Derivation of A Representative Elementary Volume (REV) for Upscaled Two-Phase Flow in Porous Media.

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Abstract. Relative permeability plays an important role in the upscaling of multiphase flow in porous media from pore scale to Darcy scale. The entire concept of relative permeability is contingent on the existence of a representative elementary volume (REV). As we move to smaller samples to measure relative permeability, such as with Digital Core Analysis, the concept of a classical REV has become increasingly unlikely when using the conventional approach to defining a representative volume. The 'conventional' understanding of an REV is that a large enough volume must be considered such that spatial variability averages out. In Digital Rock methods, such as pore-scale simulations based on micro-CT images the domain size is typically 2-4 mm. This is approximately the length scale of a single-phase flow REV using the classic REV approach. However, the single-phase perspective does not consider the complex dynamics and fluctuations often observed in multiphase flow systems even at centimeter scale experiments and/or simulations. A fundamental question is, therefore, whether the domain size commonly used in Digital Rock simulations can provide a consistent energy budget such that the concept of relative permeability exists. Based on first principles, relative permeability account for the rate of energy dissipated in a stationary process. If the dynamics are fluctuating, the energy dissipated can vary, but will average out over a long enough timescale. The key to determining the validity of the relative permeability is the timescale of the measurement, not the spatial scale. The conventional REV theory assumes that spatial, temporal, and ensemble averages are equivalent in an ergodic system, but it does not provide a way to test this assumption. We provide a formal way to identify the timescale where the relative permeability accurately captures energy dissipation as a way to validate relative permeability measurements and quantitatively assess their accuracy. This result will be tested for a practical SCAL test, determining how long a flow experiment needs to be run to accurately characterize the rate of energy dissipation by the flow. The outcome will be a best practice guide for the determination of relative permeability from core scale experiments and/or digital core simulations that ensures the energy budget is fully accounted for in the relative permeability coefficient.

1 Introduction

The representative elementary volume (REV) is a central concept when upscaling flow in porous media from the pore to the Darcy scale [1,2]. The concept has been initially used for the upscaling of single-phase flow from Stokes flow at pore to Darcy's law [3] using volume averaging or homogenization. At an intuitive level, the REV marks the length scale at which the (Darcy scale) average property such as porosity or permeability becomes independent of the averaging volume [1]. The REV might be generally different for porosity and permeability. In general, each property might have its own REV. Heterogeneity leads to hierarchy of REVs in case that heterogeneity length scales are separating [2]. The existence of an REV is not a given, for instance in the situations when heterogeneity length scales are not separating.

In multiphase flow, the REV concept may require a different interpretation than the spatial concept from single-phase flow. The reason is that in addition to the structure of rock, the immiscible fluid phases form structures themselves, for instance clusters and ganglia [4,5] which are created by a complex interplay of viscous and capillary forces as sketched in **Fig. 1**. In addition, new state variables and parameters become relevant such as saturation, curvature and fluid topology [6]. That has led to the observation that for instance a saturation REV is significantly larger than a porosity and permeability REV [5] exceeding the length scales that micro-CT scanners can still access at sufficient resolution to

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determine fluid connectivity at pore scale since non-wetting phase clusters can range up to several centimeters at the relevant capillary numbers [8].

This has led to the situation that for traditional upscaling approaches for multiphase flow from pore to Darcy scale [9] the multiphase REV is always assumed without a proof of its existence. In such approaches it is always assume that at the respective REV the spatial and also *temporal* dynamics of e.g. ganglia [5] has averaged out such that pressure, saturations and phase fluxes show stable averages without significant fluctuations for each individual property. However, experimental data shows still significant fluctuations of pressure and saturation even at length scales of several centimeters i.e. the length scale of typical experiments in special core analysis (SCAL) [10] which challenges this assumption. It either means that SCAL experiments do not cover an REV, the REV for multiphase flow may not exist, or that the REV needs to be defined differently.

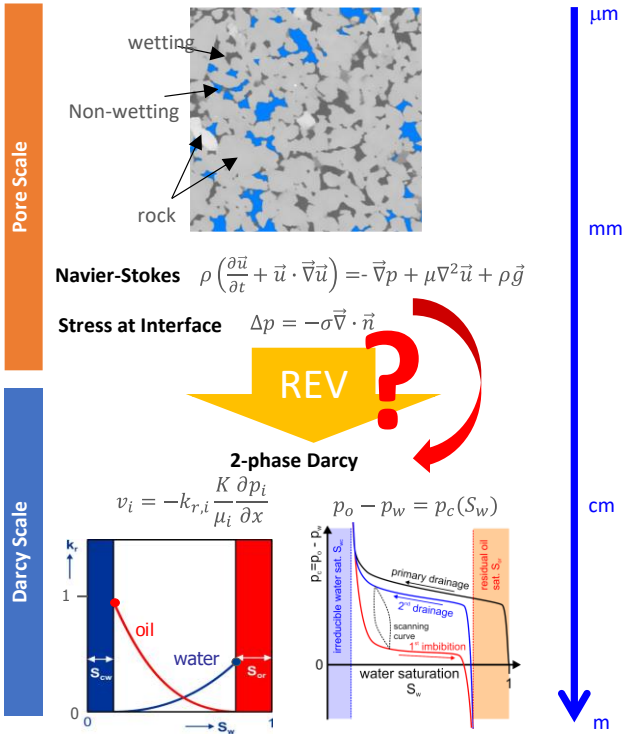

Fig. 1. Multiphase flow in porous media: from pore to Darcy scale.

