EXPERIMENTAL AND NUMERICAL INVESTIGATION OF DISPERSION IN A DUAL PERMEABILITY POROUS MEDIUM DURING MISCIBLE DISPLACEMENT

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ABSTRACT

This study presents an analytical and experimental investigation of the mixing and spreading process in a dual permeability porous medium during the miscible displacement process. The coupled two-dimensional model of high and low permeability zones that are horizontally parallel is assumed, and the interaction between two zones is manipulated by imposing the continuity of concentrations at the interface between the two zones. The mathematical model of governing equations, considering the longitudinal and transverse dispersion has been developed for each region. The equations in following have been coupled considering the continuity of concentrations and mass fluxes along the boundary of high and low permeability regions. To testify the model, the experimental study has been conducted using a two-dimensional glass micromodel, dyed and clear water and a new image processing technique. A series of miscible displacements has been implemented and taken images at known time steps have been processed using a uniquely developed image processing tool to measure the dispersion coefficients in each region at different time steps. In the following, the validity of the mathematical analysis has been tested with the experimental results. The results of this study provide a deeper understanding of the mixing process during miscible displacement, the importance of longitudinal and transverse dispersion in high and low permeability zones with different heights and the effect of injection velocity in the development of mixing zone over time.

INTRODUCTION

Miscible and near-miscible displacement processes are increasingly feasible methods for the recovery of oil from depleted reservoirs as an enhanced oil recovery (EOR) process. However, a fundamental understanding of the dependency of transport phenomenon on porous media properties and their indirect impact on oil recovery efficiency is lacking. Moreover, it is believed that miscibility may repeatedly develop and break down in a reservoir due to dispersion arising from reservoir heterogeneity and therefore the effectiveness of miscible processes as an EOR method are complicated by the heterogeneity of a porous medium. Dispersion is a transport mechanism of a substance or conserved property by a fluid due to the fluid's bulk motion [1]. A distinction is commonly made between dispersion occurring in the flow direction, longitudinal (D_L), or perpendicular to it, transverse (D_T).

The advection-diffusion equation (ADE), mass conservation law, has been commonly used [2] to describe solute dispersion in two-dimensional porous media:

$$D_L \frac{\partial^2 C}{\partial x^2} + D_T \frac{\partial^2 C}{\partial y^2} - \upsilon_x \frac{\partial C}{\partial x} - \upsilon_y \frac{\partial C}{\partial y} = \frac{\partial C}{\partial t} \quad (1)$$

where C is the solute concentration, t is time, υ_x and υ_y are the components of flow velocity, and D_L and D_T are the hydrodynamic longitudinal and transverse dispersion coefficients, respectively. The mathematical approach to modeling dispersion regularly focused on applying the partial differential equation referred to as the advection-dispersion (Eq. 1) from continuum mechanics. The dispersion mechanism is mainly characterized by contributions of molecular diffusion and mechanical dispersion expressed as a function of the Peclet number. Many mathematical studies including imperial correlations have been done. Taylor [3] (1954) described the principles of solute transport in porous media. He envisioned the porous media as a bundle of capillaries and viscous fluid flowing through a tube of circular cross section. Taylor defined dispersion D (m^2s^{-1}) as:

$$D = \frac{a^2\upsilon^2}{48D_m} \quad (2)$$

where v is the velocity (ms^{-1}), D_m is the molecular diffusion (m^2s^{-1}), and a is the tube radius (m). This study also showed that the proposed model was valid under the condition that Peclet number is within the range of 6.9 $\ll$ Pe $\ll \frac{4l}{a}$ [4]. Aris (1956) removed the restriction imposed on some of the parameters at the expense of describing the distribution of the solute in terms of its moment in the direction of flow. Aris also showed that the rate of growth of the variance is proportional to the sum of molecular diffusion and Taylor dispersion [5]. There are also some studies started with Barenblatt [6] (1960) called dual permeability approach. The dual-permeability porous medium involves two regions with different hydraulic and discursively transport properties. The term dual-permeability indicates that fluid flows in two distinct regions that one region may represent a higher permeability space with faster flow and second domain with lower permeability and therefore slower flow in the rock matrix. Other studies after, (e.g. dye tracer studies by Cislerova et al. [7], 1990; Villholth et al. [8], 1998; and Larsson et al. [9], 1999) have used this approach to investigate the miscibility development and interaction between two domains. However, most studies used probabilistic methodologies, and have not considered the analytical solutions. Leij et al. [10], in 2012, considering this lack, presented a mathematical model for dispersion development in a dual-permeability medium using analytical approach by solving Laplace transformation and matrix decomposition. The study illustrates the difference in travel time for the two regions and shows the difference in fluid velocities.

