DETERMINATION OF TWO PHASE RELATIVE PERMEABILITY FROM CORE FLOODS WITH CONSTANT PRESSURE BOUNDARIES

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This paper was prepared for presentation at the International Symposium of the Society of Core Analysts held in Avignon, France, 8-11 September, 2014

ABSTRACT

Relative permeability is one of the key SCAL measurements in reservoir engineering. A precise method is presented in this paper to interpret displacement experiments of two immiscible phases and to determine relative permeabilities dynamically using constant pressure boundaries. This method is based on a novel generalization of the classical Buckley–Leverett fractional flow theory for constant pressure boundary conditions. Under constant pressure boundaries, flow rate is determined analytically as a function of time, as well as the pressure along the length of one-dimensional core. Based on the rigorous analytical results, the relative permeability can be calculated directly from the experimental data gathered in general unsteady-state methods with constant pressure boundaries. It is demonstrated that the new method does not contain errors associated with total flow rate not being constant, as opposed to previous methods. In fact, there is significant difference between oil relative permeability interpreted by a previous method and the new method.

INTRODUCTION

Relative permeability is one of the key reservoir parameters to predict multiphase fluid flow and saturation distributions during immiscible flooding. Core flooding experiments represent the main approach to determine the relative permeabilities in immiscible flooding. The interpretation of these displacement experiments is based on the fractional flow theory, developed by Buckley and Leverett (1941) and Welge (1952). The fractional flow theory assumes that the total flow rate is constant, that the core sample is homogeneous, that the flow is immiscible and incompressible in one dimension, and that dispersion effects and capillary pressure can be ignored. Hence, core flooding experiments applying fractional flow theory traditionally use a constant injection rate and therefore constant production rate with no mass accumulation in the core.

There are two general methods to determine relative permeability. Steady-state methods aim to achieve the steady-state flow at different fractional flow ratios yielding unique core saturation at each ratio. The results are easy to interpret; however, it takes a long time to achieve steady-state conditions. In traditional unsteady-state methods, the core saturated with oil is flooded by water or gas at constant total rate until no more oil is produced. During the flooding experiments, the fractional flow ratio is recorded, as well as the pressure at both ends and the breakthrough time of the injected fluid. The two-phase relative permeability, as a function of saturation at the effluent end of the core, can then be determined based on the fractional flow theory, shown in detail by Marle (1981). The unsteady-state method is time efficient. However, the accuracy depends on the interpretation methods. As the fractional flow theory assumes a constant flow rate, the experiments are performed using constant injection rates. The general procedures of the unsteady-state displacement experiments can be found in Donaldson and Djebber (2004). One of the most popular unsteady-state methods is the Johnson, Bossler, and Naumann methods by Johnson *et al*. (1959). Jones and Roszelle (1978) presented graphical techniques to determine the point value of saturation and pressure and then to determine the relative permeabilities. These graphical techniques are equivalent to the fractional flow equations but can reduce the errors caused by the evaluation of derivatives. Toth *et al*. (2002) presented correlation formulae to determine the relative permeability using results from unsteady-state experimental methods.

The fractional flow model can be formulated using the material balance equation of the displacing phase (w), in the form of the equation:

$$\phi \frac{\partial S_w}{\partial t} + u_T(t)\frac{\partial f}{\partial x} = 0, \quad (1)$$

where S_w is the saturation of the displacing phase (w) and f is the fractional flow function of phase (w), defined by

$$f(S_w) = \frac{\lambda_w}{\lambda_w + \lambda_o}; \quad \lambda_w = \frac{Kk_{rw}}{\mu_w}; \quad \lambda_o = \frac{Kk_{ro}}{\mu_o}. \quad (2)$$

In the fractional flow theory by Buckley and Leverett (1941) and Welge (1952), the total volumetric flux u_T is a given constant. Considering the constant pressure boundaries

$$p(0,t) = p_{in}\,;\; p(L,t) = p_{out}, \quad (3)$$

the total volumetric flux u_T, is not a constant but a function of time, where p_{in} and p_{out} are the inlet and outlet pressure, respectively, and L is the length of the media. Furthermore, the initial saturation and injection saturation are assumed to be constant,

$$S_w(0,t) = 1 - S_{or}; S_w(x,0) = S_{wc}, \quad (4)$$

where S_{or} is the residual saturation of the phase (o) and S_{wc} is the irreducible saturation of the phase (w). Equation (1) is solved under the boundary and initial conditions described in Eq. (3) and (4).

Recently, an analytical solution to the fractional flow theory with constant pressure boundaries was presented by Johansen, James (2014). This solution provides a solid analytical foundation to analyze the immiscible fluids displacement experiments with constant pressure boundaries. The interpretation methods will be presented in this paper. Under constant pressure boundaries, flow rate will vary and can be determined analytically as a function of time, as well as the pressure along the length of a one-dimensional core. Based on the rigorous analytical results, the relative permeability can be calculated directly from the experimental data gathered during unsteady-state core flooding. The experimental procedures with constant pressure boundary are almost the same as with constant flow rates with the exception of the boundary conditions. The cumulative production of the two phase fluids, core properties and inlet/outlet pressure are recorded throughout the experiments. Gravity and capillary end effects can generally be ignored when the flow rates are sufficiently high, as is the general assumption for most steady-state and unsteady-state methods. The interpretation of the experimental results and calculation of the relative permeability is provided in this paper.