In laboratory experiments conducted at multiple length scales [10] demonstrate a transition from pore scale phenomena such as Haines jumps and snap-off to phenomena exhibiting fractional flow physics [16] at the centimeter length scale which are typically associated with the Darcy scale, while spatio-temporal fluctuations of significant magnitude in pressure, saturation and phase fluxes exist at any length scale. In other words, the transition in multiphase flow from pore to Darcy scale does not mean that stable averages of key variables and parameters such as saturation and pressure form as traditionally assumed.

This observation leaves us with the leading hypothesis that third option i.e. that an REV exists but needs to be defined differently, is the likely. The key questions addressed in this work are how a meaningful REV can be defined for multiphase flow in porous media and what the associated scale actually is.

2 Theoretical Framework

2.1 Traditional Representative Elementary Volume (REV) Concept

The traditional representative elementary volume (REV) concept is introduced to facilitate the transition between pore and Darcy scale in terms of *space* only [11, **Error! Reference source not found.**] as illustrated in **Fig. 2**A. Each Darcy scale property, e.g., porosity, permeability, might have their own REV. In general, this will depend highly on the porous medium [13] (although for simple sandstone outcrop rocks the porosity and permeability REV were found to be both on the scale of 2-4 mm [12]). The porosity REV can be defined to be a length scale where the average void fraction is continuous and smooth enough to be used in differential equations describing flow at the Darcy scale. That does not necessarily mean that Darcy's law is invalid at smaller length scales. Simultaneously, we want the elementary volume to be small enough to resolve features important for flow. There are obvious lower limits to the REV, e.g., it cannot be smaller than individual pores where permeability is not well defined. In practice, the REV can be determined as the length scale at which the average becomes independent of the size of the averaging volume (length scale) [7,11,12], as illustrated in **Fig. 2**B. But there are also other methods to determine REV [15,14].

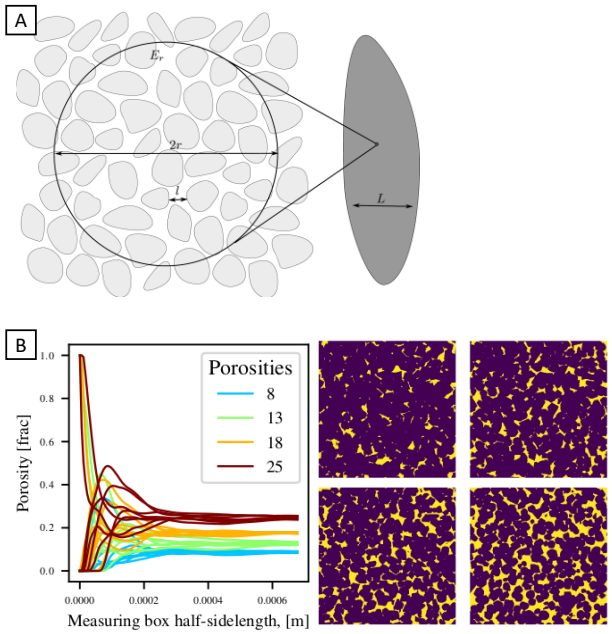

Fig. 2. Traditional representative elementary volume (REV) concept [11] marking the transition in space between pore scale

and Darcy scale (A). Illustration of how the porosity values converge for four porous media with different porosity (B).

Note that for heterogeneous rocks, in cases where the heterogeneity length scale(s) separate from the REV length scale(s), there can be a hierarchy of REVs starting from the level of lamina, lithofacies, facies etc. [**Error! Reference source not found.**]. For simplicity, here we only consider the case of homogeneous rocks. It is clear that the perspective of pore scale simulation, due to the presence of discrete pores, is different than a uniform porosity and permeability field in Darcy scale simulations. With homogeneous we mean that there is a well-defined porosity REV as displayed in **Fig. 2** and no macroscopic gradients in porosity and permeability at the REV length scale. For a more detailed definition of heterogeneity scales we refer to Ref. [2].

2.2 Averages in Space and Time

In addition to static properties, such as porosity and permeability, we are also interested in average dynamic properties at the Darcy scale, such as pressure and saturation. The source of the complication to define these averages is the dynamics of multiphase flow in space and time starting at the pore scale. Averaged could be defined as either average in space, in time or of ensembles. For ergodic systems these averages are identical. However, pore scale dynamics such as Haines jumps are non-ergodic because they travel faster, i.e., over longer distances at shorter time scales than diffusive mixing allows equilibration [17]. That means that meaningful averages need to be defined as *averages in space and time* [17]. A respective thermodynamic framework has been developed [17] and applied to the capillary fluctuations and energy dynamics in multiphase flow displacements in porous media [18]. In the context of experimental SCAL studies, a fixed spatial system will typically be considered based on the size of the core sample. Time-and-space averaging formalism can be used to identify the timescale over which flow processes can be treated as ergodic. Relative permeability coefficients measured at this scale are valid in the sense that they will accurately rate the energy dissipated by the flow.

