# EXPLICIT CORRELATION OF THE STEADY-STATE TWO-PHASE RELATIVE PERMEABILITY FUNCTIONS OF POROUS MEDIA WITH THE LOCAL FLOW RATES

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#### ABSTRACT

Steady-state two-phase flow experiments are performed on a sand column equipped with two differential pressure transducers and six ring electrodes to measure the pressure drop across each phase, and the electrical resistance across five successive segments of the column. The electrical resistance is converted to water saturation by Archie equation. The results are analyzed by considering the water saturation and oil / water relative permeability as power functions of water and oil capillary numbers which are regarded as independent variables. Results from earlier visualization studies on 2-D glass-etched pore networks are analyzed in a similar manner to correlate the estimated parameters of power functions with the 2-phase flow regimes. The new explicit relationships of relative permeabilities and water saturation with oil and water capillary numbers set the bases for a new conceptualization of the two-phase flow at reservoir-scale where the mobility of the fluids is decoupled from saturation and becomes a non-linear function of the local flow rates.

### **INTRODUCTION**

Two-phase flow of immiscible fluids in porous media has attracted the attention of many research groups during the last 30 years [1-6]. The steady-state two-phase flow is established by coinjecting two fluids through a porous medium at fixed flow rates, and has extensively be analyzed, primarily by computer simulations at pore- and network-scale [7-9], and secondarily by systematic experimental approaches in model porous media [1,2,5,6,10]. One of the main conclusions is that the wetting fluid (water) retains its connectivity along its flow path while the non-wetting fluid (oil) may move either as a connected pathway or as a population of disconnected ganglia which undergo dynamic breakup and coalescence [1,2,5,6-11]. Both the two-phase flow regime and the relative permeability functions of the two fluids depend strongly on a variety of key parameters

such as the capillary number, flow rate ratio, viscosity ratio, wettability, Bond number, etc [1,6,11]. However, when simulating the two-phase flow at field-scale, the relative permeability of wetting and non-wetting fluids are still regarded as power functions of the local saturation and their implicit dependence on other parameters, such as the local flow rates is still ignored.

#### **MATERIALS AND METHODS**

Steady-state experiments of the simultaneous flow of oil and water were performed on a long horizontal PVC column (D=5cm, L=30cm) packed with a well-sorted sand (k=25 D,  $\phi$ =0.42) and equipped with ring electrodes and a multi-point conductivity meter used to monitor the electrical conductance along the column (Fig.1, [4]). The brine was an aqueous solution of NaCl at concentration  $C_{NaCl} = 20$  g/L, while the oil was a mixture containing 61% n-C<sub>10</sub> and 39% n-C<sub>12</sub> to attain a viscosity ratio,  $\kappa = \mu_0/\mu_w = 1.0$ . First, the sand column was evacuated and filled with brine. Then, the oil was injected at a high flow rate ( $q_0=5$  ml/min) until attaining an irreducible wetting phase saturation. Afterwards, oil and brine, were coinjected at flow rates 5 ml/min and 0.5 ml/min, respectively (Fig.1). Steady-state conditions were established when the time-averaged values of oil and water injection pressures and electrical conductance were stabilized. Then, the oil flow rate was kept constant and the water flow rate varied stepwise from 0.5 to 5.0 ml/min in successive bumps. Afterwards, the oil flow rate was fixed to a lower value and water flow rate was changed stepwise from 5.0 to 0.5 ml/min, and so forth. The column-averaged steady-state relative permeability of the oil  $\langle k_{ro} \rangle$  and water  $\langle k_{rw} \rangle$ phase were calculated by using the Darcy law and the measured time-averaged pressure drops across the oil and water phase, respectively

$$q_w/A = \left(k\langle k_{rw}\rangle/\mu_w\right)\left(\Delta P_w/L\right) \qquad q_o/A = \left(k\langle k_{ro}\rangle/\mu_o\right)\left(\Delta P_o/L\right) \qquad (1)$$



Figure 1. Experimental setup of steady-state two-phase flow tests in soil columns

At the end of each steady-state step, the effluent was weighted and the total water saturation of the soil column,  $\langle S_w \rangle$ , was estimated. The measured resistivity index across the sand pack,  $I_R$ , data were commonly fitted to an Archie power law of the form

$$I_R = \langle S_w \rangle^{-n} \tag{2}$$

so that the saturation exponent, n, was estimated. This value of saturation exponent along with the values of resistivity index measured across each segment of the soil column,  $(I_R)_{ii}$ , were replaced in Eq.(2) to calculate the steady-state water saturation

averaged over each segment of the column,  $\langle S_w \rangle_{ii}$ .

