Quality Control for the Determination of the Local Saturation in Centrifuge Experiments

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Abstract. In centrifuge experiments, the calculation of the local (entrance) saturation from the effluent production has been long studied, from Hassler and Brunner in 1945 to the Forbes calculation in 1997. The SCA survey in 1997 provides the widely accepted conclusion that the best results are obtained using either the Forbes or the splines methods.

The centrifuge experiment is a key measurement in core analysis. But analyzing results provided by several service companies, the authors often discover that the calculations of the local saturations could be improved. The error is due to the limited number of points used for the Forbes calculation.

Calculation of the local saturation from the average saturation is an ill-posed problem with no unique exact solution. But the reverse calculation of the average saturation from the local value can be easily performed by numerical integration. In this paper, we propose a method where this recalculated average saturation should be compared to the actual measurement for quality control of interpretation.

This paper will provide several examples of interpretations with the standard Forbes method compared to the continuous Forbes (CF) method. This CF method uses a fit of the experimental production that allows a large number of points for the Forbes calculation, leading to a reduced error. Comparison between the experimental and the recalculated numerical average saturations, used as quality control, shows that the continuous Forbes method provides a better agreement.

1 Introduction

During a centrifuge experiment, a sample saturated with one fluid is immersed in a bucket containing another immiscible fluid. The bucket is run at a given rotation speed and the expelled fluid is recovered and measured.

Figure **1** shows a typical saturation profile at end of a step during a multistep centrifuge experiment. The measured production gives the average saturation (dashed red line) around 0.7 and the interpretation leading to the Pc curve requires the entrance (or local) saturation around 0.4.

Schematic and notations that will be used in this paper are described in Figure 2. The case corresponds to the displacement of a denser fluid by a less dense one (for instance water displaced by oil in a drainage). The indexes 1 and 2 correspond respectively to the entrance and outlet. Variables without index correspond to current position along the sample: r is the distance from the vertical axis of the centrifuge, S the saturation of the displaced fluid and Pc the capillary pressure.

The capillary pressure curve is determined at the entrance (distance r_1 from the axis). At the entrance, the pressure induced by the rotation and the difference of density of the two fluids is given by:

$$P_c = (\Delta\rho)\frac{\omega^2}{2}(r_2^2 - r_1^2) \qquad (1)$$

Where Δρ is the density difference between the two fluids and ω the angular velocity of the centrifuge.

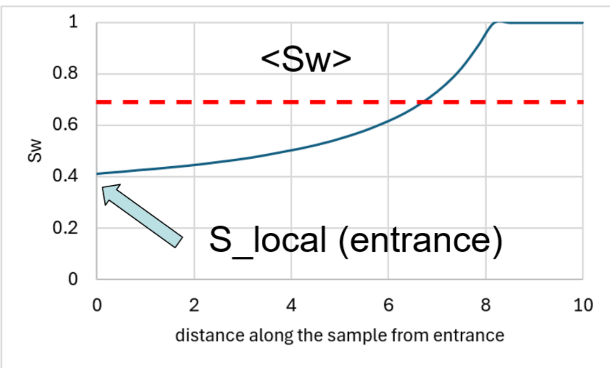

Figure 1. Typical saturation profile along the sample during a primary drainage. The measured production gives the average saturation (dashed red line) around 0.7 and the interpretation leading to the Pc curve requires the entrance (or local) saturation around 0.4.

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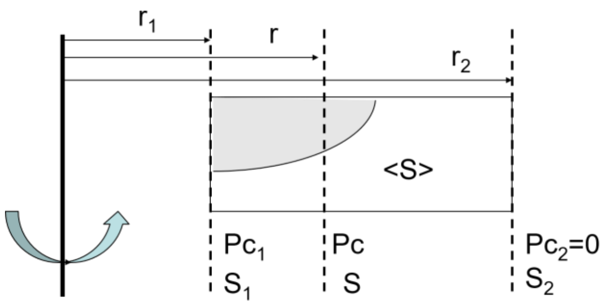


Figure 2. The centrifuge geometry (during a drainage)

The corresponding saturation at entrance cannot be measured directly. Only the average saturation can be derived from the effluent production.

2 State of the art

The calculation of the local (entrance) saturation from the average has been long studied.