In this paper we will follow derivations in [19] and use conservation of energy to obtain relative permeability as a coefficient linking time and space averages of flow velocity and pressure. The volume element we average over is not necessarily a REV in the traditional sense where ergodicity is obtained by considering a large volume element. In contrast, we find large scale representations of dynamic properties by integrating in both space and time. We therefore search an averaging time scale long enough to obtain a stationary energy balance, i.e., an averaging time for which there is no net input of energy into the system. In practice, this means an averaging time where the pressure-volume work done on the system balance the dissipated heat.

Conservation of energy can be expressed as

$$\frac{\partial}{\partial t}\frac{U}{\mathcal{V}} + \nabla \cdot \left(\frac{\mathbf{u}U}{\mathcal{V}}\right) - \boldsymbol{\sigma}\colon \nabla \mathbf{u} - \nabla \cdot \vec{q}_h = 0$$

where U is the internal energy of the volume $\mathcal{V}$, $\mathbf{u}$ is the flow field, $\boldsymbol{\sigma}$ is the stress tensor, and $\vec{q}_h$ is the heat flux. Here we assume that the constant reference volume $\mathcal{V}$ is small enough to assume ergodicity, thus any deviation from local equilibrium should be linear across $\mathcal{V}$. Thus $\mathcal{V}$ would be much smaller than the volume in our time and space average for obtaining relative permeability. Note also that we have disregarded chemical reactions in the above equation.

We now consider a volume element Ω with volume V and a time interval Λ with duration λ for our space and time average. Decomposing the stress tensor $\boldsymbol{\sigma} = -p\boldsymbol{I} + \boldsymbol{\tau}$ into the pressure p and the deviatoric stress tensor $\boldsymbol{\tau}$, the integral of the above energy conservation equation for our time and space elements yields

$$\int_\Lambda \int_\Omega \frac{\partial}{\partial t}\frac{U}{\mathcal{V}} + \nabla \cdot \left(\mathbf{u}\left(\frac{U}{\mathcal{V}} + p\right) - \vec{q}_h\right) - \mathbf{u} \cdot \nabla p - \boldsymbol{\tau}\colon \nabla \mathbf{u}\, dV dt = 0$$

The energy inputs and outputs to the system are found by using the Gauss divergence theorem on the second term in the integral above

$$\int_\Omega \nabla \cdot \left(\mathbf{u}\left(\frac{U}{\mathcal{V}} + p\right) - \vec{q}_h\right) dV = \int_\Gamma \mathbf{u}\left(\frac{U}{\mathcal{V}} + p\right) - \vec{q}_h dA$$

Here Γ is the surface of Ω. As we are seeking a stationary process, we are seeking a time interval with duration such that the energy inputs and outputs balance, thus the duration λ of the time interval Λ must be large enough to ensure that

$$\int_\Lambda \int_\Gamma \mathbf{u}\left(\frac{U}{\mathcal{V}} + p\right) - \vec{q}_h dA\, dt = 0$$

The internal work can be expressed as

$$\frac{1}{\lambda V}\int_\Lambda \int_\Omega \mathbf{u} \cdot \nabla p\, dV\, dt = \bar{\mathbf{u}} \cdot \nabla \bar{\phi}\bar{p} + \frac{1}{\lambda V}\int_\Lambda \int_\Omega \mathbf{u}' \cdot \nabla p\, dV\, dt$$

where the overline indicates time and volume average, and where we use Gray's decomposition $\mathbf{u}' = \mathbf{u} - \bar{\mathbf{u}}$ to link the local velocity to the average velocity. The porosity in front of the average pressure arises as we average the pressure inside the pore space, while the average velocity is based on the total mass and total momentum transfer.

We now seek a time interval Λ such that

$$\frac{1}{\lambda V}\int_\Lambda \int_\Omega \frac{\partial}{\partial t}\frac{U}{\mathcal{V}} - \mathbf{u}' \cdot \nabla p\, dV\, dt = 0$$

This implies that there is no net-work due to fluctuations in pressure and internal energy over the time interval Λ. The components of this equation can be studied directly from simulations.

With a time interval Λ so that two the integrals above becomes zero, we are then left with

$$-\bar{\mathbf{u}} \cdot \nabla \bar{\phi} \bar{p} = \frac{1}{\lambda V} \int_{\Lambda} \int_{\Omega} \boldsymbol{\tau} : \nabla \mathbf{u} \, dV dt$$

The right-hand side gives the viscous dissipation, which produce heat and therefore is positive. This allows us to approximate the average velocity by a linear expansion of the driving force

$$\bar{\mathbf{u}} = -\mathbf{L} \cdot \nabla \bar{\phi} \bar{p}$$

We will then find the traditional permeability tensor as $\mathbf{K} = \mu \bar{\phi}^{\mathbf{2}} \mathbf{L}$, yielding Darcy's law as

$$\bar{\mathbf{q}} = -\frac{\mathbf{K}}{\mu} \cdot \nabla \bar{p}$$

where $\bar{\mathbf{q}} = \bar{\phi} \bar{\mathbf{u}}$ is the Darcy velocity.

For Stokes flow there is no time dependence, so there is no need for including time in the average to obtain Darcy's law. However, for multiphase flow we have dynamics that require the inclusion of time into the average.

