

Derivation of Waxman-Smiths and Indonesia parameters from Co-Cw experiments: a new perspective

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Abstract. In shaly formations where water salinity is above 20-30 kppm, electrical log-derived saturations are generally obtained from the Waxman-Smiths (WS) equation. This paper presents a method to obtain the WS formation factor F^* and the density of charge in the double layer Q_v from a multisalinity test by processing the rock and water conductivity data (C_0 - C_w) in a way that is different from what is normally done. The classic interpretation of a multisalinity test is based on plotting C_0 versus C_w . Typically, four salinity steps are performed and four data points are consequently generated at the end of the experiment. A preliminary control of the alignment of these points is required and those points that do not lie in the linear region of the C_0 - C_w curve must be discarded. With the new method, instead of plotting C_0 vs C_w , at each salinity step we draw a straight line in which all salinity-dependent quantities are treated as parameters while F^* and Q_v , which are salinity-independent, are treated as variables. With four salinity steps, four lines are drawn and they mutually intersect at six points whose coordinates are $\{X=Q_v, Y=F^*\}$. In theory these six intersection points are coincident. Based on this argument, we then obtain the mean values and the uncertainties of F^* and Q_v from the centre of mass and the standard deviations of the coordinates of the six intersection points. The same approach is utilized with the Indonesia equation, which is the most widely used equation when water salinity is below 20 kppm. In this case, the six intersection points provide the expectation values and the standard deviations of the Indonesia formation factor F and clay conductivity C_{cl} . A practical example is reported, where we demonstrate the method and compare the results to those from the standard approach. As far as the WS analysis is concerned, the new method gives a similar F^* but a mean Q_v that is 10% lower than that obtained from the standard approach. In the Indonesia analysis, more substantial differences are observed: the formation factor F is 35% greater and clay conductivity C_{cl} is more than doubled compared to the values obtained with the standard interpretation. The impact on log interpretation may be significant.

1 Introduction

The Waxman-Smiths [1, 2] and the Indonesia [3] equations are widely used to estimate S_w in shaly sands. The Waxman-Smiths equation gives the fully saturated conductivity of rock, C_0 , as $C_0=[C_w+BQ_v]/F^*$, where C_w is the saturant brine conductivity, B and Q_v are the mobility of counterions and the density of charge in the double layer (the BQ_v product represents the double layer conductivity), and $F^* = \Phi_{tot}^{-M^*}$, where Φ_{tot} is the total porosity and M^* is the Waxman-Smiths cementation exponent. While B is known from published formulas, Q_v must be obtained experimentally, and this can be done in two ways: with a Cation Exchange Capacity measurement or a multisalinity test. This paper focuses on multisalinity tests, which have the advantage of being non-destructive and provide not only Q_v but also F^* .

A multisalinity test consists of measuring the electrical conductivity of the tested rock sample at fully brine saturated conditions, with different brines. Four brines are generally used. In the standard approach, the conductivity C_0 of the saturated core sample is plotted against the saturant brine conductivity C_w , and F^* is obtained from the slope of the C_0 - C_w line. Then, BQ_v is derived from the C_w intercept at $C_0 = 0$. The underlying assumption is that the C_0 - C_w curve is a

straight line. For this assumption to hold, C_w must be large enough. When C_w is low, namely when brine salinity becomes lower than 50 kppm, B starts to be dependent on C_w (**Fig.1**) resulting in a C_0 - C_w relationship that is no longer linear. Of course, all C_0 - C_w data points lying in the non-linear region must be discarded in the standard multisalinity data interpretation.

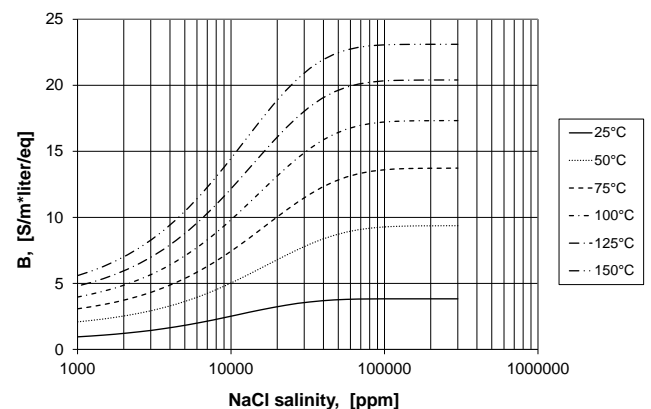


Fig.1 – Dependence of counterions mobility B on temperature and brine salinity, according to Dacy and Martin [4].

