

A NOVEL RELATIVE PERMEABILITY MODEL - A TWO-FLUID APPROACH ACCOUNTING FOR SOLID-FLUID AND FLUID-FLUID INTERACTIONS

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ABSTRACT

A novel model is presented for estimating relative permeabilities (RPs). It is derived based on a two-fluid Stokes formulation and friction terms due to phase interaction at the fluid-fluid and fluid-rock interfaces. The model gives consistent relationships between solid-fluid interaction terms, interstitial fluid velocities and corresponding RPs all vs. water saturation. Analysis of steady-state co-current flow predicts that fluid-fluid interaction gives a physical origin for RPs being dependent on viscosity ratio: A viscosity contrast can enhance the difference between the interstitial velocities and thus affect the drag exerted between the phases. The slower phase is accelerated as reflected in increased RP, and vice versa. Analysis of steady-state counter-current flow shows that counter-current RPs are lower than corresponding co-current curves for all intermediate water saturation values when fluid-fluid interaction is present. The model seems to capture essential features of two-phase flow in porous media. It looks like an attractive tool for analysis, understanding and interpretation of two-phase RP data in terms of the physical processes involved, i.e. solid-fluid and fluid-fluid interactions. Validation of the model should be pursued using high quality measurements where interaction terms and experimental artifacts, such as capillary end effects, are adequately quantified by simulation.

INTRODUCTION

The simultaneous flow of two immiscible phases (in this work assumed to be oil (*o*) and water (*w*)) through a porous medium is commonly described by the Darcy velocity for each phase v_w, v_o [m/s] as:

$$v_o = -\lambda_o \frac{dP_o}{dx}, \quad v_w = -\lambda_w \frac{dP_w}{dx}, \quad \lambda_o = \frac{kk_{ro}}{\mu_o}, \quad \lambda_w = \frac{kk_{rw}}{\mu_w}, \quad (1)$$

where k_{ri} , [-], P_i [N/m²] and λ_i [N s/m⁴] ($i=o,w$) are relative permeabilities (RPs), phase pressures and phase mobilities. k [m²] is the permeability. Hence, the flow of oil and water is assumed to take place in separate channels and no interaction between the two fluid phases is accounted for through fluid-fluid interaction terms. Experimental observations and theoretical analysis have, however, indicated that fluid-fluid interactions could be significant and important for the understanding of multi-phase flow in porous media [1, 2, 5, 7, 10]. We derive a novel RP formulation emphasizing this interaction.

THEORY

In the following we assume the oil and water saturations, s_o and s_w , are normalized between their residual values, i.e. from 0 to 1. General momentum equations (ignoring acceleration and external body forces) for oil and water are given by [4, 9]:

$$0 = \nabla \cdot (s_o \sigma_o) + P_o \nabla s_o + F_{ow} + M_{om}, \quad (2)$$

$$0 = \nabla \cdot (s_w \sigma_w) + P_w \nabla s_w - F_{ow} + M_{wm}. \quad (3)$$

F_{ow} [N/m³] denotes the drag that the water phase exerts on the oil. The oil exerts an equal and opposite drag $-F_{ow}$ on the water phase. Similarly, M_{om} and M_{wm} [N/m³] represent interaction (drag) between fluid and pore walls (solid matrix), respectively, for oil and water. The terms $P_o \nabla s_o$ and $P_w \nabla s_w$ arise from an averaging process, see [3, 4] for details.

The stress term σ_i [N/m²] is conventionally given as the sum of the stationary isotropic stress associated with the hydrostatic pressure P_i [N/m²] and the viscous stress tensor τ_i [N/m²] such that $\sigma_i = -P_i \delta + \tau_i$ (δ is the identity matrix). The viscous stress τ_i is neglected in the following since our main purpose is to gain insight into the role of the interaction terms F_{ow} , M_{om} and M_{wm} . They are modelled as follows (see also [5, 10]):

$$F_{ow} = \hat{k}(u_w - u_o), \quad M_{im} = -\hat{k}_i u_i, \quad (i = o, w) \quad (4)$$

where the terms \hat{k} , \hat{k}_i [N s/m⁴] remain to be determined. Hence, the momentum equation for horizontal, collinear flow of two phases on the macroscopic scale restricted to a one-dimensional setting can be formulated as:

$$s_o P_{ox} = -\hat{k}_o u_o - \hat{k}(u_o - u_w), \quad (5)$$

$$s_w P_{wx} = -\hat{k}_w u_w + \hat{k}(u_o - u_w), \quad (6)$$

P_{ox} and P_{wx} are partial derivatives of oil and water phase pressures with respect to space coordinate x . The effect of a $\hat{k} > 0$ is that the faster moving fluid phase will be slowed down and vice versa for the slower moving phase due to transfer of momentum. Non-coupled flow equations are regained for $\hat{k} = 0$. The interstitial velocities u_o , u_w are related to the Darcy phase velocities v_o , v_w by:

$$v_o = s_o u_o \phi, \quad v_w = s_w u_w \phi, \quad (7)$$

where ϕ is the fractional porosity. It is furthermore quite obvious that the solid-fluid friction terms \hat{k}_o , \hat{k}_w should respect the relation $\hat{k}_i \propto \mu_i / k$ ($i = o, w$), to be consistent with standard porous media flow modeling. We assume the following form:

$$\hat{k}_o = I_o s_o^\beta \frac{\mu_o}{k} \phi, \quad \hat{k}_w = I_w s_w^\alpha \frac{\mu_w}{k} \phi, \quad \hat{k} = I s_w s_o \frac{\mu_w \mu_o}{k}. \quad (8)$$

I_o , I_w [-] and I [m²/N s] are interaction term coefficients. Note that the friction terms are related to the interfacial areas [6], which here are assumed to be related to the saturations, expressed implicitly through (8). For co-current flow, we assume equal phase pressure gradients and no capillary pressure effects and obtain effective k_{ri} from solving (1)+(5-8):

$$k_{rw} = \frac{I_o s_w s_o^{\beta-1} + (I/\phi) \mu_w [s_w^2 + s_w s_o]}{I_w I_o s_w^{\alpha-1} s_o^{\beta-1} + (I/\phi) [I_w s_w^\alpha \mu_w + I_o s_o^\beta \mu_o]}, \quad (9)$$

$$k_{ro} = \frac{I_w s_w^{\alpha-1} s_o + (I/\phi) \mu_o [s_o^2 + s_w s_o]}{I_w I_o s_w^{\alpha-1} s_o^{\beta-1} + (I/\phi) [I_w s_w^\alpha \mu_w + I_o s_o^\beta \mu_o]}.$$

For co-current flow, the Darcy velocities are related to the volumetric injection rate Q :

$$v_o + v_w = v_T = \text{constant} = \frac{Q}{A}, \quad (\text{co-current}) \quad (10)$$

where A is the cross-sectional area of the porous medium. The interstitial velocities can then be calculated by combining (1)+(7)+(9) to:

$$u_o = \frac{v_T}{\phi} \frac{\lambda_o}{s_w \lambda_w + \lambda_o}, \quad u_w = \frac{v_T}{\phi} \frac{\lambda_w}{s_w \lambda_w + \lambda_o}. \quad (11)$$

Table 1 Parameters for illustration of fluid-fluid interaction

VARIABLE	WITH FLUID-FLUID INTERACTION	NO FLUID-FLUID INTERACTION
μ_{oil}	1.0	1.0
μ_{water}	1.0	1.0
Oil-solid friction coefficient I_o	1.0	1.0
Water-solid friction coefficient I_w	4.0	4.0
Fluid-fluid interaction coefficient I	1.0	0.0
Water saturation exponent α	0.3	0.3
Oil saturation exponent β	0.3	0.3