Assume a two-fluid system, where the space and time average of the mass fraction of each fluid is given as $\bar{\omega}_i = \bar{M}_i / \bar{M}$ and the fluid velocity is the average velocity of the given fluid phase $\bar{\mathbf{u}}_i$, where the subscript *i* represents the two fluid phases $i = n, w$. Splitting the average velocity $\bar{\mathbf{u}}$ into the phase velocities $\bar{\mathbf{u}}_i$, we obtain the equation

$$-(\bar{\omega}_w \bar{\mathbf{u}}_w + \bar{\omega}_n \bar{\mathbf{u}}_n) \cdot \nabla \bar{\phi} \bar{p} = \frac{1}{\lambda V} \int_{\Lambda} \int_{\Omega} \boldsymbol{\tau} : \nabla \mathbf{u} \, dV dt \geq 0$$

Here the two fluids are considered to have the same driving force $\bar{\phi} \bar{p}$. This driving force cause a corresponding response for each fluid, as they from our derivations are treated together. This gives

$$\bar{\omega}_i \bar{\mathbf{u}}_i = -\mathbf{L}_i \cdot \nabla \bar{\phi} \bar{p}$$

If the fluids are immiscible and incompressible, then the mass fraction will equal the saturation, $\bar{s}_i = \bar{\omega}_i$, and we obtain a relative permeability equation by letting $\mathbf{L}_i = k_i^r \mathbf{K} / \mu_i$.

In contrast to traditional relative permeability expression, our derived expression uses the total average pressure instead of the phase pressures. If the total pressure gradient is expanded into a sum over the phase pressures, we will pick up cross-coupling terms. Such cross-coupling terms also emerge from averaging of the momentum equations [21].

2.3 Degrees of Freedom and Energy Dynamics

The previous section details the derivation of the multiphase extension of Darcy's law, which is essentially a mechanical relationship. For this relationship to hold, the internal energy dynamics must be stationary. This imposes a constraint on the thermodynamic behavior, since the stationary system must reflect a balance between the rate of work performed and the generated heat. The form of the constraint will depend on the thermodynamic representation; these choices determine how to account for the energy within the system. A natural choice is to separately account for the energy in oil, water and system interfaces. Subject to the assumption that the oil and water phases are chemically homogeneous, the internal energy is assumed to depend on the entropy S, the volume of each fluid V_w and V_n, the meniscus area A_{wn}, the solid surface area $A_s$and the film thickness h_s

$$U = U(S, V_w, V_n, A_{wn}, A_s, h_s)$$

In the stationary (time-averaged) system, there is no net change in any extensive measure of the system state. If the system is iso-thermal and the fluid-fluid interfacial tension is constant then the fact that the change to the internal energy is zero for a stationary process is expressed based on the time-and-space integral [19],

$$\frac{\partial \bar{U}}{\partial t} = \frac{V}{\lambda} \int_{\Lambda} \left[\frac{A_s}{V} \left(\frac{\partial \gamma_s'}{\partial t} - h_s \frac{\partial \Pi_s'}{\partial t} \right) - \phi_w \frac{\partial p_w'}{\partial t} - \phi_n \frac{\partial p_n'}{\partial t} \right] dt = 0$$

The timescale at which this expression holds corresponds to the time required for the system to be treated as ergodic. Deviation terms for the fluid pressure, fluid-solid interfacial tension and disjoining pressure are defined as follows

$$p_w' = p_w - \bar{p}_w$$

$$p_n' = p_n - \bar{p}_n$$

$$\gamma_s' = \gamma_s - \bar{\gamma}_s$$

$$\Pi_n' = \Pi_n - \bar{\Pi}_n$$

At the very slow timescale where diffusion becomes dominant, these deviation terms should be zero and associated fluctuations will be Gaussian. At faster timescales the behavior may be inhomogeneous and fluctuations may be non-Gaussian. However, the stationary constraint can still be

satisfied provided that the deviation terms perform no net work. In other words, symmetries within the system (e.g., due to equal and opposite forces where one fluid is pushing against the other) can lead to a situation where the multiphase extension of Darcy's law is valid even if when strong non-linearities may be evident from the pressure signal.

3 Lattice Boltzmann Simulations

Time-and-space averages provide a particular advantage with respect to the analysis of pore-scale simulation data. Pore-scale quantities can be directly converted to core-scale quantities by integrating over the sample domain. Within this context, lattice Boltzmann methods provide a powerful tool to obtain detailed information about pore-scale flow behavior. Efficient GPU-based simulations can resolve multiphase flow dynamics based on 3D image data. The associated "digital rock physics" workflows have proven to be a useful tool to determine permeability and relative permeability from first principles [23].

The lattice Boltzmann method has also proven to capture pore scale fluctuations in a realistic manner. For instance, the fluctuations obtained during drainage by a 2-phase LBM simulation (Fig. 18 in [32]) are very similar to those obtained in pore scale experiments (Fig. 4a in [33]).

The advantage to using pore scale simulations instead of experimental data in this context is that: (1) the full pressure and velocity fields are obtained; (2) the fluid saturation can be kept constant by construction. For the first point, this information is difficult (if not impossible) to obtain experimentally; (3) the wetting condition assigned to the grain is known (local wetting state is very difficult to measure experimentally and has high uncertainty). For the second point, many experimental systems are associated with fluctuations in the saturation, and simulation provides a way to study a system where they do not occur based on the way the simulations are constructed.