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When water salinity is lower than 20 kppm, the Indonesia equation is considered to represent a valid alternative to Waxman-Smits. The Indonesia equation gives the fully saturated conductivity of rock as $\sqrt{C_0} = \sqrt{C_w/F} + V_{cl}^{1-V_{cl}/2} \sqrt{C_{cl}}$, where V_{cl} is the clay volumetric fraction, C_{cl} is the clay conductivity and the Indonesia formation factor is defined as $F = \Phi_{\text{eff}}^{-M}$ (with Φ_{eff} = effective porosity and M = Indonesia cementation exponent).

The presence of square roots makes the Indonesia equation different from most shaly sand equations (whose general form is $C_0 = C_w \Phi^u + X$) and relates it more to CRIM (Complex Refractive Index Model), a widely used model which describes the average dielectric permittivity in composite systems [5].

In log interpretation practice, it is commonly assumed that the M exponent coincides with the m exponent of Archie's equation (Archie's equation for the fully water saturated case is $C_0 = C_w \Phi_{\text{tot}}^m$). The argument supporting the $M = m$ assumption is that the Indonesia equation reduces to Archie's equation when V_{cl} tends to 0, as the term containing V_{cl} vanishes and Φ_{eff} tends to Φ_{tot} . Because of the convergence of the two equations in the $V_{cl} \rightarrow 0$ limit, not only it is assumed that $M = m$ but it also became common practice to obtain the Indonesia parameters for log interpretation by fitting the laboratory data with the Archie equation even when the tested samples are not clay-free. This generates errors in the log-derived water saturation profiles. The method discussed in this article provides a solution to this problem.

2 Method and experimental data

2.1 Waxman-Smits parameters extraction

According to the Waxman-Smits model, the electrical conductivity of a fully water saturated rock is expressed as

$$C_0 = \frac{1}{F^*} [BQ_v + C_w] \quad (1)$$

where:

- C_0 [S/m] is the conductivity of fully water saturated rock. It depends on the characteristics of the rock, the conductivity of the saturant brine and the temperature. Thus: $C_0 = f(\text{rock}, C_w, T)$.
- C_w [S/m] is the conductivity of the saturant brine. It depends on the salinity of the brine and the temperature. Thus: $C_w = f(\text{salinity}, T)$.
- F^* is a dimensionless quantity. It depends on the characteristics of the rock but is independent of the characteristics of the saturant fluid. Thus: $F^* = f(\text{rock})$.
- B [S/m-liter/eq] is the mobility of exchange cations. Charge mobility is defined as the ratio v/E between the average velocity of the charge carriers and the intensity of the applied electric field. B is a function of the water conductivity and temperature. Thus: $B = f(C_w, T)$. B increases as the water salinity increases and reaches a plateau at high salinity values. Although the Juhasz equation [6] is the most used in log interpretation, probably the most accurate formula for B is that reported by Dacy

and Martin [4]. Expressing B in [S/m-liter/eq], C_w in [S/m] and T in [°C], it gives:

$$B = \left\{ 1 - 0.83 \cdot \exp \left[-C_w \cdot \exp \left(-2.38 + \frac{42.17}{T} \right) \right] \right\} \cdot [-3.16 + 1.59 \cdot \ln(T)]^2 \quad (2)$$

- Q_v [eq/litre] is the exchange cations concentration (i.e. the charge in the double layer per unit pore volume of water). It depends only on the characteristics of the rock. Thus: $Q_v = f(\text{rock})$. Namely, Q_v is a function of Cation Exchange Capacity (CEC), total porosity and grain density and is given by $Q_v = CEC(1-\Phi)\rho_g/\Phi$. Quantity BQ_v represents the contribution of surface conduction mechanisms to the overall electrical conductivity. This reflects the fact that electrical conductivity is the product of charge carrier concentration by charge carrier mobility.