On the other hand, mixing processes that occur during miscible injection have been studied at different scales, from a laboratory to field scale, using various experimental approaches.

One of the initial leading studies was published by Brigham et al. [11] in 1961 describes the result of experiments on miscible displacement in different porous media. Both glass bead packs and natural cores were used to represent a porous medium. Since the required experimental parameters were not directly measurable, appropriate assumptions were made to obtain such values by Brigham. After analyzing the error function, he proposed Eq. 3 to calculate the longitudinal dispersion.

$$D_L = \frac{1}{t}\left(\frac{x_{90} - x_{10}}{3.625}\right)^2 \tag{3}$$

The variables are time t (s) and x_{10} is defined as the distance to the point of 10 percent displacing fluid and x_{90} is the distance to the point of 90 percent displacing fluid. However, Brigham related the dispersion coefficient to the error function by drawing the best straight line through the values of U (the error function parameter). The modified equation based on this method is given in Eq. 4 and 5.

$$D_L = \frac{1}{t}\left(\frac{x_{90} - x_{10}}{3.625}\right)^2 \tag{4}$$

$$U = \frac{V_p - V}{\sqrt{V}} \tag{5}$$

The parameters include the pore volume V_p of the porous medium (cm^3) and the volume of the recovered fluid at any time, V (cm^3). For transverse dispersion, Grane and Gardner [12] (1961) carried out a set of experiments in homogeneous porous media to study the influence of fluid flow velocity and pore structure properties on transverse dispersion. They observed that by increasing the difference in density the mechanical dispersion significantly decreased and therefore density difference has a significant effect. Perkins and Johnston [4] (1963) performed experimental investigations using the Brigham method. Perkins and Johnston studied the impact of various parameters including density contrast and pore geometry. Fried and Combarnous [13] (1971) published the result of the extensive laboratory experiments performed to investigate the influence of the increase in velocity on dispersion in porous media. Most of the later experimental studies applied the same methodology to refine the understanding of longitudinal and transverse mixing and effective parameters by directly considering the physical mechanisms governing solute transport. The current study uses an analytical solution for miscible displacement in a dual-permeability two-dimensional porous medium. A new experimental approach is applied to validate the analytical results.

MODEL DESCRIPTION

We assume a two-dimensional porous medium as in Fig. 1, consisting of high permeability (1) and low permeability (2) regions combined in one glass etched pore network micromodel. In this model, inlet pressure (P_1) and outlet pressure (P_2) are constant. The high permeability region is very low in width (w) (assumed 1D) and region 2 is higher in width (W) and considered two-dimensional (w << W). It is assumed that for y = 0, v = v_{x1}, for y > 0, v(x, y) = $(\upsilon_{x2}, \upsilon_{y2})$. The concentration in region 1 (y = 0) is assumed C(x, y) and in region 2 is assumed C^*(x, y). Initial conditions for ∀x,y is C(x, 0) = C^*(x, y, 0) = 0 and boundary conditions are assumed as Eq. 6.