Previous comparisons have been made between the LBM simulator and experimental systems, including micromodel systems (see [34]) and also for Bentheimer sandstone (which is used in the present work [23]). The LBPM simulator has been used before to simulate the full range of flow regimes from connected pathway flow to ganglion dynamics and has been cross validated against experimental data in terms of pore scale fluid topology [24]. The LBPM software package includes several simulation protocols targeted towards SCAL workflows [23]. Among these is a "fractional flow" protocol designed to measure relative permeability based on steady-state immiscible displacement in digital rock images. Each point on the relative permeability curve is obtained by performing a flow simulation with a constant driving force and constant fluid saturation. Fully periodic boundary conditions are used to limit the influence of boundary effects. Each simulation is performed with a target capillary number, which is typically chosen such that results are obtained within the capillary dominated flow regime (generally Ca < 1e-5). Once a stationary steady-state is achieved for a particular fixed-saturation flow process, an internal flow adaptor is applied to change the saturation [23,24]. The fractional flow protocol has been validated against experimental data for a range of wetting conditions. Provided that digital rock image data is sufficiently well-resolved, simulations are able to recover reasonable trends for the endpoint behavior and the influence of rock wetting property on fractional flow curves.

4 Results

In this work, the 3D pore structures extracted from micro-CT images of Bentheimer sandstone with a resolution of 1.66 micrometer were used to define interior boundary conditions of flow modeling in a pore-scale lattice Boltzmann simulator, LBPM, an open-source software package that is developed to simulate fluid flow through porous media. Bentheimer sandstone was chosen for this study because it is a rock that has widely studied experimentally with a relatively uniform pore-size distribution. The specific digital image of Bentheimer rock used in these simulations is publicly available online [35] along with selected fluid configurations from different wetting states that were obtained from LBPM simulations [36]. The simulations were performed with two computational domains: one has a maximum dimension of 1494 x 1494 x 2656 micrometer after removing the region outside of the core holder; the second domain one has a smaller dimension of 747 x 747 x 1328 micrometer extracted from the center of the larger domain. In the simulation, the target capillary number was Ca = 1e-5 (which is just below the onset of capillary de-saturation and in a previous study only minor differences to Ca = 1e-6 were observed for Gildehauser rock [24] which is very similar to the Bentheimer used in this study), interfacial tension is 25 mN/m, and the density and viscosity ratio between oil and water are one. Morphological opening is applied to establish initial conditions and instantiate two surface wetting boundary conditions: water-wet and oil-wet. With respect to contact angle θ , for "water-wet" cases $\cos(\theta) = 0.9$, and for "oil-wet" cases: $\cos(\theta) = -0.6$ was used. Additional details on the simulation wetting conditions are provided in [23].

The simulated saturations are listed in Table 1.

Table 1. LBPM Simulated saturations for water-wet and oil-wet conditions.

	Saturations
Water-wet	0.22, 0.43, 0.58, 0.68
Oil-wet	0.18, 0.37, 0.53, 0.72

Simulations were performed until the dynamics were sufficiently stationary, meaning the measured flow rate does not drift with time.

4.1 Examples of Fluctuations

From the theoretical standpoint, time-and-space averaging provides practical guidance to the application of SCAL methods. Capillary noise is a common feature for both computational and experimental results. As fluids move through the pore structure, capillary forces cause interfaces to pin in the pore throats before moving in a "bursty" manner as a consequence of Haines jumps [25, 10]. These events are directly evident from the experimentally measured pressure signal. Pore-scale events are frequently large enough to cause significant pressure fluctuations; time-averaging the pressure signal provides a straightforward way to remove this noise so that an average pressure can be clearly identified, which is in turn needed to determine the relative permeability based on the pressure drop across the sample.

Different behavior is observed for these two wetting cases (water-wet and oil-wet). The fluctuations depend not only on the wetting condition, but also the fluid saturation. Within the same rock and at the same wetting condition, the structure of capillary fluctuations will change dramatically as the saturation changes. This is because the amount of intermittency is strongly saturation dependent.

Our theory provides a clear quantitative interpretation of pressure fluctuations in the context of relative permeability. We assert that the standard relative permeability provides a valid representation of the energy dissipation provided that the stationary timescale is chosen such that the net work done by pressure fluctuations is zero. Given a stationary flow process, the net work done by pressure fluctuations will tend toward zero as the averaging timescale increases. A fluctuation condition can be derived to describe this situation [19]

$$\frac{1}{\lambda}\int_{\Lambda}\left[\frac{A_s}{V}\left(\frac{\partial\gamma_s'}{\partial t}-h_s\frac{\partial\Pi_s'}{\partial t}\right)-\phi_w\frac{\partial p_w'}{\partial t}-\phi_n\frac{\partial p_n'}{\partial t}\right]dt\rightarrow 0$$

The first term corresponds to changes in the solid wetting energy based on movement of the contact line and changes in the film disjoining pressure. The second two terms are associated with fluid pressure fluctuations. The form expressed above has units of power per unit volume and characterizes the energy dynamics associated with fluctuations to pressure and wetting state. In a stationary process, the pressure and wetting energy will fluctuate around a stable value. If a process is observed for a sufficiently long period of time, the integral fluctuation criterion will be zero. Generally, one expects that a physically larger sample will require a longer period of time to achieve stationary conditions based on this criterion. However, a stationary process in a finite-sized sample should be associated with a finite timescale beyond which the contribution of fluctuating modes is negligible in comparison with the overall flow dynamics.