As we mentioned earlier, a multialinity test typically consists of four steps. After reaching equilibrium at each brine injection step, the conductivity of the sample under examination is measured. According to the best practices [7], the sample is flooded with at least 10 pore volumes of brine and brine salinity is progressively reduced during the injection steps. Brine is injected at low rate (less than 0.1 ml/min) to prevent fines movement. Conductivity is considered as stabilized when the change in the C_w/C_0 ratio between successive measurements is less than 2%. The experiment described here was performed using these procedures. We did not try the opposite brine sequence, i.e. from the lowest brine conductivity to the highest, but in theory the same results should be obtained.

While brine salinity and conductivity change from one step to another, the petrophysical characteristics of rock (pore space geometry, mineralogy, etc.) are assumed to remain the same. This means that parameters F^* and Q_v , which are dependent on the rock characteristics only, can be treated as constants.

In equation (1) the conductivities C_0 and C_w are known because they are measured during the multialinity test; the B parameter is known because it can be calculated from equation (2) or a similar equation [1, 2, 6, 8]. The only two unknowns, therefore, are F^* and Q_v . By re-naming such quantities Y and X , respectively, it is possible to re-write equation (1) as:

$$Y = \frac{1}{C_0} [B \cdot X + C_w] \quad (3)$$

Equation (3) defines a straight line in the Y - X plane whose slope is B/C_0 . The $X=0$ intercept is C_w/C_0 . Since C_0 , B and C_w depend on the conductivity of the saturant brine, each salinity step will be assigned a characteristic straight line (both the slope and the intercept will change). F^* and Q_v , on the other hand, depend only on the characteristics of the rock and do not change during the multialinity experiment. That implies that all the straight lines intersect at the same point, in theory. This is the key point of the new method. The coordinates of the intersection point are Q_v and F^* . In practice, a bundle of straight lines is drawn for each multialinity test and the coordinates of the centre of the bundle give F^* and Q_v . Compared to the classic method, this new interpretation

method uses all the available data, including those acquired at a low brine conductivity. That should improve the quality of the results: since F^* and Q_v are obtained from a greater amount of experimental observations, the accuracy should be higher. Moreover, the new procedure is less subjective, as no judgement is required from the analyst about which data should be used.

Of course, a bundle of straight lines with a single centre is to be expected only in theory. In practice, the straight lines will intersect at different points, due to experimental errors and the approximations inherent in the Waxman-Smiths model. The centre of mass of the intersection points gives the expectation values of F^* and Q_v , while the dispersion of the points (more precisely the standard deviation of their X and Y coordinates) represent a measure of the uncertainty in the F^* and Q_v estimation.

2.2 Indonesia parameters extraction

Even though this does not occur frequently in practice, it seems obvious that the model to be used to fit the laboratory data in order to obtain the Indonesia parameters should be the Indonesia model. The Archie model cannot be used unless the analysed plug is totally free from clay minerals. This is a very rare fact. For a brine saturated rock, the Indonesia equation is:

$$\sqrt{C_0} = \sqrt{\frac{C_w}{F}} + V_{cl}^{1-V_{cl}/2} \sqrt{C_{cl}} \quad (4)$$

According to [3], we consider the form of the Indonesia equation that uses clay and not shale. Besides C_0 and C_w , which have already been defined, the other variables in equation (4) are F , V_{cl} and C_{cl} .

- F is a dimensionless quantity that depends only on the characteristics of the rock. Thus: $F = f(rock)$.
- V_{cl} [dimensionless] is the clay fraction and depends only on the characteristics of the rock. Thus: $V_{cl} = f(rock)$.
- C_{cl} [S/m] is the clay conductivity. It depends on the type of clay that is present and, to a lower extent, on C_w . Here we will assume that the C_w dependency can be neglected or, equivalently, that C_w is high enough. With this assumption, also C_{cl} can be considered as a function of the rock characteristics only. Thus: $C_{cl} = f(rock)$.