Several decades ago, each core laboratory was using a different model without any guarantee for the accuracy. In 1993, the SCA launched a first survey to compare the various models based on twin samples sent to various laboratories[1]. As shown in Figure 3 and Figure 4 there were a significant spread in the results. However, concerns remained regarding samples homogeneity, and the observed differences may have been due to variations in the samples' properties.

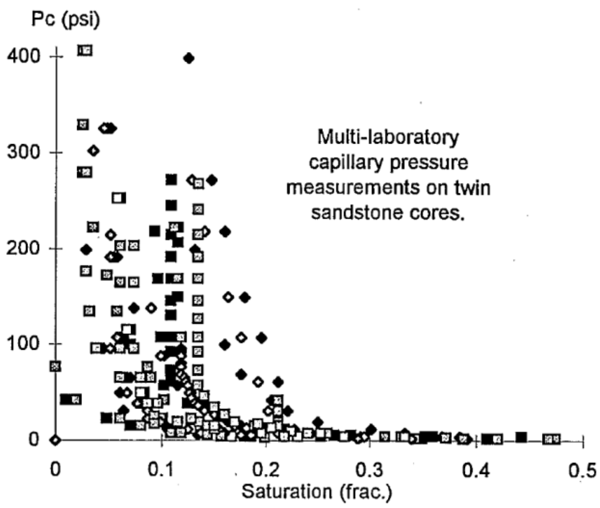


Figure 3. Results for the calculation of the Pc curves on twin sandstone plugs (from the 1993 SCA survey).

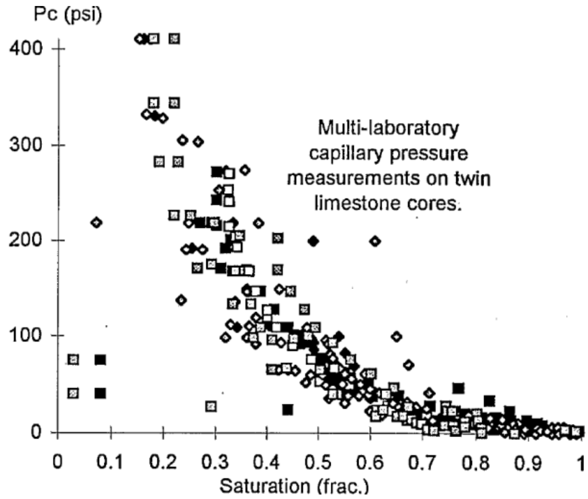


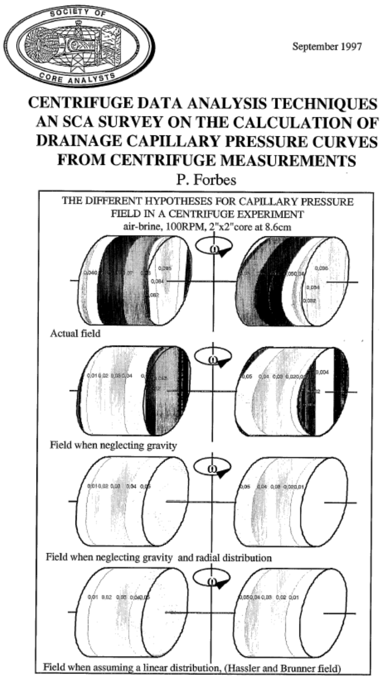
Figure 4. Results for the calculation of the Pc curves on twin limestone plugs (from the 1993 SCA survey)

A second survey was launched in 1997 [2]. This survey is still a reference document for the SCA community (Figure 5). To avoid the problem of heterogeneity, the data were synthetic with both the local and average saturations were exactly known analytically. Forbes performed the interpretation using most of the models published in the literature:

- Hassler-Brunner
- Hassler-Brunner second solution,
- Domselaar
- Hoffman
- Rajan
- Forbes 1st solution $S\alpha$
- Chen and Ruth
- Clinch
- Forbes 1st solution $S\alpha\beta$
- Ruth and Wong 1st solution
- Ruth and Wong 2nd solution
- Hermansen
- Bentsen
- Spline Nordtvedt
- Skuse
- Direct solution King.

There were 10 cases with different geometries and shapes of the Pc curves. Figure 6 gives the result for a simple shape for Pc (case 2). Symbols correspond to the various interpretations and the solid line to the "exact" analytical solution.

The spreading of the results was similar to the first survey demonstrating that the problem was not due to the heterogeneity of the samples but the lack of accuracy of the models used for the interpretation of the raw data.