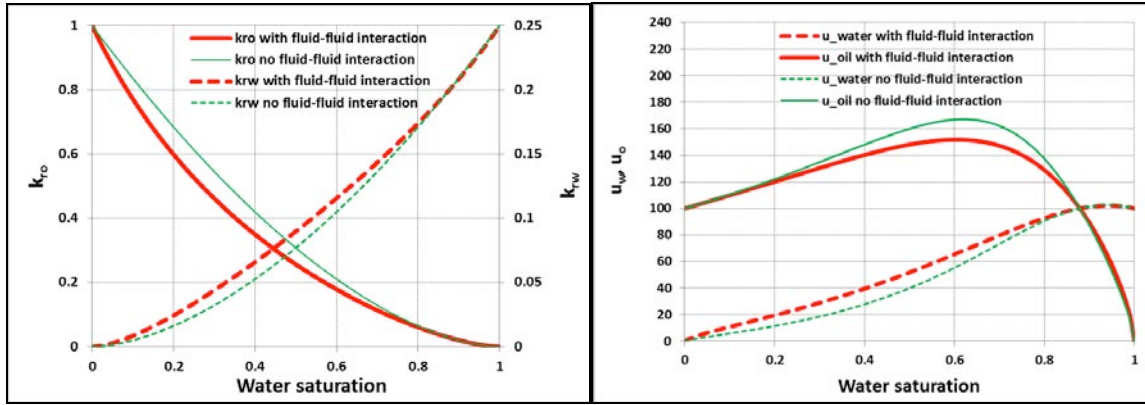


Fig. 1. Illustration of fluid-fluid interaction based on data in Table 1. Left: RP curves from (9) with (red) and without (green) fluid-fluid interaction ($I=1$ or 0). Right: Corresponding fluid phase interstitial velocities from (11).

For counter-current flow, and now using equal, but *opposite* phase pressure gradients (assuming no capillary pressure effects) we obtain effective k_{ri} from solving (1)+(5-8):

$$k_{ro} = \frac{I_w s_w^{\alpha-1} s_o + (I/\phi) \mu_o [s_o^2 - s_w s_o]}{I_w I_o s_w^{\alpha-1} s_o^{\beta-1} + (I/\phi) [I_w s_w^\alpha \mu_w + I_o s_o^\beta \mu_o]}, \quad (12)$$

$$k_{rw} = \frac{I_o s_w s_o^{\beta-1} + (I/\phi) \mu_w [s_w^2 - s_w s_o]}{I_w I_o s_w^{\alpha-1} s_o^{\beta-1} + (I/\phi) [I_w s_w^\alpha \mu_w + I_o s_o^\beta \mu_o]}.$$

With no fluid-fluid interactions ($I=0$) the expressions both in (9) and (12) simplify to the standard (ST) Corey formulation:

$$k_{roST} = \frac{s_o^{2-\beta}}{I_o}, \quad k_{rwST} = \frac{s_w^{2-\alpha}}{I_w}. \quad (13)$$

NUMERICAL INVESTIGATIONS

Two sets of parameters are listed in Table 1, the only difference being that the fluid-fluid interaction is 0 in one case. Without fluid-fluid interaction ($I=0$) the RPs (9) correspond to standard Corey formulation, see (13). Considering Fig. 1, for water saturations $s_w < 0.88$ the interstitial velocity of oil u_o is greater than that of water u_w (calculated from (11)), while for larger saturations the water travels faster. Including fluid-fluid interaction induces a momentum transfer between the phases and for water saturations less than 0.88 the oil phase is slowed down, while the water velocity has increased. This is reflected in increased RP of water at low saturations and reduced oil RP. The opposite occurs at saturations larger than 0.88, where the water velocity is higher (although less visible).

Table 2 Parameters for illustration of viscosity effects

VARIABLE	OIL VISCOSITY = 1	OIL VISCOSITY = 10
μ oil	1.0	10.0
μ water	1.0	1.0
Oil-solid friction coefficient l_o	1.0	1.0
Water-solid friction coefficient l_w	4.0	4.0
Fluid-fluid interaction coefficient I	1.0	1.0
Water saturation exponent α	0.3	0.3
Oil saturation exponent β	0.3	0.3

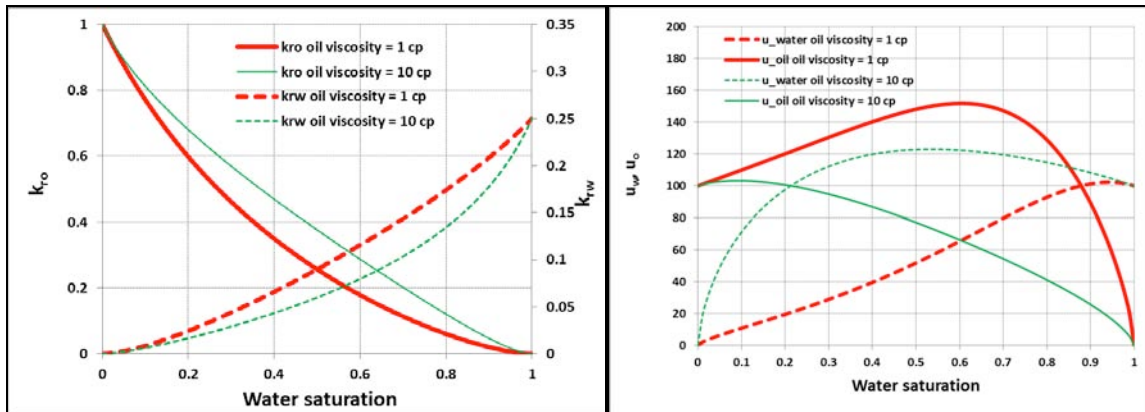


Fig. 2. Illustration of viscosity effects based on data in Table 2. Left: RP curves from (9) for oil viscosity of 1 cP (red curves) and 10 cP (green curves). Right: The corresponding interstitial velocities from (11).