This interpretation method for constant pressure boundary conditions is an important supplement to the existing method for constant flow rates. It is necessary to perform core flooding experiments under constant pressure boundaries in the laboratory since these conditions are common in field practices. Furthermore, if the displacing phase is gas, constant pressure boundary experiments are easier to conduct than constant flow rate experiments in the laboratory environment.

RELATIVE PERMEABILITY DETERMINATION

During a displacement experiment, for example water flooding, the inlet and outlet pressure are kept constant. The core is first saturated with oil and then flooded with water until no more oil is produced at the outlet end of the core. The connate water saturation S_{wc} and irreducible oil saturation S_{or} can be determined from the experiment at conditions when no more oil can be saturated and when no more oil can be produced, respectively. Both conditions approach steady state. The end points of the relative permeabilities curve, $k_{ro}(S_{wc})$ and $k_{rw}(1 - S_{or})$, can be determined by Darcy's law correspondingly.

The relative permeabilities can only be calculated at saturations larger than the frontal water saturation ($S > S^*$, S^* is the frontal water saturation) from dynamic flooding experiments. To obtain values for low water saturation, a drainage experiment must be conducted, i.e. displacement of water by oil. However, the values obtained from this are hampered by hysteresis effects. During the experiments, the frontal breakthrough time t_{BT} is recorded and a series of points in time t_S after water breakthrough is chosen. At this series of time points ($t_{BT} < t_{S_1} < \cdots < t_{S_n}$), a series of corresponding saturation values ($S^* < S_1 < \cdots < S_n$, $i = 1,2, \ldots n$) arrives at the outlet of the core and the injection and production flow rates of each phase are recorded.

The relative permeabilities at saturation S can be calculated by Darcy's Law at time of frontal breakthrough as:

$$k_{ro}(S) = -\frac{u_T(t_{BT})\left[1 - f_w(S)\right]\mu_o}{K\dfrac{\partial p}{\partial x}}; k_{rw}(S) = -\frac{u_T(t_{BT})\, f_w(S)\,\mu_w}{K\dfrac{\partial p}{\partial x}}. \quad (5)$$

Here, the total velocity u_T at time t_{BT} can be read and the viscosity of oil and water and the absolute permeability are known directly from experimental measurements. Given all the measurements, three steps are needed to calculate a series of relative permeabilities at different saturations. One set of data from the series of measurements ($S = S_i;\ t = t_i$) is shown below. The first step is to calculate the water fractional functions $f_w(S)$. The produced fluids, oil and water in this case, will be separated and the water fraction $f_w(S)$ is then calculated as the volume ratio of water production and total production. The second step is to calculate the saturation S, which arrives at the end of the core at time t_S. Based on mass balance equation we have

$$S = \bar{S} - \frac{1 - f_w(S)}{\phi L} \Psi(t_S). \tag{6}$$

Here, $\Psi(t) = \int_0^t u_T(t)dt$ and $\bar{S}$ is the average water saturation in the core, which can be determined from the mass balance since $\bar{S} = S_{wc} + N_p/V_p$; where N_p and V_p are the volume of oil produced and pore volume, respectively.

The third step is to determine the pressure profile along the core in order to calculate the pressure gradient $\frac{\partial p}{\partial x}$ for saturation S. It is derived step by step based on solutions given in Johansen, James (2014). First, the position of any saturation behind the front saturation can be determined at time of frontal breakthrough (Johansen, James, 2014):

$$x(S, t_{BT})^2 + \frac{2u_T(t_S)(t_S - t_{BT})L}{\Psi(t_{BT})} x(S, t_{BT}) - L^2 = 0. \tag{7}$$

Knowing the position of saturation S at time t_{BT}, we then can determine $f'(S)$ following Johansen, James (2014), which gives

$$f'(S) = \frac{x(S\ ; t_{BT})\phi}{\Psi(t_{BT})}. \tag{8}$$

The error associated with numerical differentiation to get $f'(S)$ is avoided since Eq. (8) is exact. Next, applying the solution to pressure distribution (Johansen, James, 2014) at time t_{BT} results in

$$p(x(S, t_{BT})) = p_{in} - \frac{\Delta p\, f'(S)\Psi(t_{BT})}{\phi L}, \tag{9}$$

where Δp is the pressure difference between the inlet and outlet of the core. Finally, the pressure gradient $\frac{\partial p}{\partial x}$ can be calculated using numerical differentiation given the pressure profile in Eq. (9). Therefore, the unknowns in Eq. (5) are all calculated through the three steps and the relative permeabilities can then be easily calculated.