Time series of fluctuations for the water-wet and oil-wet cases are shown in **Fig. 3** in terms of contribution to the energy dynamics e.g. the pressure-volume terms $-\langle\frac{\phi_w\partial p_w'}{\partial t}\rangle$ and $-\langle\frac{\phi_n\partial p_n'}{\partial t}\rangle$.

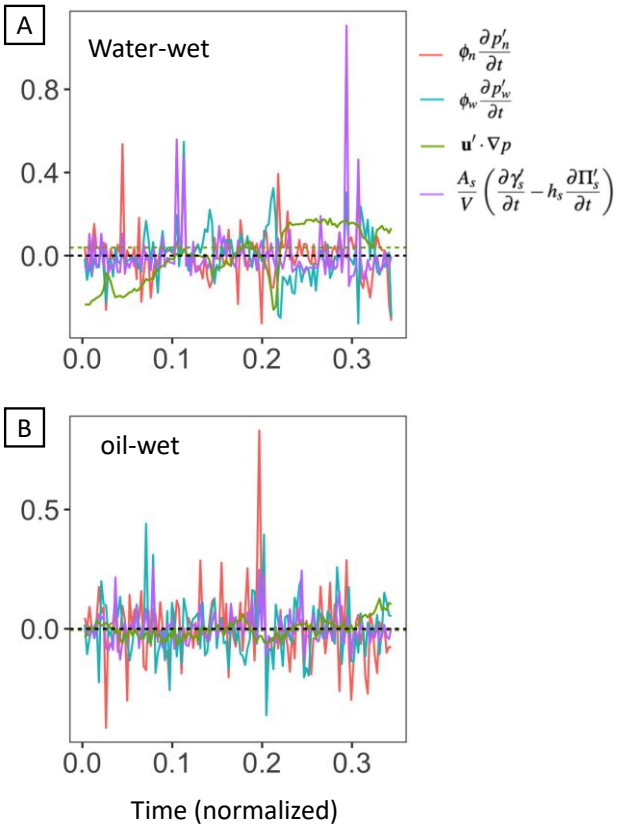

Fig. 3. Time series of fluctuations obtained in LBPM simulations for water-wet (A) and oil-wet (B) conditions. The red and blue lines refer to pressure-volume work fluctuations. The dashed black line denotes "0" and the dashed green line is the average of all contributions. [19]

4.2 Energy Dynamics of Fluctuations

In **Fig. 4** histograms for the contribution for each term is shown in, considering two different saturation values in each of the two domain sizes (columns). The averaging time is $\leq 1\ s$ which is shorter than a typical experiment.

In general, fluctuating contributions are expected to follow a Gaussian distribution in the limit of infinite time. If this is the case, each fluctuating term will be individually zero. We argue that the relative permeability provides a valid representation of the Darcy-scale flow process provided that the sum of the terms is zero, which is less restrictive and may be reached more quickly. Due to the existence of intermittent flow pathways, distributions are frequently multi-modal. Individual fluctuating contributions are not necessarily zero mean, since energy transfers between modes can result in a

net gain or loss of energy. However, because the energy lost by one mode is gained by another, the sum of the fluctuating modes will tend to be closer to zero. Since a system will not accumulate energy internally based on a stationary flow process, the relative permeability is sufficient to represent the energy dissipated.

