INVESTIGATION OF THE PHYSICS OF INERTIAL EFFECTS AND THE FORCHHEIMER EQUATION UTILIZING THE LATTICE BOLTZMANN METHOD

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ABSTRACT

The Darcy law provides the basic linear equation for fluid flow in porous media in an ideal, simplified condition. The Darcy's equation has been shown to be valid in flow processes happening in sufficiently low Reynolds number regimes. At higher Reynolds numbers, the inertial effect causes extra pressure drop and a decrease in the apparent permeability of the porous media. The Forchheimer's equation is a semi-empirical relationship which accounts for the inertial effects on the flow characteristics.

In this research, high Reynolds number flow through two-dimensional pore-throat combinations were simulated by the Lattice Boltzmann method. The effect of the geometry on the extent of the inertial effects was studied based on the outputs of the simulation. The validity and sensitivity of the Forchheimer's correlation was tested in this work.

1. INTRODUCTION

Permeability is defined as the capability of a porous medium to pass a single phase, single component flow. For a natural porous medium such as a natural hydrocarbon reservoir, the porous medium consists of a wide range size of pores and the connecting pore throats.

The first basic mathematical model to study fluid flow in porous media is the empirical relationship known as the Darcy's law. This equation relates the fluid viscosity μ, the rock permeability k, the flow area A, the sample length L and the piezometric pressure difference $\Delta\Phi$ of the fluid flow rate through the sample Q as

$$Q = \frac{kA}{\mu}\frac{\Delta\Phi}{L}, \qquad \Delta\Phi = \Delta P + \rho g \Delta z \tag{1}$$

At higher Reynolds numbers, the relationship between the pressure gradient and flow velocity becomes non-linear. Forchheimer (1901) was one of the first people to provide a quadratic empirical correlation for the velocity and pressure gradient relationship

$$-\frac{dP}{dx} = \frac{\mu u}{k_F} + \beta \rho u^2 \tag{2}$$

where β is termed the Forchheimer coefficient, u is the average velocity of the fluid, k_F is the Forchheimer permeability and ρ is the fluid density. There has been some research works that propose a cubic relationship for the velocity and pressure gradient (Mei and Auriault 1991; Coulaud et al. 1998; Balhoff and Wheeler 2009). However, the range of the applicability of the cubic relationship is not wide.

Ruth and Ma (1992) proposed an alternative form of representing the inertial effects on the permeability as

$$\frac{1}{k} = \frac{1}{k_o}\left(1 + \frac{\beta k_o \rho u}{\mu}\right) \tag{3}$$

where k is the permeability, and k_o is the apparent permeability. The Forchheimer coefficient β is measured experimentally for each type of fluid and porous media by multi-rate flow tests and there is no generally accepted theory to predict its value. However, there are empirical correlations relating the Forchheimer coefficient to permeability and porosity.

Considering a porous medium to be a bead pack, Ergun (1952) derived a correlation for the Forchheimer coefficient as

$$\beta = ab^{-0.5}(10^{-8}k)^{-0.5}\emptyset^{-3/2} \tag{4}$$

where a and b are constants depending on the porous structure surface, and ϕ is the porosity. MacDonald et al. (1979) modified Ergun's correlation and defined ranges for a and b. There are also some other correlations obtained for natural porous media. Table 4 presents a few of the correlations found in the literature.

Flow test experiments on regular shaped sphere packs have shown the applicability range of the Forchheimer equation (Dybbs and Edwards 1984; Fand et al. 1987). This ranges differ for each type of the packings.

2. MODEL ANALYSIS AND RESULTS

Natural porous media consists of a wide range of pores and pore throats of different sizes and shapes. In natural porous media the pores could be connected to any number of throats. In this study a simple circle in 2D is chosen to represent the pore and channels are assumed to represent the pore throats. Figure 1 shows the schematic of the simplified pore.

The purpose of this study is to investigate the onset of the non-Darcy flow and the pressure loss and velocity relationship in the pore-throat combination by simulating the hydrodynamics of flow in pore bodies by the Lattice Boltzmann method. The Lattice Boltzmann method is a popular fluid dynamics simulation method. For a detailed explanation of the method, the paper by Arabjamaloei & Ruth (2016) is suggested.

It is known that the sudden change in the flow path width causes a shift of the laminar flow from the Darcy regime (creeping flow) to non-Darcy regime (Forchheimer or turbulent). Turbulent flow happens at very large velocity and rarely innatural porous media flow. The dimensionless Reynolds number (*Re*) is typically the main indicator of the onset of the flow regime change. Studies on the onset of non-Darcy flow in sphere packs have shown that assigning a unique *Re* onset for all types of porous media is not possible due to the sensitivity of the fluid flow to the geometrical properties of the porous medium (Hassanizadeh and Gray 1987).

