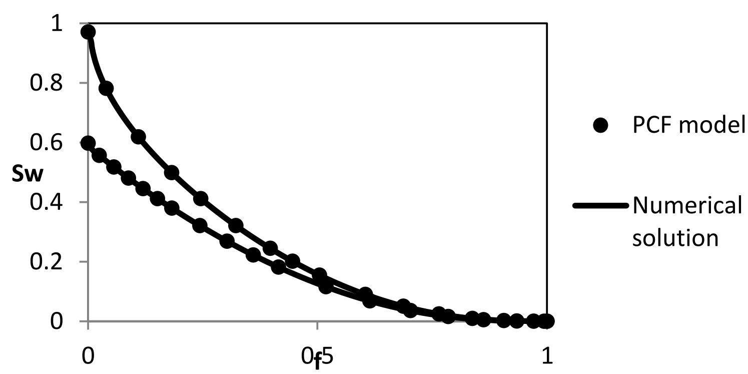

Figure 4. Comparison of water saturation distribution obtained by numerical simulation and tube bundle model

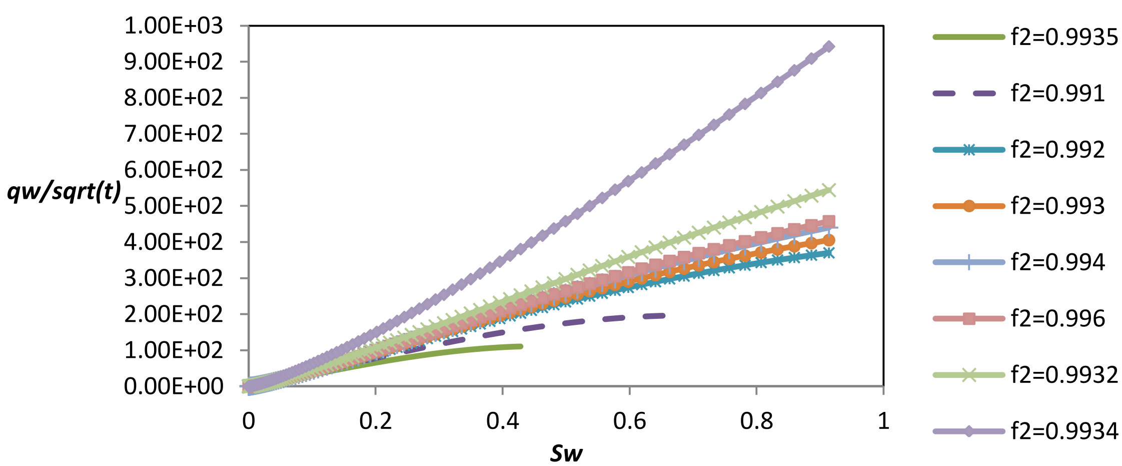

Figure 5. Wetting phase flow rate for the curves in figure 4