Visualization studies have revealed that the relative permeability functions are correlated strongly with the dominant steady-state two-phase flow regime which is classified with respect to the distribution and mobility of non-wetting oleic phase [1,2]: (1) over the Connected Pathway Flow (CPF) regime, the oil retains its hydraulic continuity across the porous medium, whereas the oil relative permeability is relatively high and does not change sensibly with the flow rate of wetting fluid; (2) over the Ganglion Dynamics (GD) regime, the oil is disconnected into large ganglia (LGD), small ganglia (SGD), and drops of size less than that of a single pore (drop traffic flow, DTF), whereas the oil relative permeability is relatively by the water flow rates.

A new formulation is introduced where the water saturation,  $\langle S_w \rangle$ , and steady-state oil,  $\langle k_{ro} \rangle$ , and water,  $\langle k_{rw} \rangle$ , relative permeability, all averaged over the soil column, can be regarded as functions of two independent variables: the water,  $Ca_w$ , and oil,  $Ca_o$ , capillary numbers defined by

$$Ca_{w} = \mu_{w}q_{w}/(A\gamma_{ow}) \qquad \qquad Ca_{o} = \mu_{o}q_{o}/(A\gamma_{ow}) \qquad (3)$$

where  $\mu_i$ =viscosity of phase i (i=w,o),  $q_i$ =flow rate of phase i (i=w,o)),  $\gamma_{ow}$ =interfacial tension, A=cross-sectional area of porous medium. The dependence of  $\langle k_{rw} \rangle$ ,  $\langle k_{ro} \rangle$ ,  $\langle S_w \rangle$  on  $Ca_w$ ,  $Ca_o$ , is described by power laws of the form

$$\langle k_{rw} \rangle = a_w C a_w^{b_w} C a_o^{b_o} \qquad \langle k_{ro} \rangle = a_o C a_w^{e_w} C a_o^{e_o} \qquad (4)$$

$$\langle S_{w} \rangle = a_{s} C a_{w}^{c_{w}} C a_{o}^{c_{o}} \tag{5}$$

The experimental results were fitted to Eqs. (4) and (5) to estimate all unknown parameters by using the Athena Visual Studio [4,10]. The same equations were used to fit earlier datasets of steady-state two-phase flow experiments performed on a 2-D glass-etched pore network for two values of the capillary number,  $\kappa = \mu_o/\mu_w = 1.45$ , 3.35 [1].

#### **RESULTS AND DISCUSSION**

From the fitting of  $I_R - \langle S_w \rangle$  datasets to Archie equation, Eq.(2), the saturation exponent  $n = 1.06 \pm 0.0041$  [12] was obtained and used to convert the electrical resistivity to water saturation. We can assume that water saturation of each segment changes independently with each capillary number and follows power laws with unaltered exponents, namely

$$\langle S_w \rangle_{ij} = (a_s)_{ij} C a_w^{c_w} C a_o^{c_o}$$
(6)

where the coefficients  $(a_s)_{ij}$  are governed by all other pertinent parameters (e.g. pore space morphology, wettability, etc) and may differ among the various segments due to the different boundary conditions. The column-averaged coefficient,  $a_s$ , is expected to be equal to the arithmetic mean of local coefficients,  $(a_s)_{ij}$ , namely

$$a_s = \sum_{ij} \xi_{ij} (a_s)_{ij} \tag{7}$$



Figure 2. Soil column-averaged water saturation and water / oil relative permeability as functions of  $Ca_w$  at various values of  $Ca_o$  ( $\kappa$ =1.0).