In equation (4) the unknowns are F and C_{cl} , whereas the other quantities (C_w , V_{cl} and C_0) are known. As a matter of fact, V_{cl} is either directly measurable or indirectly derivable from the logs around the plug; C_w and C_0 are directly measured during the multialinity test. The way we proceed is similar to the Waxman-Smiths analysis. In practice, the quantities that vary from a salinity step to another (C_w and C_0) are treated as parameters, while the fixed quantities that do not vary during the entire multialinity experiment (F and C_{cl}) are treated as variables. By putting $C_{cl}^{1/2} = X$ and $F^{-1/2} = Y$, we re-write equation (4) as:

$$Y = -X \frac{V_{cl}^{1-V_{cl}/2}}{\sqrt{C_w}} + \sqrt{\frac{C_0}{C_w}} \quad (5)$$

Equation (5) represents a straight line in the Y - X plane. One peculiar straight line is associated with each salinity step. All the straight lines are different because C_w and C_0 change from one salinity step to another, generating a slope that is equal to $-V_{cl}^{1-V_{cl}/2} \cdot C_w^{-1/2}$ and an intercept equal to $(C_0/C_w)^{1/2}$ that are characteristic of that particular step. However, since C_{cl} and F depend only on the characteristics of the rock, the straight lines must intersect at the same point. The coordinates of this point are $X = C_{cl}^{1/2}$ and $Y = F^{-1/2}$. From these we can derive the most probable values of C_{cl} and F and their associated standard deviations.

2.3 Experimental procedures and data

Tab.1 reports a set of multialinity data measured on a 1.5 inch diameter sample having an ambient porosity of 9.3% and an ambient Klinkenberg permeability of 0.27 md (sample length is 5 cm).

Tab.1 – Multialinity data set used to extract the F^* and BQ_v parameters of the Waxman-Smiths model, and the F and C_{cl} parameters of the Indonesia equation.

	NaCl Salinity [ppm]	C_o [S/m]	C_w [S/m]
Step 1	120000	0.164	15.60
Step 2	90000	0.138	12.59
Step 3	60000	0.104	8.93
Step 4	30000	0.064	4.90

The mineralogical composition of the sample includes quartz, feldspars, micas and chlorite. The clay volume assigned to the sample on the basis of mineralogical analysis is 0.112. The multialinity test was performed at ambient conditions (temperature = 25°C). Four salinity steps were performed (equivalent NaCl salinities: 120, 90, 60 and 30 kppm). Brine composition included NaCl, CaCl₂ and KCl (the latter was added to prevent adverse brine-rock reactions). At each salinity step, the sample was flooded at a constant injection rate of 0.1 ml/min for several hours and conductivity was monitored. The C_o data reported in **Tab.1** were taken after conductivity stability was achieved. Equilibrium was considered as reached when C_o changes were less than 2% after injecting an additional half pore volume of brine. All conductivity measurements were done with the Novocontrol Alpha impedance analyzer.

We will use this dataset to show how the proposed method works and to compare the results of the new method to those obtained with the standard approach.

3 Results and discussion

3.1 Waxman-Smiths analysis

Fig.2 and **Fig.3** show two interpretations of the multialinity data reported in **Tab.1**. **Fig.2** illustrates the classic Waxman-Smiths interpretation. The data point that was measured during step 4 has been discarded because it is not perfectly lined up with the first three points and is considered to lie in the low water conductivity region where the C_0 - C_w curve is not linear.

This subjective judgement introduces a certain level of uncertainty in the result. The BQ_v and F^* values that are obtained by interpolating the three valid data points are 2.66 S/m and 111, respectively (the slope of the C_0 - C_w line is $1/F^*$ and so the 111 value correspond to the reciprocal of the slope). Note that the classic interpretation does not provide any indication on the accuracy of these values.

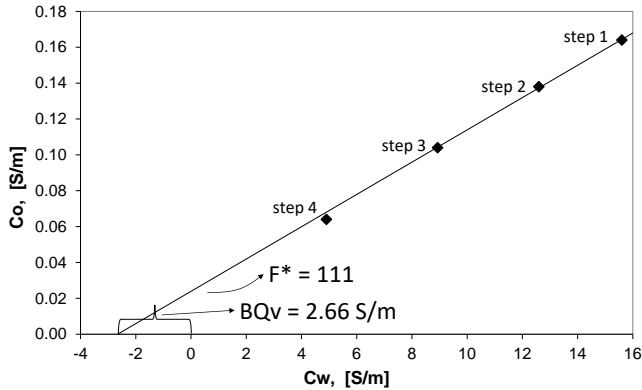


Fig.2 – Classic interpretation of the multivalinity data reported in Tab.1. The F^* and BQ_v parameters of the Waxman-Smits model are obtained from the slope and the intercept of the straight line fitting the C_o - C_w data points.

