Fractal dimension of pore space in carbonate samples from Tushka Area (Egypt)

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ABSTRACT

The permeability of a porous rock is an intrinsic petrophysical parameter that depends on the geometry of the pore space. The pore radius distribution provides a simplified description of the pore space geometry that can be used to investigate the fractal nature of the pore space or to determine a fractal dimension. The fractal dimension is used to describe the size of geometric objects as a function of resolution. Pore volume and pore surface are typical quantities that increase with higher resolution.

A set of eight Upper Cretaceous carbonate rock samples from the Tushka area in Egypt was used to compare the pore radius distribution that was determined by mercury porosimetry (MP), nuclear magnetic resonance (NMR), and spectral induced polarization (SIP). Additionally, the parameters porosity, permeability, formation factor, and specific internal surface were available. The values of fractal dimension were determined from the pore radius distributions of the different methods. Additionally, the fractal dimension of the internal surface was derived using the effective hydraulic radius and the specific surface area per unit pore volume. The relationships between the fractal dimensions and the effective hydraulic radius are compared.

INTRODUCTION

The microgeometry of pore space in carbonates is much more complicated than that in sandstone due to the existence of biotic constituents such as shells, fossils and corals [1]. Therefore the prediction of permeability of carbonate rocks is always a challenge [2]. An accurate experimental permeability determination requires expensive methods based on the observation of fluid flow through the rock driven by a known pressure gradient.

Alternatively, physical methods that provide insight into the distribution of pore sizes enable permeability prediction using more easily accessible parameters including the fractal dimension.

The pore radius distribution provides a simplified description of the pore space geometry that can be used to investigate the fractal nature of the pore space or to determine a fractal dimension. The fractal dimension is used to describe the size of geometric objects as a function of resolution. Pore volume and pore surface are typical quantities that increase with higher resolution. The study [3] applied fractal dimension to predict the permeability of a set of sandstone samples.

We present here the pore radius distribution determined by mercury porosimetry (MP), nuclear magnetic resonance (NMR), spectral induced polarization (SIP) and specific internal surface of eight Upper Cretaceous carbonate rock samples from the Tushka area in Egypt. The parameters porosity, permeability, formation factor and the specific internal surface were available. The values of fractal dimension were determined from the pore radius distributions of the different methods. The different values were compared.

METHODS AND RESULTS

This study investigates a set of limestone samples taken from seven shallow wells in the Tushka area in south-eastern part of Egypt [4]. The Tushka basin is located in the southern part of Western Desert, approximately 250 km away from Lake Nasser and the Aswan High Dam, between the latitudes 22° 30′-22° 45′ N and longitudes 31° 45′-32° 0′ E. The region is bordered by the Sen EI-Kaddab Eocene limestone plateau to the north, by Nasser Lake to the east, by the Nubian pediplain and the political border between Egypt and Sudan to the south.

We selected eight samples from a larger set [4] for our study. All samples are cylindrical in shape and with a diameter of 25 mm and a length of about 30 mm. The measurements of petrophysical parameters such as grain density, porosity, permeability, and internal surface were performed according to acknowledged procedures. The MP experiments cover a pressure range between 0.004 and 400 MPa. The NMR T2 relaxations times were determined with a MARAN 7 equipment operating at a Larmor frequency of 7.05 MHz at room temperature and ambient pressure. The complex conductivity spectra were acquired in a frequency range between 3 mHz and 100 Hz using an impedance spectrometer with high phase accuracy. The measurements of all samples utilized the same protocols. The reproducibility of the resulting parameters has been checked.

Fractal dimension of the pore space can be characterized by the curve of cumulative volume fraction of the pores versus pore radius. For MP method, the capillary pressure shows an inverse proportionality with the pore radius. For NMR method, the proportionality between the transversal relaxation time and pore radius is used.

For SIP method, we transformed the relaxation time distribution of complex conductivity spectra determined by Debye decomposition [5] into a distribution of pore radii *r*. We adopt an equation proposed by [6] and applied by [7] for the Stern layer polarization model:

$$r = \sqrt{2\pi D_{(+)}} \tag{1}$$

with τ being the relaxation time and $D_{(+)}$ the diffusion coefficient of the counter-ions in the Stern layer. Originally, this equation describes the relation between the radius of spherical particles in an electrolyte solution and the resulting relaxation time [6]. Though it is questionable whether the radius of spherical grains can be simply replaced by the pore radius, we follow this approach. Additionally, we assume a constant diffusion coefficient $D_{(+)} = 3.8\times10^{-12}$ m²/s as proposed for clayey material [8]. The total chargeability is attributed to the total pore volume. The volume fraction $V_c = V(<r)/V$ is the cumulative volume fraction of pores with radii less than r, which corresponds to the ratio of cumulative intensity to total intensity.