September 1997

CENTRIFUGE DATA ANALYSIS TECHNIQUES : AN SCA SURVEY ON THE CALCULATION OF DRAINAGE CAPILLARY PRESSURE CURVES FROM CENTRIFUGE MEASUREMENTS

P. Forbes

Figure 5. Front page of the 1997 SCA survey.

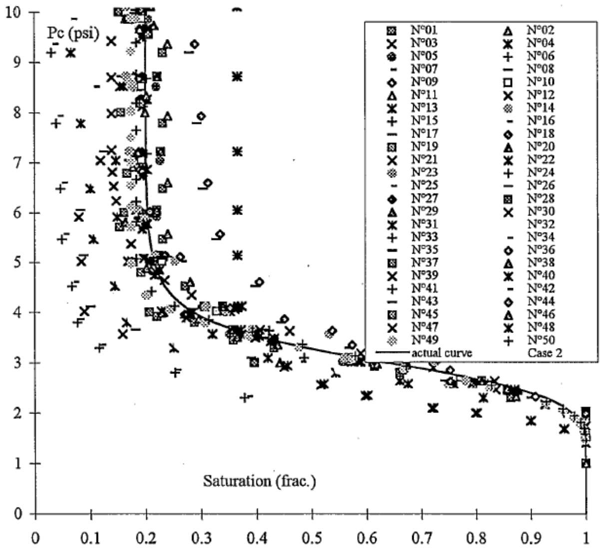


Figure 6. Capillary pressure determination for case 2 (after 1997 SCA survey [2].

The conclusion of this second survey was clear: only two models were accurate for all the 10 cases:

- The Forbes model proposed by Forbes in 1991 [3].
- The splines model developed by Nortvedt *et al*. [4] in 1991.

These conclusions were accepted by the profession. Forbes provided an Excel template for his calculation that is still used by many companies. After this benchmark, Forbes proposed some improvements to his model [5], to include gravity effect at low speed (the sample is not perfectly horizontal), and radial effect (all the points on the entry face are not at the same distance of the centrifuge axis). But the corrections have little effects and are generally not considered. Doug Ruth commercialized a software based on the improvement of the Forbes method [6] and Cydarex commercialized the software CYDAR using an improved version of Forbes as a first guess, followed by a splines optimization to improve accuracy. We are not aware of other models used today in the SCAL community to convert the average saturation to local saturations. Another approach is used in several core laboratories: the determination of both the relative permeabilities and the Pc curve by history matching the production data with a numerical simulator like Sendra. This approach is not concerned with the Forbes calculation and will not be discussed in this paper.

However, there is still a problem since we observed that the two main models used in the companies, Forbes and CYDAR, sometimes lead to different results when used to interpret the same experiment. We have not been able to evaluate Doug Ruth's model.

Figure 7 shows an example of interpretations of the same experimental data for an imbibition provided by a service company using Forbes method compared to CYDAR: symbols and LET fit in red for Forbes and curve in green for CYDAR. The experimental points are the symbols in blue with a fit with an LET function (blue line). Obviously, there is a difference between the two interpretations. Which one is more accurate?

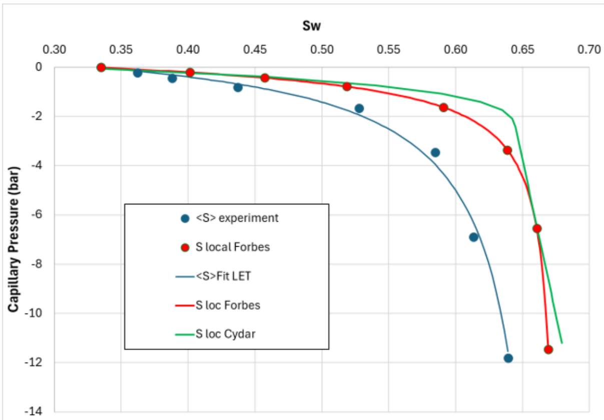


Figure 7. Comparison of local saturation calculated using the Forbes method: symbols in red and Cydar curve in green. The experimental points are the symbols in blue. The experimental and Forbes calculated points are fitted with LET function (blue and red curve).

In this paper we will first show how we can easily discriminate between these two results by numerically recalculating the average saturation from the local one. We will then explain the origin of this difference by describing the models.