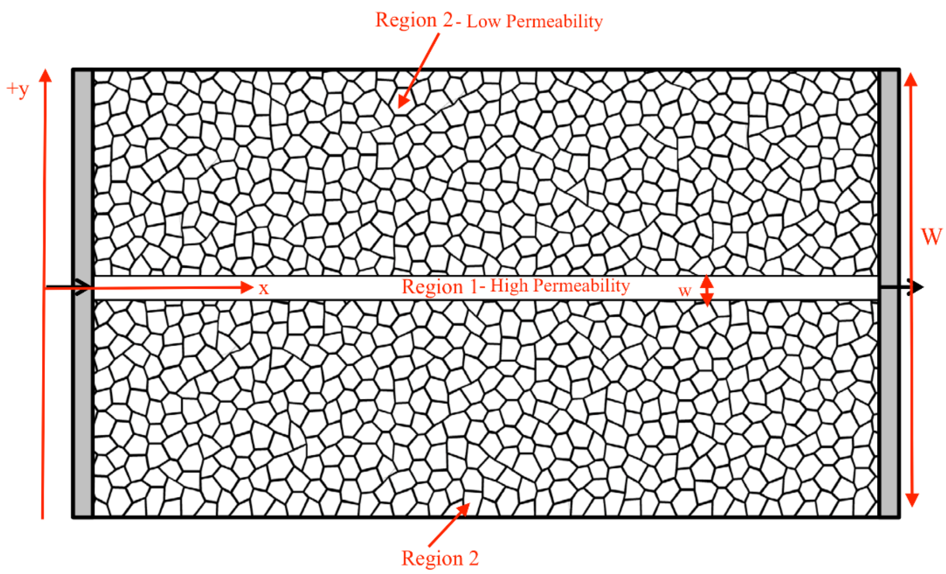

Figure 1. Assumed dual-permeability porous medium

$$\begin{cases} \forall t \ \Rightarrow \ C(0,t) = C_0 \\ \forall t \ \Rightarrow \ C^*(0,t) = 0 \\ \forall x,t \ \Rightarrow \ lim_{\Delta y \to 0}\, C^*(x,\Delta y,t) = C(x,t) \end{cases} \tag{6}$$

Dominant velocity is in x-direction and only in high-permeability region and advection in low-permeability region is neglected. Considering the advection-diffusion equation (Eq. 1) in each region, the equations below are assumed:

$$\begin{aligned} &y = 0: \ \frac{\partial C}{\partial t} + \upsilon \frac{\partial C}{\partial x} - D_{x1}\frac{\partial^2 C}{\partial x^2} = -q(x,t) \\ &y > 0: \ \frac{\partial C^*}{\partial t} - \upsilon_{x2}\frac{\partial C^*}{\partial x} - \upsilon_{y2}\frac{\partial C^*}{\partial y} - D_{x2}\frac{\partial^2 C^*}{\partial x^2} - D_y\frac{\partial^2 C^*}{\partial y^2} = q(x,t) \\ &\forall t, x \text{ and } y = 0: q(x,t) = D_y lim_{\Delta y \to 0} \frac{C^*(x,\Delta y,t) - C(x,t)}{\Delta y} \\ &\forall t, x \text{ and } y > 0: q(x,y,t) = 0 \end{aligned} \tag{7}$$

The model examines single phase flow (clear water injected into dyed water where the fluids are the same viscosity and we have single phase flow. When an increased concentration C_i^n enters the first cell, due to the fluid velocity in the x-direction, concentration distribution in the entire system reforms. To keep it simple, we assumed that the concentration in each cell has a single value. We need to solve for the concentration in the i^{th} cell at time step n (C_i^n), depending on time and transport properties.

EXPERIMENTAL PROCEDURE

Micromodel flooding experiments are conducted to evaluate the validity of the obtained mathematical model and gain a deeper understanding of miscible displacements in a 2D medium. To avoid the effect of density difference in driving dispersion, clear and dyed water are used as a displacing and displaced water, respectively. The experimental apparatus include a syringe and hand pump, micromodel holder, burette, graduated cylinder, weighing paper, moisture trap, digital camera connected to computer with appropriate lens, pressure transducers, a mass balance, pressure gauges, temperature transducers, a temperature gauge, tubing, valves, Swagelok® fittings (tees, caps, plugs, nuts, ferrules, tube adapters, connectors, and a pressure relief valve), and tools.