Reforming equation 2, for gravity driven flow, by inserting the kinematic viscosity v instead of the dynamic viscosity μ, results in another form of the Forchheimer equation as

$$g = \frac{vu}{k_F} + \beta u^2 \tag{5}$$

The flow processes in all the pore-channel combinations at varying gravity force was simulated. The plot of gravity (g) and the average exit velocity (u) for 4 different pore-throat combinations is provided by figure 2. As it is seen in figure 2, a second order Forchheimer type polynomial fits the data well but the trend of the velocity profile is more like a third order polynomial with an obvious critical point that could be related to the critical *Re*. The velocity and gravity force in lattice Boltzmann units show a well behaved cubic relationship for each combination. However, the effectiveness of this relationship depends on its applicability to the whole range of aspect ratio combinations. To investigate this issue, the plot of the velocity and gravity for 10 different pore-channel combinations with varying aspect ratio was produced, as shown in figure 3. The mass flow rate is also a characteristic of the flow process and the inertial effects extent. The permeability for all the different cases was scaled by dividing the calculated permeability to the absolute permeability for all the data 10 combinations. The absolute permeability was calculated at vanishing Reynolds number. Scaled permeability (*Ks*) was plotted versus the mass flow rate in figure 4. Comparing figures 3 and four illustrates that the mass flow rate and scaled permeability provide a well behaved relationship for the whole ranges of pore-throat geometries while the velocity and head loss relationship foe all the geometries is not correlating well.

4. DISSCUSSIONS AND CONCLUSIONS

- As it is seen in figure 3, neither third order nor second order polynomial can perfectly predict the velocity and gravity relationship. This indicated the weakness of the Forchheimer type equation for the pore network combinations.
- A second order polynomial precisely predicts the permeability change as a function of mass flow rate due to the inertial effects in the simplified geometries used for this research.
- The Forchheimer equation seems to work for some pore-throat combinations, however it doesn’t work well for all size combinations.
- The third order form of the Forchheimer equation works better than the second-order form in the two dimensional models studied (Figure 2).

ACKNOWLEGEMENTS

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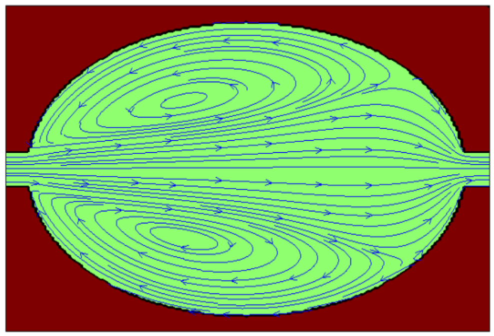

Figure 1: Schematic of the velocity streamlines in simple pore (the green circle) and two connecting channels (the green channels) surrounded by solid impermeable medium (the red color)