3 Test for Quality Control of the local saturation

Calculation of the local saturation from the average saturation is an ill-posed problem with no unique exact solution. But the reverse calculation of the average saturation from the local value can be easily performed with a very good accuracy by a standard numerical integration. We propose here that this recalculated average saturation should always be compared to the actual measurement for quality control.

Assuming uniform porosity, the average saturation <S> is easily derived from the local saturation S, by integration along the sample (variable r, distance from the vertical axis of the centrifuge).

$$< S >= \frac{1}{L}\int_{r2}^{r1} S(r)dr \qquad (2)$$

Where L is the length of the sample ($L=r_1-r_2$).

Since S(r) is not explicitly known, the variable r is written as function of Pc using the standard expression:

$$P_c(r) = (\Delta\rho)\frac{\omega^2}{2}(r_2^2 - r^2) \qquad (3)$$

This equation satisfies the boundary conditions Pc=0 at outlet ($r = r_2$).

We introduce the geometrical parameter b:

$$b = 1 - \frac{r_1}{r_2} \qquad (4)$$

Now, with Pc as variable of integration, and since $Pc_2=0$:

$$< S >= (2-b)\frac{1}{2Pc_1}\int_0^{Pc_1} \frac{S(Pc)}{\sqrt{1-\frac{Pc}{Pc_1}b}}dPc \qquad (5)$$

A numerical integration using the standard trapezoidal method with 1000 points leads to the result shown on Figure 8. The recalculated <S> using the CYDAR model (green dashed line) is in better agreement with the experimental data (blue symbols) than the Forbes model (red dashed line).

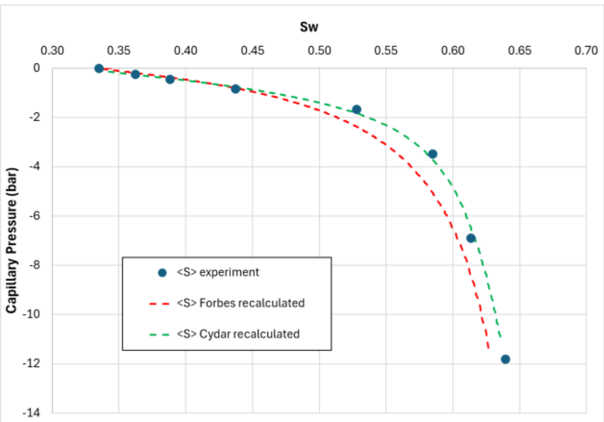


Figure 8: Re-calculated average saturation for Forbes (dashed red) and Cydar (dashed green) interpretations. The CYDAR model is in better agreement with the experimental data.

Now we will explain this difference by describing the various models.

4 Actual models for calculating the local saturation

We will describe the historical Hassler-Brunner model and the models that are currently used in industry. Other previous models were described by Doug and Chen [7].

In all this section, the examples of interpretation are based on a real experiment.

4.1 Hassler-Brunner [8]

This model is based on the approximation that the sample is short compared to the centrifuge arm, leading to r_1 close to r_2 and consequently the parameter b close to 0.

Equation (5) reduces to:

$$< S >= \frac{1}{Pc_1}\int_0^{Pc_1} S(Pc)dPc \qquad (6)$$

Now taking the derivative with respect to Pc_1:

$$\frac{\partial < S >}{\partial Pc_1} = -\frac{1}{{Pc_1}^2}\int_0^{Pc_1} S(Pc)dPc + \frac{1}{Pc_1}S_1(Pc_1) \qquad (7)$$

Rearranging this equation:

$$S_1(Pc_1) =< S > +Pc_1\frac{\partial < S >}{\partial Pc_1} \qquad (8)$$

Since Pc_1 is proportional to the centrifuge velocity n (row per minute), the equation simplifies into:

$$S_1(Pc_1) =< S > +\frac{\partial < S >}{\partial Log(n)} \qquad (9)$$

The local saturation is then derived from the derivative of the plot of <S> as function of Log(n).