The water saturation datasets were fitted simultaneously to Eq.(6) to estimate the parameters  $c_w, c_o, (a_s)_{ij}$  ij = 12,23,34,45,56 and calculate  $a_s$  through Eq.(7). On the other hand, the datasets of 2-D experiments [1] were fitted to Eq.(5) to estimate the parameters  $c_w, c_o, a_s$  (Fig.2). Due to the boundary effects, the spatial distribution of fluids may change over the various soil column segments and this is reflected in different relative permeability functions expressed by

$$\langle k_{rw} \rangle_{ij} = (a_w)_{ij} C a_w^{b_w} C a_o^{b_o} \qquad \langle k_{ro} \rangle_{ij} = (a_o)_{ij} C a_w^{e_w} C a_o^{e_o} \qquad (8)$$

The relative permeability of each phase is proportional to its hydraulic conductivity and inversely proportional to its hydraulic resistance. The soil column can be regarded as resistors (segments) in series and therefore the column-averaged relative permeability is given by relationships of the form

$$1/\langle k_{rw}\rangle = \sum_{ij} \xi_{ij} / \langle k_{rw}\rangle_{ij} \qquad 1/\langle k_{ro}\rangle = \sum_{ij} \xi_{ij} / \langle k_{ro}\rangle_{ij} \qquad (9)$$

which are equivalently written as  $1/a_w = \sum_{ij} \xi_{ij} / (a_w)_{ij}$ 

$$1/a_{o} = \sum_{ij} \xi_{ij} / (a_{o})_{ij}$$
(10)



Figure 3. 2D network-averaged water saturation and water / oil relative permeability as functions of  $Ca_o$  at various values of  $Ca_w$  for oil to water viscosity ratio  $\kappa$ =1.45.

According to Eqs.(9) and (10), the column-averaged oil/water relative permeability and coefficients  $a_w, a_o$  are the geometric means of the segment-averaged relative permeabilities and local coefficients,  $(a_w)_{ii}$ ,  $(a_a)_{ii}$ . The oil and water relative permeability datasets were fitted to Eqs. (4) and the results are shown in Fig.2. The data resulting from the fitting of earlier datasets of 2-D glass-etched pore networks to Eqs.(4) are shown in Fig.3. The water relative permeability is more sensitive to  $Ca_w$  and less sensitive to  $Ca_{a}$  (Figs.2,3). As  $Ca_{w}$  increases, water starts occupying a gradually increasing number of non-interrupting and pore network spanning paths that enhance substantially the water hydraulic conductivity or equivalently the value of  $\langle k_{rw} \rangle$  (Fig.2). The viscous coupling between oil and water is reflected in the weak but negative effect of  $Ca_w$  on  $\langle k_{ro} \rangle$  (Fig.2). The oil flow regime is dominated by CPF and GD, and the decrease of  $\langle k_m \rangle$  is due to the decrease of the oil flow fraction transferred by GD. Likewise, the oil relative permeability is more sensitive to  $Ca_o$  and less sensitive to  $Ca_w$ (Figs.2,3). As Ca<sub>o</sub> increases, a higher number of network spanning clusters are occupied by connected pathway flows of oil with result that  $\langle k_{ro} \rangle$  increases substantially (Fig.3). On the other hand,  $\langle k_{rw} \rangle$  is affected weakly by  $Ca_{a}$  because the water, as wetting fluid, can surround oil ganglia in the form of thin films or pendular rings and retain its hydraulic continuity even over high  $Ca_o$  values.

#### CONCLUSION

It becomes evident that, for a given porous medium, the steady-state relative permeability functions change with the capillary numbers. Therefore, when simulating transient problems of two-phase flow at the macroscopic scale of a homogeneous reservoir, the local relative permeability of each fluid should be regarded a function of the local flow rates of both fluids rather than a function of the local saturation. In this manner, the enhanced non-linearity of the problem is expected to affect substantially the spatialtemporal evolution of the displacement.

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