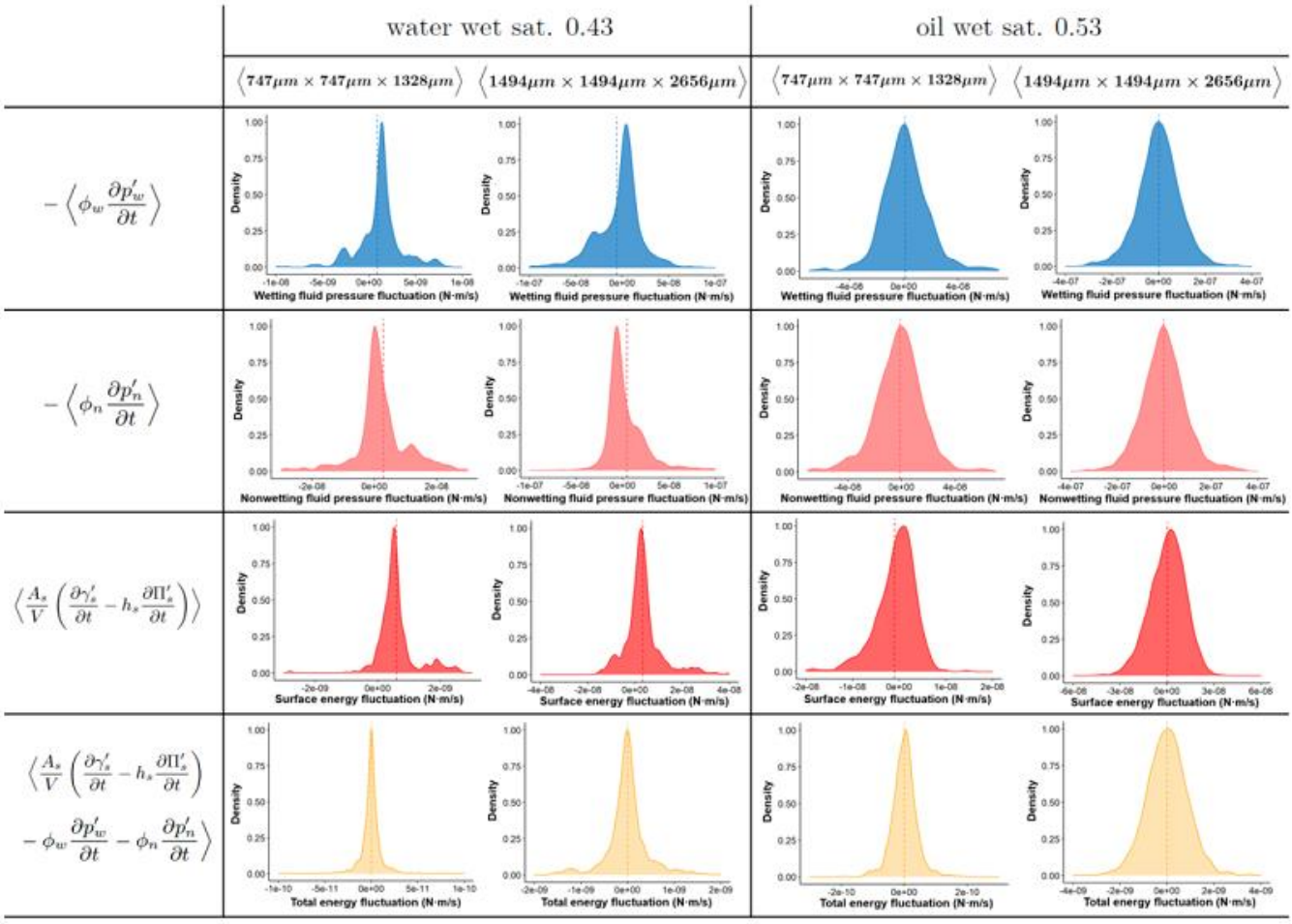

Fig. 4. Histograms for fluctuation terms for two different saturations ($S_w = 0.43$ and $S_w = 0.53$) for two different systems sizes in a Bentheimer sandstone [23]. Stationary behavior requires that the power of associated with fluctuation terms is zero over the timescale for relative permeability measurement. Lattice Boltzmann simulation provides a way to evaluate dynamic effects due to the fluid pressures and wetting state to assess how these contributions influence the overall energy dynamics of flow processes.

The representative timescale for a stationary process is linked with the length of time needed to observe cancellation between the fluctuating contributions. At faster timescales, the system dynamics may include apparently non-stationary contributions that are due to these fluctuations. In other words, it is necessary to identify the timescale where the flow process can be treated as stationary to be able to homogenize the non-linear contributions. From a spectral standpoint, this means that the frequency for the fluctuations (and the timescale for underlying pore scale events) must be fast in comparison with the averaging timescale. In a digital rock simulation, all contributions can be evaluated directly, including the energy due to wetting state. Average pressure values can be determined using volume averaging, consistent with theoretical developments presented in the literature [30,31]. In the computational setting it is straightforward to separately identify the pressure values for water and oil, since these are clear based on the spatial sub-division of the system. In SCAL experiments these measurements will usually be obtained from pressure transducers located at the boundary. These measurements will not necessarily correspond to the volume averaged pressures, and it may not be straightforward to associate measured pressure values with one fluid or another. Even so, boundary fluxes must necessarily be consistent with the interior flow dynamics, since these are the energy inputs to the system. Since a stationary system should not accumulate energy, internal fluctuations will be closely correlated with those measured at the boundary based on the flow rate and pressure.

5 Discussion

5.1 Implications for Digital Rock

The really important observation in Fig. 3 is that already at a length scale of 1-2 mm we observe that the histogram of the combined fluctuations has a zero mean. This length scale roughly coincides with porosity and permeability REV for simple sandstone rock such as Bentheimer rock [12] (the rock in that study was Gildehauser which is from the Bentheimer family). This result shows that the domain size of several mm can already lead to meaningful results for multiphase flow parameters. That is really good news for the concept of Digital Rock because the alternative would have been to base an REV the size of the largest non-wetting phase cluster sizes which would be at the relevant capillary numbers more of the order of several cm [26] which is also largely consistent with the length scale at which a (single-time snapshot) saturation REV converges [7]. An REV at that length scale would have rendered Digital Rock approaches which are all based on micro-CT samples of few mm size (because only these provide sufficient resolution to resolve the pore structure as input for e.g. direct numerical simulation [27-29]) as highly impractical. It would have been very difficult to image a several cm sized sample at 1-2 micrometer resolution and also simulate multiphase flow on such a computational domain, at least for direct numerical simulation approaches.

The main implication of our work is that we can measure a valid relative permeability coefficient even if there is no spatial REV. What matters is that the coefficient accurately matches the rate of dissipation for the process. To be able to do this (1) the process needs to be stationary; and (2) we need to measure the process for a sufficiently long period of time, long enough that the noise sources cancel out. In this context, capillary fluctuations are a noise source and measurements need to be averaged over a timescale that is large compared to the scale of the fluctuations. In general, as the system becomes physically smaller less time is required based on diffusive scaling ($[L]^2 \sim [t]$). Measurements in a smaller system should require less time.

The ergodic requirement ultimately requires a long enough timescale for the process of interest. The size of the spatial system constrains this timescale. As long as we measure the process for "long enough" then we can characterize the rate of dissipation for the process with a single scalar coefficient (since dissipation is a scalar). A spatial REV is then the scale at which the coefficient you measure becomes invariant with respect to changes in system size.