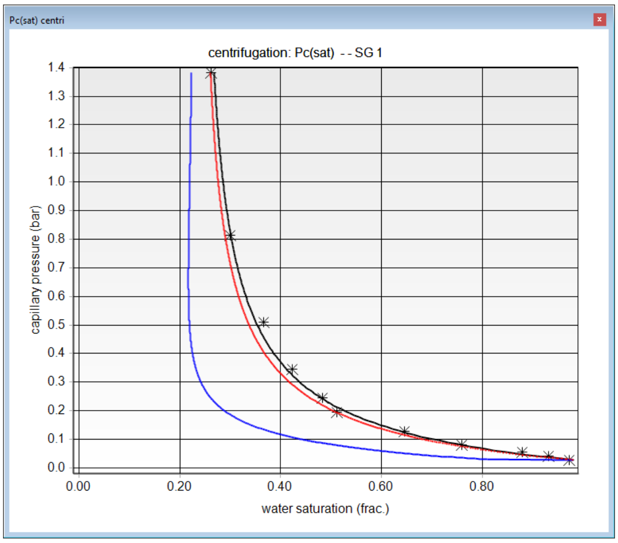


Figure 9. Example of Hassler-Brunner calculation using smoothing with a hyperbolic function (black line) of the experimental data (symbols). Local saturation in blue line and the recalculated <Sw> in red line.

Figure 9 is an example of Hassler-Brunner calculation using smoothing of the experimental data (symbols) with a hyperbolic function (black curve). The recalculated saturation (red line) is not perfectly superimposed to the fit (black line). The difference is due to the error introduced by the assumption b=0.

4.2 First kind Forbes model [2]

This model proposed by Forbes is based on two assumptions:

First, in equation (5), the denominator under the integral is a square root (power ½) and in the Hassler-Brunner equation (6), the denominator is 1 (power 0). Forbes assumed that in a general case, the power can take an adjustable value ν.

The second point is to fit the experimental <S> points with a polynomial. The polynomial has the advantage to allow an analytical calculation of the integral.

According to the article [2], there were drawbacks in the model due to the constrain to fit the experimental data with a polynomial: "*its drawback is linked to this smoothing, whose suitability can not be checked, exactly as in other methods using smoothing, fitting or averaging*".

Then Forbes decided to develop a method based only on the points, without smoothing.

It is interesting to note that the possibility of an analytical calculation of the integral with a fit by a polynomial is at the base of the spline method (see below § 4.4).

4.3 Second kind Forbes model [2]

The solution called $S_{\alpha\beta}$ is based on two solutions S_α and S_β that are combined with a weight function of the geometrical factor b, equation (4). For drainage, the equations are the following, where S_{HB} is the Hassler-Brunner solution, equation (7):

$$S_\alpha(P_c) = <S>(Pc) + \frac{P_c}{1+\alpha}\frac{d<S>(Pc)}{dPc} \qquad (10)$$

$$\alpha = \frac{1-\sqrt{1-b}}{1+2\sqrt{1-b}} \qquad (11)$$

$$S_\beta(P_c) = (1+\beta)\int_0^1 x^\beta\, S_{\mathrm{HB}}(xP_c)\,dx;\ \beta = \frac{2}{\alpha} \qquad (12)$$

$$S(P_c) \approx S_{\alpha\beta} = \frac{B}{2}S_\beta + \left[1-\frac{B}{2}\right]S_\alpha; \qquad (13)$$

From Forbes, the main advantage of this set of equations is that the calculation can be performed directly from the experimental points without the need of smoothing [2]: "*The solutions contain integration and differentiation, as do numerous solutions proposed. But this solution can be evaluated with high accuracy from discrete <S> data, using a simple differencing scheme*".

Forbes provided an Excel template for this calculation (Figure 10) still in use in many companies. We will see that the use of the differencing scheme is not accurate.

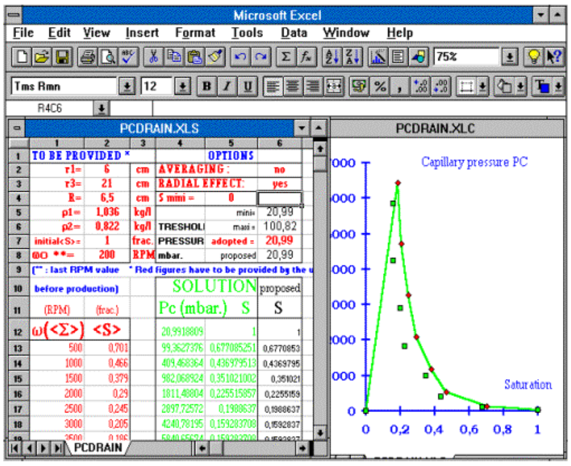


Figure 10: Excel sheet provided by Forbes for the calculation of the local saturation using the second kind solution.