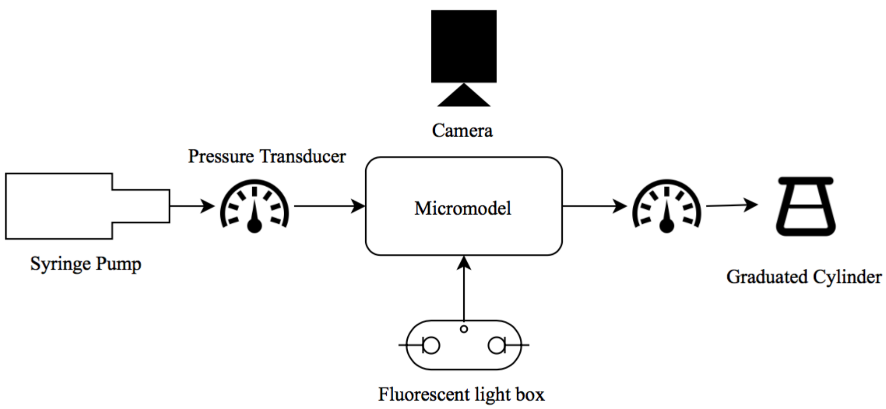

Figure 2. Experimental Setup Schematic

Fig. 2 shows the schematic of the experimental set up. Table 1 shows the micromodel properties. The micromodel is placed between two frames with a port at each end (injection and production). A syringe pump (Cole-Pamer dual syringe pump RS-232) is used to inject the water. Injection of dyed water and full saturation of micro model associated with image processing is needed to calculate the porosity and absolute permeability. Miscible displacement tests were conducted, an image processing technique developed to find the concentration profile, and the longitudinal and transverse dispersion coefficients were calculated.

Table 1. Properties of the dual-permeability micromodel

K_1	56 D	$\emptyset_1$	0.52	L_1	32 cm	W_1	0.5 cm
K_2	18 D	$\emptyset_2$	0.39	L_2	32 cm	W_2	6.0 cm

During the miscible displacement tests, we have single phase flow and we do not have a distinct interface between the displacing and displaced fluids. Clear water is injected into coloured/dyed water. However, a mixing zone exists between the pure displacing fluid and the displaced fluid. The scaled concentration in the mixing zone ranges from zero (clear displacing fluid) to 100 (pure displaced fluid, red). Images taken during the miscible injection at known times are used to approximate the concentration profile. The most challenging part is distinguishing the concentration based on colour gradient (clear to red) and this requests robust calibration of color intensity and image processing. To relate the different colors to corresponding concentrations, several images were taken to obtain a unique correlation on a per pixel basis. After extracting the concentration profile, the

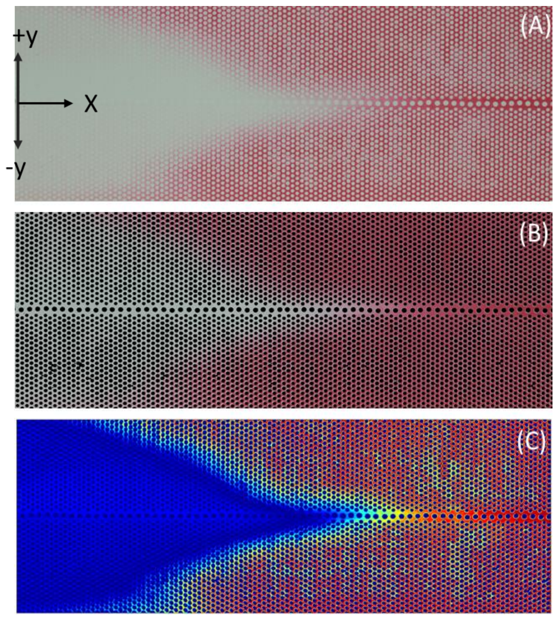

Figure 3. Image processing technique

longitudinal and transverse dispersion will be estimated by fitting the experimentally measured transient solute concentration profiles with the aid of the Bayesian estimator to an analytic solution of Eq. 1. The cropped raw image (A), selected-analyzed mask image (B) and processed image (C) is illustrated in Fig. 3.

RESULTS AND DISCUSSION

To study the dispersion in a dual-permeability porous medium using experimental and numerical investigations the following results have been accomplished.

Experimental Observations

During the experiment, we observed a developing mixing zone over time. As Fig. 4 shows, the mixing zone in high permeability region is caused by longitudinal dispersion, and in lower permeability porous medium longitudinal and transverse dispersion are coupled. From the Taylor Eq. 2, two factors affect miscible displacement: advection and dispersion, especially in the case of lower injection rate. Comparing the longitudinal dispersion in each region shows that dispersion is greater in the porous medium region. This may be caused by a presence of dominant flow velocity and lower height of the region 1. As shown in Fig.