In summary, it is important to see meaningful results for 2-phase flow direct numerical simulations can be obtained for domain sizes of typical micro-CT sample sizes, at least for typical rocks (we cannot exclude that for more complicated or more heterogeneous rocks the domain size would be different, but so would be conventional porosity and permeability REV).

The main reason why the domain size is smaller than a single-time snapshot saturation REV [7] is that we need to employ both time and space averages. It is also important to point out that these are not arbitrary choices, but consequences of a well-founded theoretical framework [17-19] which shows that for exactly this averaging methodology the functional form of the 2-phase Darcy equations is obtained. In other words, the concept including theoretical framework, average procedure, flow equations and length + time scales are consistent. This does, in the end, provide the theoretical basis for conducing Digital Rock in the multiphase flow space.

5.2 Defining uncertainty for relative permeability

In a practical sense, one can also use the fluctuation condition as a measure of uncertainty for the relative permeability coefficient. This can be considered to define a process-dependent representative timescale." This is obtained by considering the ratio of the power associated with fluctuating modes in the system relative to the rate of work done by Darcy flow,

$$\varepsilon_\lambda = \frac{\left\langle \frac{A_s}{V}\left(\frac{\partial \gamma_s'}{\partial t} - h_s \frac{\partial \Pi_s'}{\partial t}\right)\right\rangle - \left\langle \phi_w \frac{\partial p_w'}{\partial t}\right\rangle - \left\langle \phi_n \frac{\partial p_n'}{\partial t}\right\rangle}{\left(\bar{\omega}_w \bar{\mathbf{u}}_w + \bar{\omega}_n \bar{\mathbf{u}}_n\right) \cdot \nabla\left(\phi \bar{p}\right)}$$

This quantity defines the accuracy of a relative permeability measurement in terms of the fluctuations. From the standpoint of non-equilibrium theory, the coefficient embedded in the relative permeability must account for the energy dissipated by the flow process, which determines the efficiency. Fluctuations can confound this measurement because some of the energy in the system is contained within the fluctuating modes. This energy cannot be directly attributed to dissipation. For example, some of the pressure-volume energy that accumulates prior to a Haines jump is reversibly transferred to surface energy, which can be stored and then later released in future Haines jumps. The net effects can only be homogenized if the timescale for averaging is larger than the timescale for the underlying pore-scale events. For a stationary process, this quantity will go toward zero as the time interval becomes sufficiently large in comparison with the timescale for pore-scale events and other relaxation timescale associated with the flow behavior.

Therefore, in essence, there is no simple answer for the question "how long do I need to measure". Ideally the measurement time would be determined until the uncertainty in the relative permeability coefficient decreased below a desired value based on the ϵ_λ criteria. In a simulation one might be able to directly measure it and translate it in a convergence criteria. But in an experiment the wetting contributions will be very difficult to infer, so there will be additional measurement uncertainty which is difficult to assess. The other angle to this question is that there is probably no general estimate because the averaging time depends on the size and type of porous media (rock) and also the flow process. The underlying reason why we can simulate relative permeability coefficients relatively quickly in terms

of physical time are (1) the sample size is small and (2) the conditions are often chosen such that fluid saturation does not fluctuate. In an experiment where saturation fluctuates, the associated timescales will be much longer, and the timescale for those slower fluctuations will determine how long one would need to average. That is one of the reasons why the convergence time may be in "steady-state" experiments a strong function of the fractional flow f_w because for different fractional flows different flow regimes (connected pathway flow vs. ganglion dynamics [5]) are encountered [10].

6 Summary and Conclusions

We present an approach to characterize the representative timescale for immiscible fluid flow processes in porous media. The proposed quantities can be measured directly from pore-scale simulation or SCAL experimental data and provides a way to characterize the uncertainty associated with relative permeability coefficients. Conceptually, the idea is related to the spatial representative elementary volume (REV) for the system, but with important practical differences. In the laboratory setting, relative permeability measurements will always be performed based on a particular fixed sample. Sample sizes may be significantly smaller than a spatial REV, or a spatial REV may not even exist in the reservoir-scale system. Even if spatial REV requirements are not met, valid relative permeability coefficients can still be obtained to characterize flow processes at the laboratory scale. In this sense, a coefficient is valid in the sense that it accurately represents the rate of energy dissipation. Our theory provides rigorous support for such measurements provided that a flow process is stationary. Stationary conditions are achieved by considering a sufficiently long timescale such that fluctuating contributions to the energy dynamics become negligible. We show that cancellation between fluctuating modes is not unusual, e.g. fluctuations in the water phase pressure may be closely mirrored by fluctuations in the oil phase pressure. This symmetry can reinforce the accuracy of a relative permeability measurement, allowing for accurate measurements even in situations where the fluctuations are not Gaussian. An explicit criterion is suggested to measure the accuracy of relative permeability coefficients based on the ratio of the work due to fluctuations relative to the work of the overall flow process. This result is linked with an explicit method to derive the conventional relative permeability based on time-and-space averaging theory and supports rigorous upscaling of pore-scale simulations as well as laboratory SCAL experiments.

Signe Kjelstrup, Dick Bedeaux, and Alex Hansen are acknowledged for helpful discussions.

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