4.4 Splines model

Splines (meaning Souple Lines) is a very powerful tool for smoothing curves with complicated shapes. It consists in polynomial functions of a given order (generally 3) applied on several adjacent intervals. At each boundary of the intervals (called knots), the parameters of the polynomial are automatically adjusted for continuity of the function and continuity of all the derivatives in order to give very smooth curves.

As already mentioned by Forbes, a polynomial function for the local saturation allows analytical integration of the basic equation (5). This property is also used in the spline model developed by Nordtvedt [4].

4.5 CYDAR model

The society Cydarex developed the software CYDAR based on Forbes results with two main improvements: "continuous Forbes" model and physical constrains imposed with a Spline fit of the local Pc curve, as in the Nordtvedt model.

Since the discretization on a small number of points can introduce errors (see discussion below), the experimental points are fitted by an analytical curve, and the fit is discretized in a large number of points (typical 1000). Then the Forbes method is applied to all the points. This procedure is called the "continuous Forbes model".

Depending on the type of function used for fitting, the local Pc curve may not be regular. Figure 11 shows the result with a fit using an hyperbolic function. The curve is regular. The recalculated <S> agrees perfectly with the fit. Figure 12 shows the same example with Hermite interpolation of the experimental data. The interpolation is very close to the hyperbolic fit, but the Pc with local saturation is wavy. This is normal and the consequence of the centrifuge equations, already observed and discussed in Forbes publication. In his Excel spreadsheet, Forbes proposed a method to move the experimental points to remove these oscillations. However, even with oscillations, the recalculated average saturation is also superimposed to the fitted original data. This solution is mathematically accurate, but not physical.

These two examples show that the solution of the centrifuge equation is not unique. From all the possible local curve, we must select those which are in agreement with physical behavior like continuous decreasing saturation when Pc is increasing for a drainage. In CYDAR, this constraint is imposed using a spline fit of the calculated local Pc and optimization on the <S> curve. The result is shown in Figure 13. The recalculated curve (red line) is not exactly superimposed to the fit (black curve) but is still in good agreement with the experimental points.

These examples show how the local Pc curve, and especially the final saturation depends on the function used for fitting. With the hyperbolic fit, the final local saturation is 0.225 and with the Hermite interpolation, it is 0.198. This is the range of errors linked to the centrifuge method. This error could be reduced by using a larger number of experimental points at high pressure.

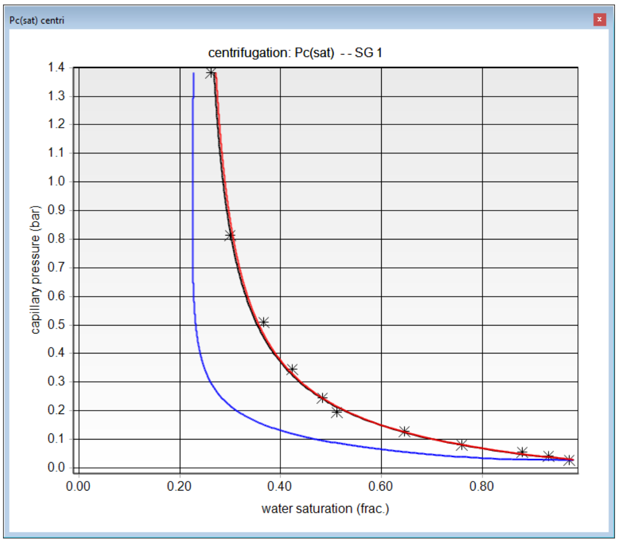


Figure 11. Example of Forbes continuous calculation using smoothing with a hyperbolic function (black curve) of the experimental data (symbols). Local saturation in blue line and the recalculated in red line superimposed to the fit.

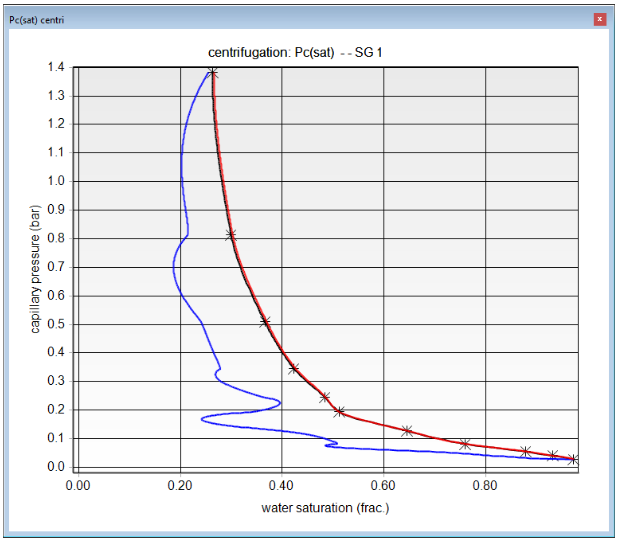


Figure 12. Example of Forbes continuous calculation using smoothing of the experimental data (symbols) with a Hermite interpolation (black curve). Local saturation in blue wavy line and the recalculated in red line superimposed to the fit.