4, in the high permeability zone (channel), the fluid advances from left to right at a higher rate than in the low permeability porous medium.

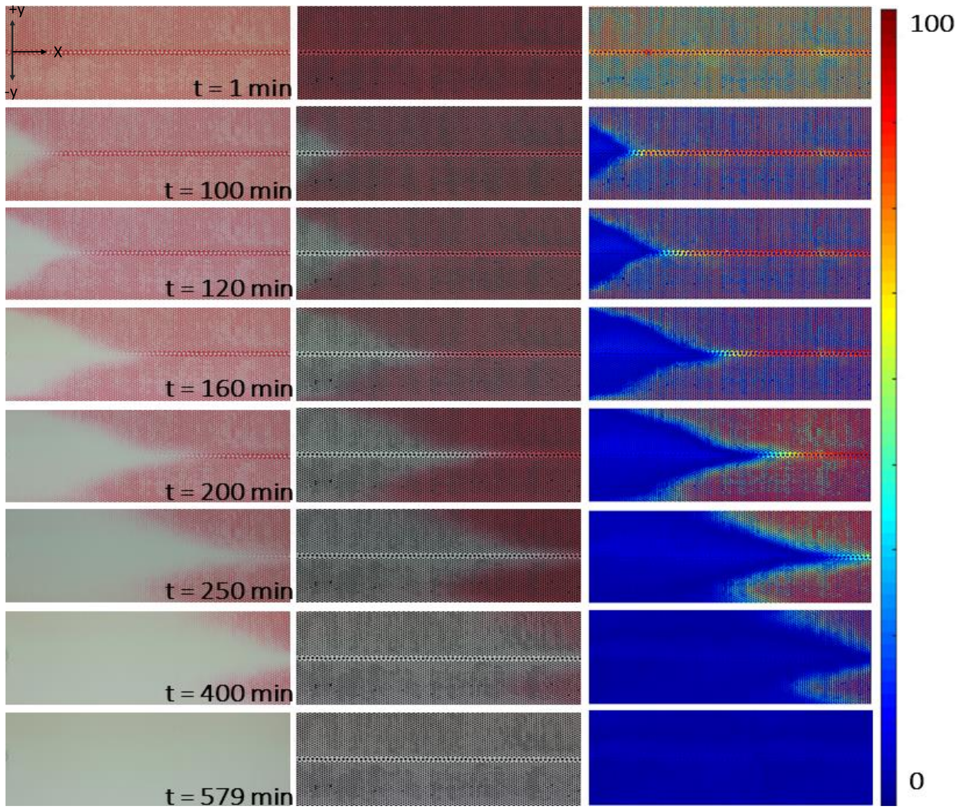

Figure 4. Miscible displacement progress over time

Analytical Solution

Eq. 7, presents the assumed mathematical model for miscible displacement in the dual-permeability medium with defined assumptions. The set of equations is solved using first and second order upwind scheme. In high permeability region, due to very low height for the medium, advection and longitudinal dispersion terms have been considered, and in the lower permeability medium, advection, longitudinal and transverse dispersion have been assumed. Although both regions have longitudinal dispersion terms, the longitudinal coefficient in high permeability region is considered 20 times larger than the lower permeability zone. Table 2 shows the assumed parameters and coefficients for the set of equations. The assumptions appear reasonable in circumstances of the relatively smaller contribution of dispersion terms to spreading of displacing fluid when there is a clear difference in velocity between the two regions. Fig. 5 shows that the predicted longitudinal and transverse dispersion coefficients in the low permeability region in Table 2 are in a good agreement with the experimental results. This leads to same dispersion coefficients in both component in low permeability region in the absence of a dominant velocity.

Table 2. Assumed parameters and coefficients for the set of equations

$\upsilon_x\ (m/s)$	$\upsilon_{x2}(m/s)$	$\upsilon_{y2}(m/s)$	$D_{x1}(m^2/s)$	$D_{x2}(m^2/s)$	$D_y(m^2/s)$
3×10^{-5}	1.5×10^{-6}	2×10^{-6}	2×10^{-6}	1×10^{-7}	1×10^{-7}

Model Validation

The simulation results are used to find the displacement process in both regions over time. Fig. 5 illustrates the miscible displacement flooding in the same dual permeability pattern as the one used in the micro model experiment.