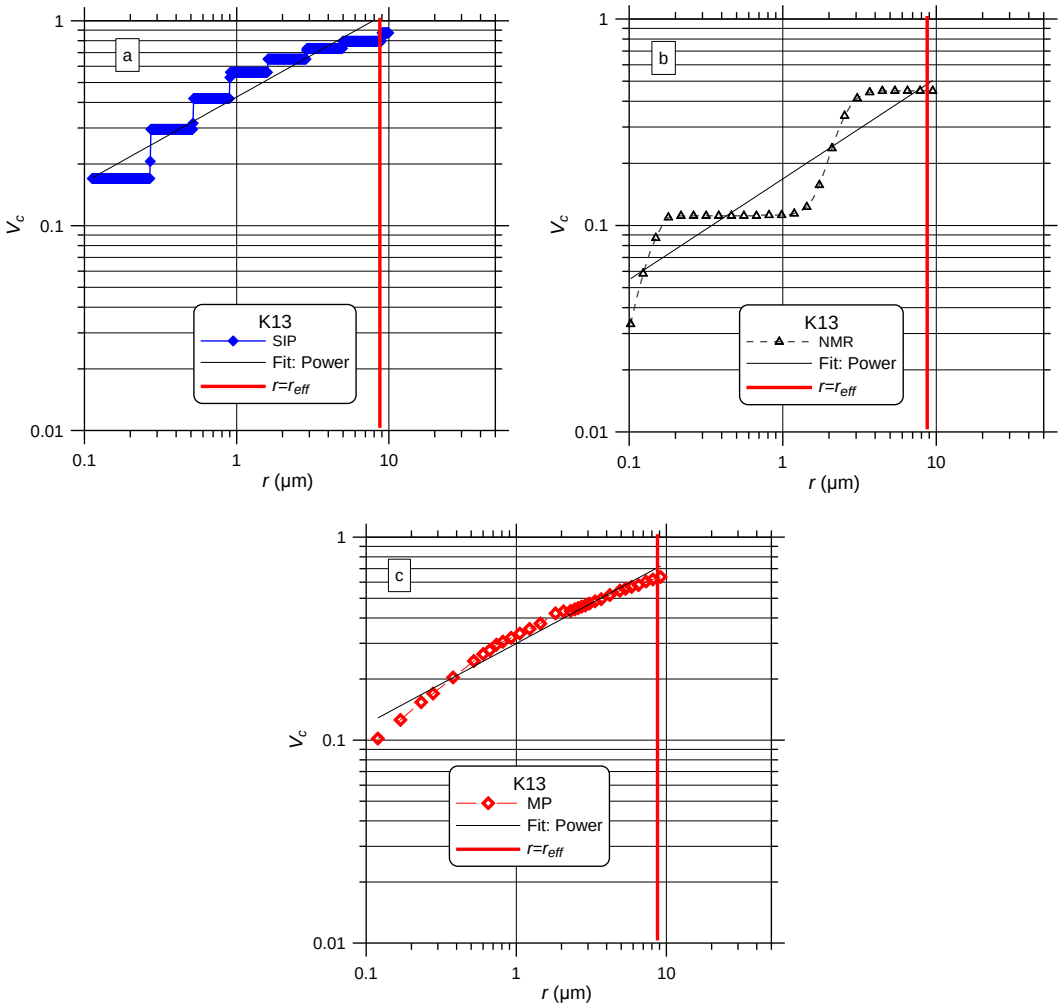

Figure 1: a) The fractal dimension D_{SIP} of sample K13. D_{SIP} is determined from the slope of the linear fitting equation $\log(V_c) = 0.416 \log(r) - 0.857$ ($R^2 = 0.915$) with $D_{SIP} = 3 - 0.416 = 2.584$, b) The fractal dimension D_{NMR} of sample K13 is determined from the slope of the linear fitting equation $\log(V_c) = 0.488 \log(r) - 1.783$ ($R^2 = 0.847$) with $D_{NMR} = 3 - 0.488 = 2.512$. c) The fractal dimension D_{MP} of sample K13 is determined from the slope of the linear fitting equation $\log(V_c) = 0.397 \log(r) - 1.207$ ($R^2 = 0.967$) with $D_{MP} = 3 - 0.397 = 2.603$. The vertical red lines indicate $r = r_{eff} = 8.44$ µm.