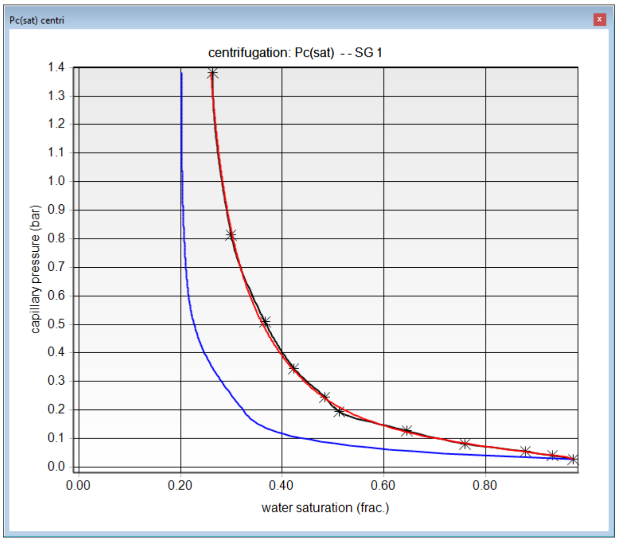


Figure 13. Example of Forbes continuous calculation using Hermite interpolation (black curve) followed by a spline fit of the Pc curve. The Pc curve with local saturation in blue line and the Pc curve with the recalculated <Sw> in red line.

5 Discussion

We have briefly recalled the principles of the Forbes and Cydar models. We will now explain why the Forbes model is less accurate than the CYDAR model when tested with recalculation of the average saturation (recalled in Figure 14).

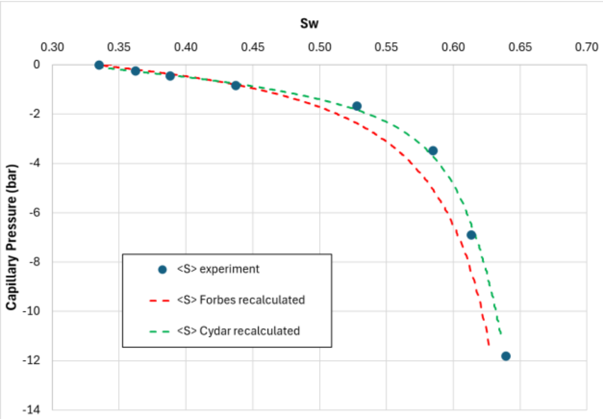


Figure 14. Re-calculated average saturation for Forbes (dashed red) and Cydar (dashed green) interpretations. The CYDAR model is more in agreement with the experiment.

The problem of Forbes model is due to the calculation based on a discrete differentiation using a small number of points. In his papers, Forbes was always convinced that smoothing introduces errors and that the experimental points are the exact results. In Cydar, and most of other approaches like Splines, it is recognized that the purpose is to provide a smooth curve with physical constrains like absence of "waves" (monotone saturation as function of pressure). Experimental points are never perfect, subject to local heterogeneity or, most of the time, non-stabilized production. This is why it is necessary to smooth them.

Forbes had studied the effect of the discretization. In Figure 15, he compared the results of his model with the analytical case (solid line) for different numbers of experimental points. His conclusion was: "*it appears that the method is not very sensitive to the number of points provided a lower limit of about 5 is reached*". However, from the figure, we can also conclude that the result with 7 points is not in perfect fit with the theoretical curve.