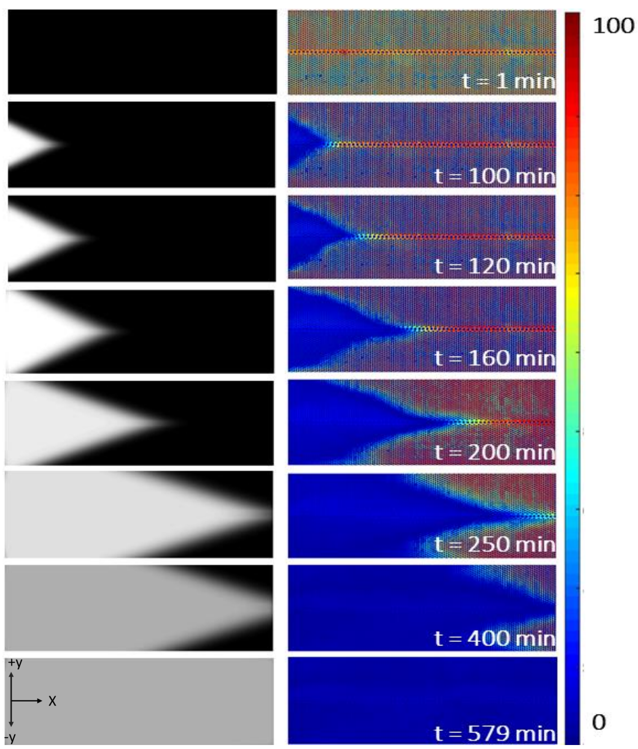

Figure 5. Mathematical vs experimental results of miscible displacement progress over time

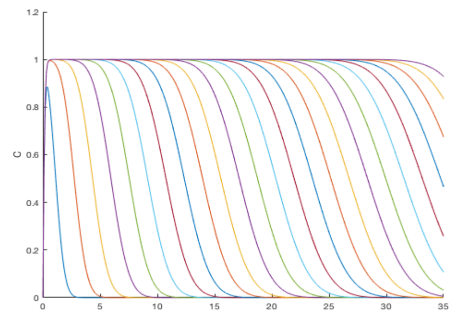

Figure 6. Concentration profile of displacing fluid versus x-direction

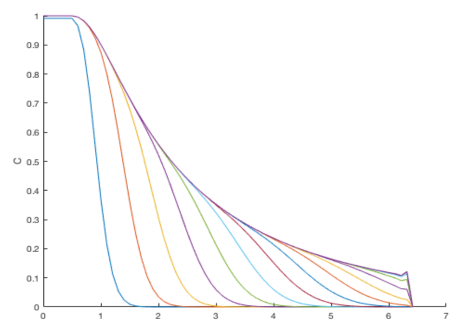

Figure 7. Concentration profile of displacing fluid versus y-direction

Fig. 6 and 7 shows the concentration profile versus positions in the x and y-directions for different times. Fig. 6, showing the concentration profile in the direction of displacement indicates a convection dominated mass transfer whereas Fig. 7 showing the concentration profile in the y-direction. The shape of the concentration profile is indicative of conduction dominated mass transfer. The analytical solution indicates a solid agreement with experimental images. If a more viscous miscible fluid was injected we would likely see the convective terms reduced due to the more viscous flow but conductive mass transfer should not be affected.

CONCLUSIONS

Experimental results showed that the mathematical models fit the miscible displacement concentration profiles obtained from miscible tests. Experimental results illustrated some typical mixing zones that are larger in high permeability zone than lower permeability region. This observation also, concludes higher longitudinal dispersion coefficient in high permeability region. Despite the significantly dominant longitudinal dispersion in high permeability zone, in the low permeability region, longitudinal and transverse dispersion coefficients are in same order of magnitude which indicates the effect of the presence of dominant velocity in the x-direction on the amplitude of the longitudinal dispersion. The simulation and experimental results show that the longitudinal and transverse dispersion in low permeability region are in the same order of magnitude. This is most likely caused by the absence of dominant velocity in the region.

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