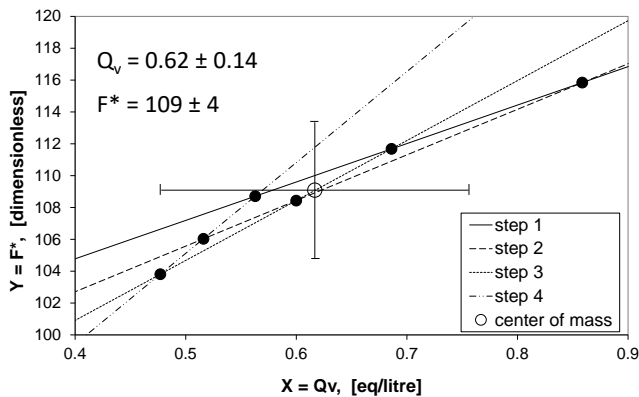


Fig.3 – Interpretation of the multivalinity data reported in Tab.1 using the new method. Each salinity step is assigned a straight line and the F^* and Q_v parameters of the Waxman-Smits model are obtained from the intersection points of these straight lines.

Fig.3 presents the analysis of the same multivalinity data, but using the new method. The six black points define the mutual intersections of the four straight lines that are associated with the four salinity steps made during the experiment. The straight lines are defined by equation (3). The coordinates of the centre of mass of the six points give the most probable values of Q_v and F^* . The standard deviations of the X and Y coordinates of the points give the uncertainty of Q_v and F^* , respectively. In this case we obtain $Q_v = 0.62 \pm 0.14$ eq/l, and $F^* = 109 \pm 4$. If we multiply Q_v by the B value given by equation (2), we find an average BQ_v of 2.39 ± 0.54 S/m. These results are achieved by using all the available data. No subjective, preliminary analysis of the alignment of the data points is needed.

3.2 Indonesia analysis

The data set used for the Indonesia analysis is again that reported in **Tab.1**. We first see the classic interpretation,

which obtains the F parameter from Archie, as $F = C_w/C_0$. As we said, this procedure is conceptually correct only if $V_{cl} = 0$. The sample under examination, however, has a value of V_{cl} of 0.112. Because of this shaliness, the sample deviates from a strict Archie behaviour. The four salinity steps give C_w/C_0 values that increase as the salinity of the saturant brine increases. They are 76.6, 85.9, 91.3 and 95.1, respectively. Note that in an Archie rock the C_w/C_0 ratio should be independent of C_w . Given that Archie rocks are quite rare, similar problems should be expected in most laboratory results. Of the four F values found here, that closest to the true F is probably 95.1. This value was obtained with the most saline brine and therefore it is the least affected by the clay presence. We will consider $F = 95$ as the result of the classic analysis.

An input parameter of the Indonesia equation is the clay conductivity, C_{cl} . One of the classic ways to estimate C_{cl} is to consider the logs and to plot log resistivity R_t versus V_{cl} . **Fig.4** shows such a plot for the well where the sample under test comes from.

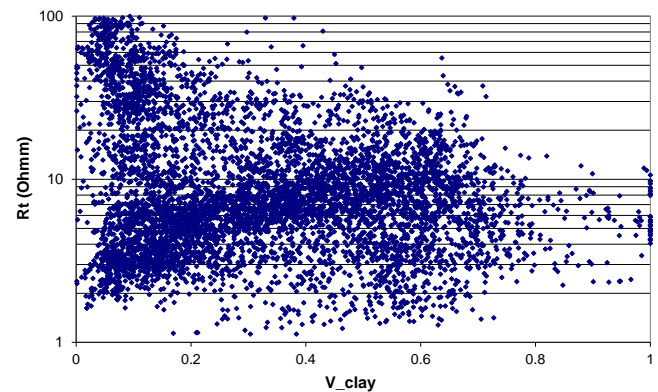


Fig.4 – True resistivity versus clay volume from the GR log in the well where the analysed sample was taken. R_t ranges from 2 to 10 Ohm-m for $V_{cl} \approx 1$. The C_{cl} parameter of the Indonesia equation is usually derived from a plot like this.