Another set of input parameters to the RP model (9) is given in Table 2. The only difference is the oil viscosity (1 or 10 cP). Fluid-fluid interactions are included. Fig. 2 shows the RP curves and interstitial velocities. Both RP curves are affected by the viscosity change. The model predicts reduced oil and increased water interstitial velocities for the 10 cP case compared to the 1 cP case. This is expected since oil interstitial velocity is inversely proportional to oil viscosity for the same value of k_{ro} . Reduced oil velocity will at the same time increase the water interstitial velocity due to the condition of constant total Darcy velocity v_T . The more viscous fluid, which travels slower, is however, accelerated by the momentum transfer between the fluids and obtains a larger RP, while the opposite happens to the low viscous fluid. The expressions for k_{ro} and k_{rw}

(9) show that an influence from fluid viscosities is possible only if there is fluid-fluid interaction ($I > 0$). Such trends have been observed experimentally [10]. Further, it was seen in [8] that high oil viscosity increased the residual oil saturation, s_{orw} , and lowered the water RP in this point, $k_{rw}(s_{orw})$. Such end point effects are currently not included in the model. The extent at which RP curves in fact are viscosity dependent must be substantiated by simulation-evaluation accounting for artifacts such as capillary end effects.

Table 3 Parameters for illustration of flow direction effects

VARIABLE	COUNTER-CURRENT	CO-CURRENT
μ oil	3.0	3.0
μ water	1.0	1.0
Oil-solid friction coefficient I_o	1.0	1.0
Water-solid friction coefficient I_w	4.0	4.0
Fluid-fluid interaction coefficient I	1.0	1.0
Water saturation exponent α	0.3	0.3
Oil saturation exponent β	0.3	0.3

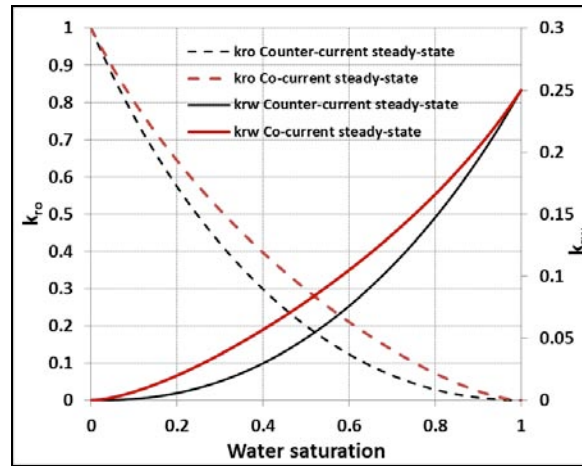


Fig. 3. Illustration of flow direction effects based on data in Table 3. Co- and counter-current steady-state RP functions predicted from (9) and (12), in red and black color, respectively.

Reported data in the literature [1, 2] indicate that counter-current RP curves are lower than the corresponding co-current curves. Table 3 contains equal parameter sets for simulation of co- and counter-current flow setups. It is seen in Fig. 3 that the predicted counter-current RPs are lower than the co-current permeabilities. By examining the expressions (9) and (12) it can be shown that this will always be the case, as long as fluid-fluid interaction is accounted for, i.e. $I > 0$. This observation naturally results from the increased drag obtained when the fluids flow counter-currently and decelerate each other.

CONCLUSIONS

A model for flow in porous media was derived from momentum balance equations accounting fluid-fluid and fluid-rock interactions. Relative permeabilities were predicted.

- The momentum transfer between fluids increases the velocity of the slowly traveling fluid and slows down the fast traveling fluid. This is reflected in increased and reduced relative permeabilities of the respective phases.
- Viscosity changes can affect the relative permeabilities. The more viscous fluid slows down, but obtains higher relative permeability due to the drag from the faster traveling fluid, and vice versa.
- Counter-current relative permeabilities are lower than co-current relative permeabilities due to the greater flow resistance between the phases.
- The model should be validated further by comparison with high quality data and interpretation able to distinguish physical effects from experimental artifacts.

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