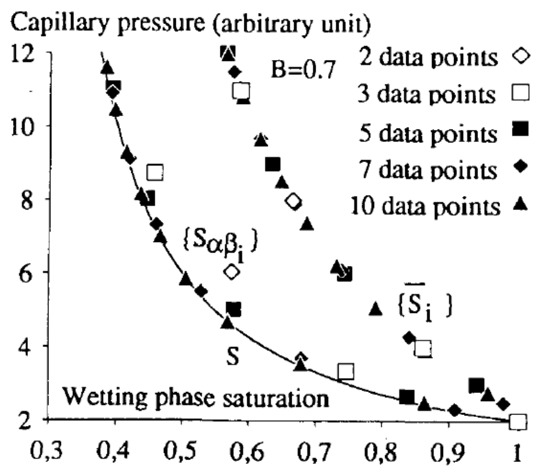


Figure 15. Study of the effect of the number of points (from Forbes 1991).

The limited number of points introduces an error, like any finite difference method compared to analytical calculation. Forbes gave a limit of about 5 points [5]. When we use Forbes Excel template, we are confident since the calculation of the average Pc (<S>) points gives back the experimental ones. But this is misleading since the integration is performed using the same discrete method that compensate the error introduced during the differentiation.

The error introduced by the limited number of points was already analyzed by Bauget *et al.* [9] using the analytical case presented by Chen and Ruth [10] where both local and average Pc are known analytically For the local Pc in psi:

$$S(P_c) = \begin{cases} \frac{1.5}{P_c} + 0.25 & \text{if } P_c \geq 2 \\ 1 & \text{if } P_c \leq 2 \end{cases} \tag{14}$$

Figure 16 shows the local P_c and average $P_c(<S>)$ curves obtained using both methods and the theoretical curves. The continuous approach implemented in the software CYDAR gives good results when the discrete solution gives a local P_c slightly shifted. The error with the discrete solution does not seem significant but if we look at the production curve using the three local P_c curves (Figure 17) it appears that the discrete method is away from the exact solution. This difference has a large impact on the determination of the relative permeability, since the plateaus at each step cannot be fitted.

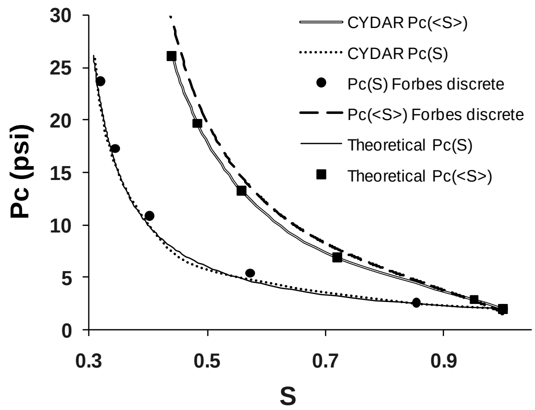


Figure 16: Analytical case with 6 points, comparison with Forbes SCA 1991 and CYDAR optimization tool (after Bauget et al. [9]).

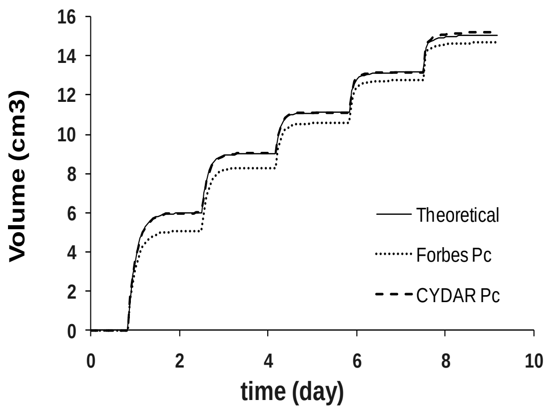


Figure 17. Production simulation using a theoretical local Pc, the Forbes' discrete solution and the continuous approach (after Bauget et al.[9]).

6 Conclusions

We have demonstrated that using the discrete Forbes calculation of the local saturation from the average saturation is not accurate.

Accurate calculation can be performed using the continuous Forbes model, or the Spline model like in CYDAR. Without a specific software, it is always possible to use the standard Forbes method and his Excel template but with an increased number of points. The experimental points should be fitted with a smooth function (LET for instance). Then several points are chosen on this curve, at least 10, and used as experimental points in Forbes excel spreadsheet.

For quality control, we cannot rely on the backward method proposed by Forbes, since the discrete integration compensate the error introduced by the discrete differentiation. The only valid test is to perform numerically the integration leading to the average saturation, equation (5), recalled below:

$$< S >= (2-b)\frac{1}{2Pc_1}\int_0^{Pc_1}\frac{S(Pc)}{\sqrt{1-\frac{Pc}{Pc_1}b}}dPc \qquad (15)$$

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