As V_{cl} tends to 1, R_t approaches values that lie in the 2-10 Ohm-m range, so C_{cl} is between 0.1 and 0.5 S/m. However, where logs give $V_{cl} = 1$ it is very unlikely that there is pure clay there, and a certain amount of silt must be expected. The C_{cl} value obtained from this procedure, therefore, may be different from the true value.

In order to compare these C_{cl} values with those that we will obtain using the new method, we must transform them to laboratory conditions. This operation is not simple with the Indonesia model, but as a first approximation we will multiply C_{cl} by the B_{lab}/B_{res} ratio provided by equation (2), analogously to what is normally done in the Waxman-Smits analysis. Given that the reservoir brine conductivity is 20.4 S/m, the reservoir temperature is 145°C and the lab temperature is 25°C, the resulting clay conductivity ranges from 0.02 to 0.1 S/m at laboratory conditions.

Another way to estimate C_{cl} at lab conditions is to use Arp's equation [9]. For the above reported temperatures, Arp's equation predicts a ratio between reservoir and lab brine conductivities equal to 3.58, and this would translate into C_{cl} values at lab conditions ranging from 0.03 to 0.14 S/m. Of course, the underlying assumption in this application of Arp's equation is that clay bound water exhibits the same

temperature dependence as NaCl brine, which may not be true. However, the agreement with the C_{cl} values obtained from the B_{lab}/B_{res} approach is good in this case.

We now see how the same data set can be analysed using the new method. The analysis is presented in **Fig.5** and **Fig.6**. **Fig.5** shows the four straight lines defined by equation (5). Each straight line represents one of the salinity steps made in the multisalinity experiment. The straight lines mutually intersect at six points, whose coordinates are $X = C_{cl}^{1/2}$ and $Y = F^{-1/2}$. The points are plotted in **Fig.6** in terms of C_{cl} and F , along with their centre of mass (open circle). The uncertainty bars were obtained from the standard deviations of the X and Y coordinates of the points. In this case, $C_{cl} = 0.24 \pm 0.06$ S/m and $F = 132 \pm 7$.

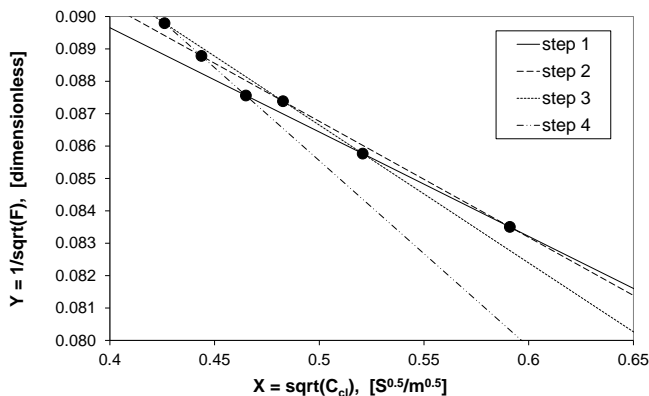


Fig.5 – Interpretation of the multisalinity data reported in Tab.1 using the new method. Each salinity step is associated with a straight line and the intersection points of these lines give the C_{cl} and F parameters of the Indonesia equation.

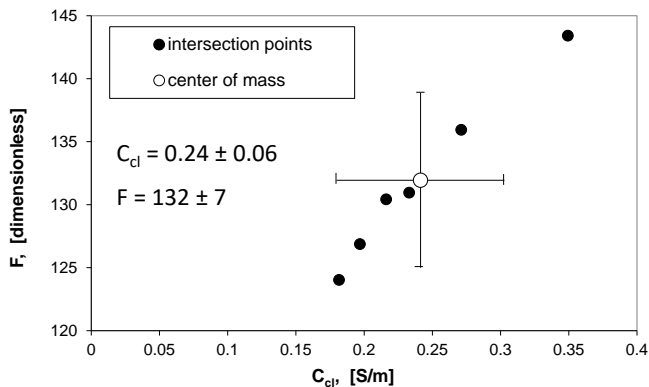


Fig.6 – Values of C_{cl} and F , and associated uncertainty bars, derived from Fig.4 for the sample under examination. C_{cl} and F are the coordinates of the centre of mass of the black points (open circle).