CALCULATION EXAMPLE

In order to verify the applicability of the proposed method, data from a water flooding experiment under constant pressure boundaries, given by Jones and Roszelle (1987), has been re-interpreted by the proposed method. The experimental conditions, fluid and rock properties are reproduced in Table 1 and the experimental data is shown in Table 2.

Following the calculation procedure outlined above results in saturation profiles and the pressure profile at time of water breakthrough. Then the relative permeabilities are calculated based on Eq. (5) and plotted in Fig. 1 along with the results by Jones and Roszelle (1987). The relative permeability determined by the new method shows a favorable agreement to that by Jones and Roszelle. The water relative permeability is almost the same for both methods. The oil relative permeability interpreted by the new method is larger than that by Jones and Roszelle (1987). The Jones and Roszelle method uses classical fractional flow theory with the constant flux assumption but the experiment is performed under constant pressure boundaries, introducing inaccuracy. Both methods result in different frontal saturations. The new interpretation uses the same mass balance as the JBN method, see Eq. (6), whereas Jones and Roszelle use a graphical technique to determine the frontal saturation.

Future work will compare steady-state results to verify the accuracy of both interpretation methods. Simulation methods under constant flux and constant pressure boundaries will be compared to core flooding results with in-situ saturation monitoring (as applicable) for verification. Since the core sample is heterogeneous in real experiments, effects of heterogeneity on the calculation results should be investigated.

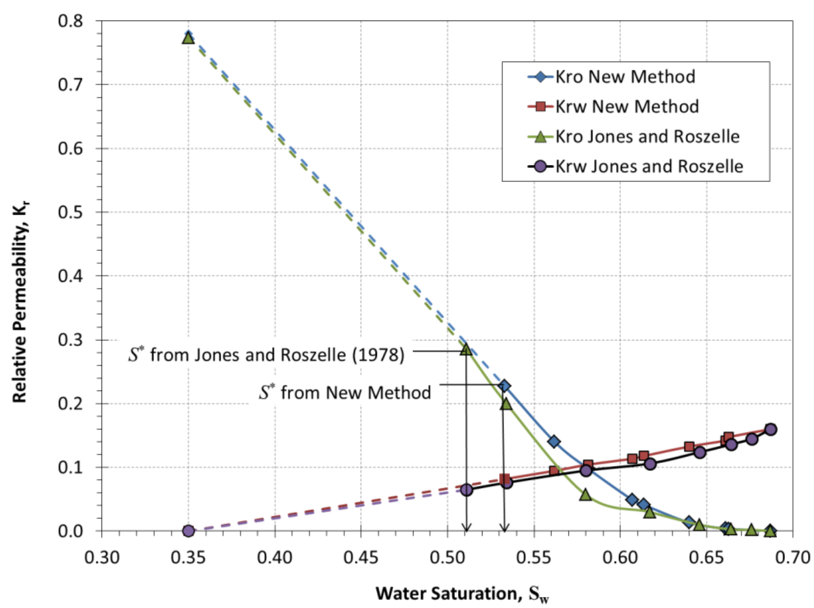

Fig. 1. Comparison of Relative Permeabilities using the New Method and Jones and Roszelle

CONCLUSION

A precise interpretation method is presented in this paper to determine the relative permeability from two immiscible phase displacement experiments. The newly developed fractional flow theory with constant pressure boundary established solid theory base for this interpretation method. The analytical nature of the new method promises the accuracy of the new interpretation methods. The calculation example demonstrates a straightforward and convenient calculation procedure. Further experimental investigations on the new interpretation method are needed as part of the future work.

TABLE 1-BASIC PARAMETERS USED IN EXPERIMENTS

Parameter	Symbol	Value (SI unit)
Pressure difference	ΔP	6.89×10^5 pa
Connate water saturation	S_{wc}	0.35
Oil viscosity	μ_o	1.045×10^{-2} $Pa \cdot s$
Water viscosity	μ_w	0.97×10^{-3} $Pa \cdot s$
Core sample length	L	0.127 m
Core sample diameter	D	0.0381 m
Core sample porosity	ϕ	0.215
Absolute permeability	K	3.54×10^{-14} m^2

TABLE 2- DATA FROM CONSTANT-PRESSURE WATERFLOOD DISPLACEMENT

Elapsed Time (minutes)	W_i (ml)	N_p (ml)	$Qt = W_i/V_p$
0.00	0.00	0.00	0
2.00	3.09	3.09	0.099
6.20	7.00	7.00	0.225
9.00	10.90	7.80	0.35
12.00	15.28	8.33	0.491
15.00	19.89	8.70	0.639
20.00	27.90	9.01	0.896
26.00	37.80	9.32	1.214
60.00	99.50	9.90	3.196
100.00	176.80	10.09	5.679
150.00	276.90	10.31	8.895

ACKOWLEDGEMENT

The authors thank the Hibernia Management and Development Company (HMDC), Chevron Canada, the Natural Sciences and Engineering Research Council of Canada (NSERC), and the Research and Development Corporation (RDC) for the support without which this work could not have been performed.

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