The cumulative curve is presented in a double logarithmic plot showing the relation log(V_c) versus log(r). Figure 1 displays the curves from SIP, NMR and MP data for sample K13. In the case of fractal behaviour of the pore volume distribution, the slope s of the fitting line is used to get the fractal dimension $D = 3 - s$. The vertical red lines in Figure 1 mark the effective hydraulic radius, which has been determined from permeability k and formation factor F:

$$r_{eff} = \sqrt{8Fk}\,. \qquad (2)$$

Following a procedure described in [3], the fractal dimension of the internal surface (D_{Spor}) was derived using the effective hydraulic radius and the specific surface area per unit pore volume determined from the nitrogen adsorption method.

Figure 2 displays the relation between pore radii determined at V_c = 0.5 (with $r = r_{50}$) from MP method and r_{eff}. The comparison indicates a good agreement between r_{eff} and r_{50}. The median pore radius r_{50} from MP can be regarded as suitable parameter to estimate the effective hydraulic radius r_{eff}. This estimate can be used if the values of permeability and formation factor are not available for the rock samples.

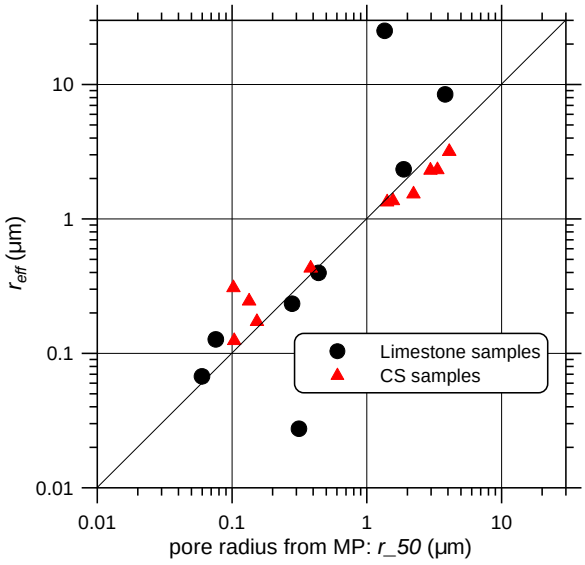

Figure 2: Relation between r_{eff} and r_{50} for the limestone samples in this study and 11 sandstone samples (CS) in [3]. The diagonal line indicates $r_{eff} = r_{50}$.

For the carbonate samples with effective pore radius r_{eff} > 1 µm, the V_c-r curves of the methods NMR, MP and SIP show similar characteristics. The best agreement exists for sample P3 with all cumulative pore radii curves indicating the steepest slope close to r_{eff} as shown in Figure 3a. The smallest effective pore radius was determined for sample K34 with r_{eff} = 0.028 µm. A shift between the different curves becomes visible in Figure 3b. The curves of MP and SIP indicate much larger pore radii. The curve of SIP can be shifted to smaller pore radii if a lower value of the diffusion coefficient is used in equation 2. But it should be noted that a shift does not affect the slope of the curve and the resulting fractal dimension.

Figure 4a displays the relation between the effective pore radius r_{eff} and the fractal dimension derived from NMR, MP, SIP and the specific internal surface. The values of fractal dimensions D_{NMR}, D_{MP}, and D_{SIP} indicate a slight decrease with rising effective pore radius, while the fractal dimension D_{spor} shows an increasing trend as the effective pore radius rises.

Figure 4b displays the comparison between the fractal dimension D_{SIP} with the fractal dimension derived from MP and NMR. In order to enable an unbiased comparison, the same *r*-range of 0.1 to 10 µm was applied for all methods. The fractal dimension derived from SIP and NMR are close to each other. The pore radius distribution of MP results in slightly larger values of fractal dimension.