As can be seen, quite different numbers are obtained compared to the classic procedure. We believe that the new method is more correct and provides values of C_{cl} and F that are closer to the truth. The C_{cl} and F values from the new method are higher than those obtained using the classic procedure ($F = 95$ and $C_{cl} < 0.1$ S/m). This tendency is rather general and has a simple explanation. F is underestimated with the classic method because of the extra-conductivity associated with clay. The Indonesia F equals Archie's F only in a clay-free rock. Now, imagine we add a clay component

to this clay-free rock: the Archie F will decrease because the conductivity of the sand+clay system tends to be higher than that of clay-free rock, but the Indonesia F will remain the same. Actually, **Fig.6** shows that a correlation between F and C_{cl} exists, at least for the data set analysed in this paper, and this would indicate that the Indonesia F is not independent of the clay conductivity, similarly to Archie F . However, while the former increases as C_{cl} increases, the latter decreases. If multisalinity test results are available for a certain number of core plugs, one can plot the Indonesia F obtained with the above described procedure against the effective (humidity dried) porosity Φ_{eff} of the analysed plugs: fitting the (Φ_{eff}, F) data points with a curve of the form $F = \Phi_{eff}^{-M}$ will give the “true” Indonesia M for use in the analysis of logs. In general, this will be different from the classic Archie m .

3.3 Summary of results

The analysis described in the previous section gave the results reported in **Tab.2**.

Tab.2 – Values of F^* , BQ_v , F and C_{cl} obtained with the classic method and the new method

	<u>Waxman-Smits</u>		<u>Indonesia</u>	
	F^*	BQ_v [S/m]	F	C_{cl} [S/m]
Classic method	111	2.66	95	< 0.1
New method	109 ± 4	2.39 ± 0.54	132 ± 7	0.24 ± 0.06

3.4 Evaluation of the impact of electrokinetic effects

We now discuss the occurrence of electrokinetic effects and evaluate the impact they may have had on test results. Electrokinetic effects are originated by the motion of the ions contained in the double layer. Double layers form in the vicinity of clay surfaces (which are negatively charged) and occupy the region of the brine saturated pore space where charge density is non-zero due to the attraction of positive ions to the clay surface and the repulsion of negative ions. A double layer consists of a layer of adsorbed, immobile water molecules+positive ions (called the Stern layer), and a diffuse layer where ions (+ the solvation water) are mobile. The limit between the Stern and the diffuse layer is a surface characterized by an electrical potential that is called Zeta-potential. Beyond the diffuse layer, positive and negative ion concentrations are balanced, and so charge density is zero and there is electrical neutrality. The thickness of the diffuse layer is controlled by the Debye length and is of order of 1-10 nanometers. It decreases as brine salinity increases and vanishes for salinities above 30 kppm.

When water flows through a shaly rock and salinity is below 30 kppm, part of the positive ions in the diffuse layers may be mobilized by the flow. An electric current (called streaming current) is consequently generated along the flow

path [10]. This generates an electric potential gradient (the streaming potential) which drives the movement of ions to generate an opposite conduction current. When steady-state conditions are reached, the streaming current is balanced by the conduction current, meaning that the total current is zero, while the streaming potential gradient will maintain a certain non-zero value.