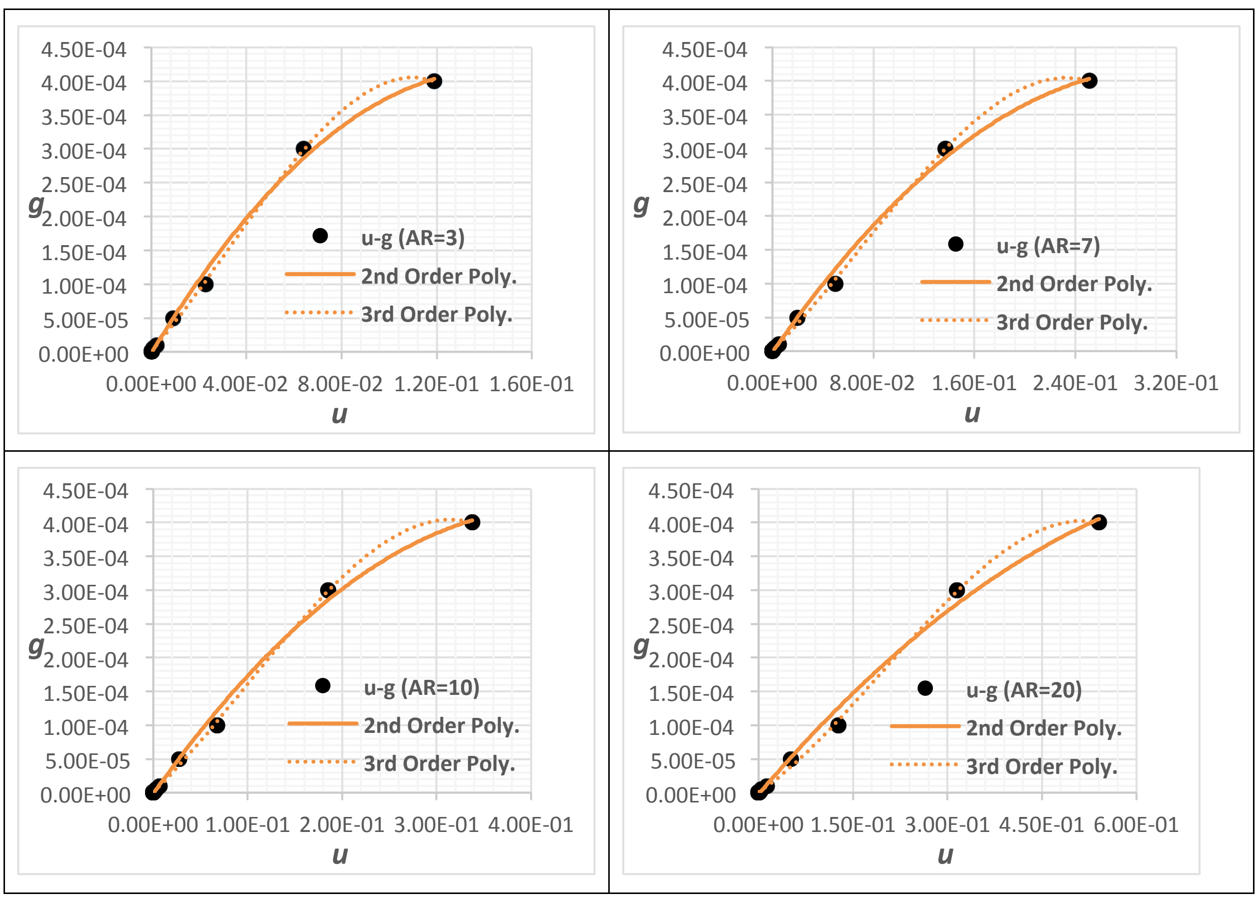

Figure 2: Average exit velocity (u) versus dimensionless gravity acceleration and two orders of polynomial fits for 4different pore-throat combinations. Data points show the simulation results and the line shows the fitted polynomial.

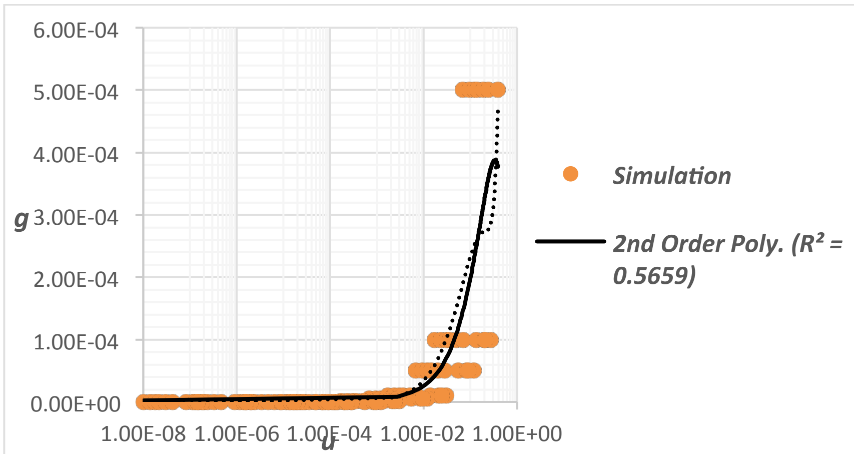

Figure 3: Plot of average exit velocity (u) versus dimensionless gravity (g).

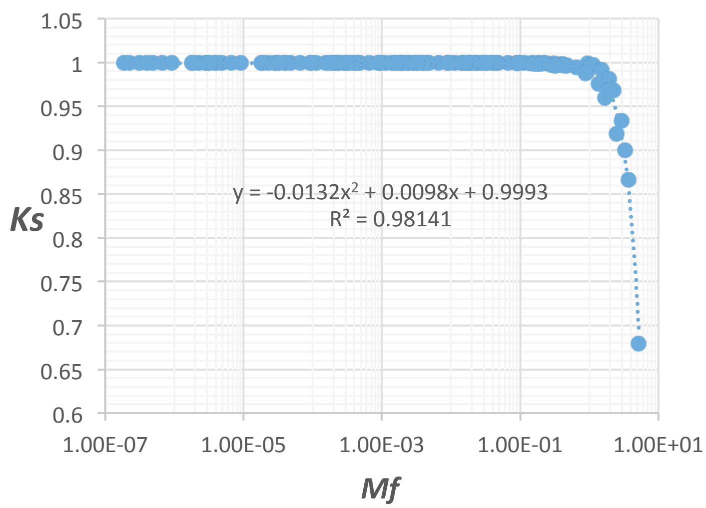

Figure 4: Scaled permeability (Ks) and mass flow rate (Mf) relationship for 10 different combinations of pore and throat with different aspect ratios and the polynomial fitting all the data points