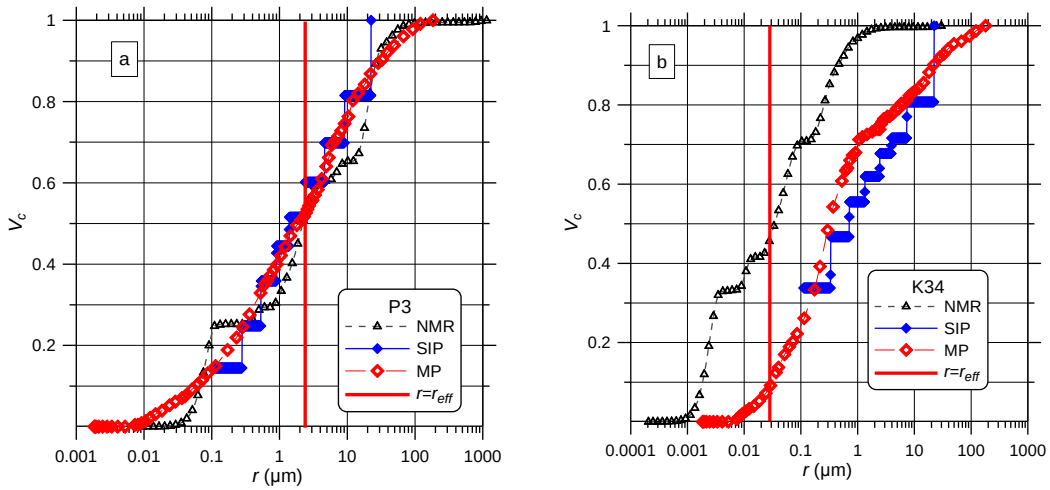

Figure 3: a)The comparison of V_c-r curves determined from NMR, MP, and SIP for sample P3. The vertical red line indicates $r = r_{eff}$ = 2.34 µm. b) The comparison of V_c-r curves determined from NMR, MP, and SIP for sample K34. The vertical red line indicates $r = r_{eff}$ = 0.028 µm.

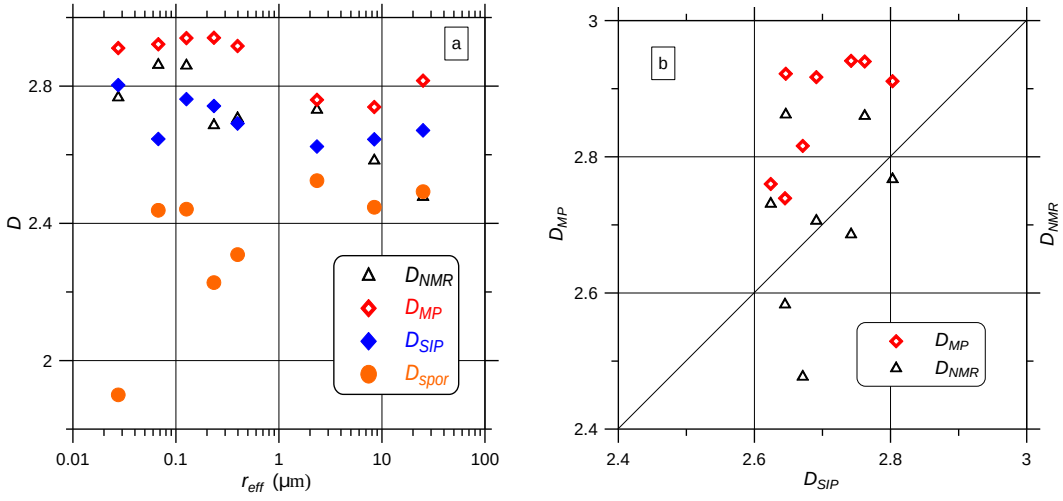

Figure 4: a) Relation between effective pore radius and fractal dimension determined from NMR, MP, SIP and nitrogen adsorption method. b) Comparison of fractal dimensions determined from NMR, MP and SIP. The diagonal line indicates $D_{SIP} = D_{NMR}$ and $D_{SIP} = D_{MP}$.

CONCLUSION

Our study demonstrates that the relaxation time distribution of complex conductivity spectra can be used to derive the fractal dimension. The resulting values, which vary in a range between 2.62 and 2.80, are comparable with those derived from NMR and MP for a set of eight limestone samples. The fractal dimension that is determined from the specific internal surface and the effective hydraulic radius indicates lower values in a range between 1.90 and 2.52. The comparison of fractal dimension determined by the different methods indicates a differentiation into “surface dimension” and “volume dimension”. The fractal dimension resulting from NMR, SIP, and MP reflects a volume dimension, while the fractal dimension determined from the specific internal surface data represents the roughness of pore surface.

The fractal dimension that describes pore space geometry as a function of resolution is a useful parameter for upscaling and downscaling pore size for different applications.

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