The streaming potential adds to the voltage difference that is applied to the tested core sample in a conductivity measurement, and so it may affect a C_0 measurement performed while injecting brine. The lower the brine salinity, the more critical this effect may be. The streaming potential gradient is proportional to the water pressure gradient that drives the flow. The proportionality factor, C_{SP} , is named "coupling coefficient". According to [11], C_{SP} can be expressed as:

$$C_{SP} = -\frac{k\hat{Q}_v}{\mu C_0} \quad (6)$$

where k is the rock permeability, μ is the brine viscosity, \hat{Q}_v is the effective excess charge density dragged by the pore water (this is lower than Q_v) and C_0 is the rock conductivity. We now want to find the voltage difference ΔV_s across a flowed core sample, which arises from the streaming potential. To obtain it, we consider the relation $\Delta V_s = C_{SP} \cdot \Delta p$ and replace quantity k/μ in (6) by quantity $qL/(A \cdot \Delta p)$, where L is the sample length, A is the cross-sectional area, q is the water injection rate and Δp is the water pressure difference across the sample. Thus:

$$\Delta V_s = \frac{L \hat{Q}_v}{A C_0} q \quad (7)$$

The effective excess charge density \hat{Q}_v must be estimated from empirical relationships. We utilize a relationship found in [12], which is simple because it contains only rock permeability and is based on a large set of published data for various lithologies:

$$\log(\hat{Q}_v) = -9.23 - 0.82 \log(k) \quad (8)$$

The sample analysed in this study has the following values of the quantities involved in eq (7) and eq (8):

- $L = 5 \times 10^{-2}$ m
- $A = 11.3 \times 10^{-4}$ m²
- $k = 2.7 \times 10^{-16}$ m²
- $\hat{Q}_v = 3.4 \times 10^3$ C/m³
- $C_0 = 6.4 \times 10^{-2}$ S/m (value obtained at step 4)
- $q = 1.6 \times 10^{-9}$ m³/s

Substituting the above numbers into (7) and (8) gives a streaming voltage difference ΔV_s of 3.8 mV. In this calculation we used the conductivity measured during step 4, which is the most critical, as we said. The other steps should not have been affected by electrokinetic effects because brine salinities were above the critical value of 30 kppm (above 30 kppm, the diffuse layer collapses). Considering that our conductivity measurements were performed by applying a voltage amplitude of 0.5 V, that is 2 orders of magnitude

greater than ΔV_s , we argue that the streaming potential played a negligible role and did not affect the results.

Below 30 kppm, the streaming potential increases because the thickness of the diffuse layer increases and rock conductivity decreases, meaning that the \hat{Q}_v/C_0 ratio in eq (7) increases. A question therefore arises, namely down to what brine salinity it is still possible to perform accurate C_0 measurements in a multialinity test, during brine injection. Answering this question is not trivial, because the calculation of ΔV_s requires knowledge of \hat{Q}_v and formula (8), which gives \hat{Q}_v only as a function of permeability but not of salinity, is not accurate enough for this purpose. Although there are more complex formulas which give \hat{Q}_v as a function of salinity (see for example [13]), they require knowledge of Zeta-potential, which is usually not known. So, the difficulties in determining ΔV_s remain.

In trying to understand the effect of electrokinetic phenomena on a multialinity test, we might say that since brine is injected at a constant rate and ΔV_s is proportional to the injection rate, ΔV_s must stabilize once brine salinity is stabilized; we might therefore think that the presence of a constant ΔV_s which adds to the applied ΔV should not affect the detection of equilibrium. However, despite all these considerations, it is advisable that ΔV_s stays below the voltage amplitude used in the C_0 measurements. The salinity corresponding to the ΔV_s value that exceeds the applied voltage amplitude represents the lowest salinity that can be used in a multialinity test.

4 Conclusions

By making use of the same multialinity data set, we derive the F^* and Q_v Waxman-Smits parameters and the F and C_{cl} Indonesia parameters by using both the classic interpretation procedure and the new method described in this paper. As shown in **Tab.2**, certain differences are seen. As far as the Waxman-Smits parameters are concerned, the differences are essentially due to the fact that the classic procedure does not allow us to use all the available data. The classic method requires a preliminary control of the linearity of the C_0 - C_w curve, and the data points that do not lie in the linear region cannot be used. That causes a loss of information and introduces a degree of subjectiveness in the data analysis process.

The extraction of the Indonesia parameters is more critical. **Tab.2** shows that the differences between the classic and new method results are bigger. We think that they are due to conceptual errors inherent in the classic method, which lead to underestimating both F and C_{cl} . The new method seems more consistent and reliable. Moreover, it provides not only the average value of F and C_{cl} but also their uncertainties.

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